

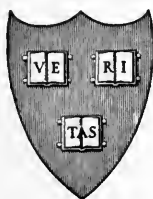
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TABLES  
OF  
COMPLEX HYPERBOLIC AND  
CIRCULAR FUNCTIONS

BY  
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*SECOND EDITION*  
*REVISED AND ENLARGED*



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## PREFACE

THE tables in this book present hyperbolic and circular functions of a complex variable, both in polar and rectangular coördinates. Such complex functions have not hitherto been published, except over a very restricted range. They have important applications in electrical engineering. For instance, it is possible with their help to find in a few minutes the potential, current and power, at any point of an alternating-current line-conductor of known constants and terminal conditions; whereas the same problem, to a like degree of precision, without aid from these functions, and by older methods, would probably occupy hours of labor and cover several sheets of computing-paper.

Although the principal application of these functions at the present time is in dealing with alternating-current lines, especially those of either great length or high frequency; yet it seems likely that other uses will develop for them.

The author desires to acknowledge his indebtedness, for suggestions and help, to a number of workers, both in mathematical and practical fields; and particularly to Messrs. C. L. Bouton, W. Duddell, E. V. Huntington, F. B. Jewett, John Perry, H. J. Ryan, and E. B. Wilson.

A. E. K.

HARVARD UNIVERSITY  
January, 1914.

## PREFACE TO THE SECOND EDITION

IN preparing the second edition of this book, six new tables have been computed. These are actually extensions of the tables I to VI already incorporated. It has been considered advisable to add the new material in new tables at the end of the volume rather than to recast the original tables in such a manner as to include the new matter. The new matter has been found necessary in certain departments of electrical engineering to which complex hyperbolic functions may be advantageously applied.

A. E. K.

HARVARD UNIVERSITY  
June, 1920.

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## TABLE OF CONTENTS

|                                                                                                              | PAGE |
|--------------------------------------------------------------------------------------------------------------|------|
| I. HYPERBOLIC SINES. $\sinh(\rho/\delta) = r/\gamma$ . . . $\delta$ $45^\circ$ to $90^\circ$ . . . . .       | 2    |
| II. HYPERBOLIC COSINES. $\cosh(\rho/\delta) = r/\gamma$ . . . . .                                            | 8    |
| III. HYPERBOLIC TANGENTS. $\tanh(\rho/\delta) = r/\gamma$ . . . . .                                          | 14   |
| IV. CORRECTING FACTOR. $\frac{\sinh \theta}{\theta}$ . . . . .                                               | 20   |
| V. CORRECTING FACTOR. $\frac{\tanh \theta}{\theta}$ . . . . .                                                | 26   |
| VI. FUNCTIONS OF SEMI-IMAGINARIES. $f(\rho/45^\circ)$ . . . . .                                              | 32   |
| VII. HYPERBOLIC SINES. $\sinh(x + iq) = u + iv$ . . . . .                                                    | 42   |
| VIII. HYPERBOLIC COSINES. $\cosh(x + iq) = u + iv$ . . . . .                                                 | 58   |
| IX. HYPERBOLIC TANGENTS. $\tanh(x + iq) = u + iv$ . . . . .                                                  | 74   |
| X. HYPERBOLIC SINES. $\sinh(x + iq) = r/\gamma$ . . . . .                                                    | 90   |
| XI. HYPERBOLIC COSINES. $\cosh(x + iq) = r/\gamma$ . . . . .                                                 | 106  |
| XII. HYPERBOLIC TANGENTS. $\tanh(x + iq) = r/\gamma$ . . . . .                                               | 122  |
| XIII. FUNCTIONS OF $4 + iq$ . $f(4 + iq) = u + iv$ . . . . .                                                 | 138  |
| $f(4 + iq) = r/\gamma$ . . . . .                                                                             | 139  |
| XIV. SEMI-EXPONENTIALS. $\frac{e^x}{2}$ and $\log_{10} \left( \frac{e^x}{2} \right)$ . . . . .               | 140  |
| XV. REAL HYPERBOLIC FUNCTIONS. $f(x + i0) = u + i0$ . . . . .                                                | 144  |
| XVI. SUBDIVISIONS OF A DEGREE . . . . .                                                                      | 150  |
| EXPLANATORY TEXT . . . . .                                                                                   | 151  |
| XVII. HYPERBOLIC SINES. $\sinh(\rho/\delta) = r/\gamma$ . . . . . $\delta$ $0^\circ$ to $45^\circ$ . . . . . | 214  |
| XVIII. HYPERBOLIC COSINES. $\cosh(\rho/\delta) = r/\gamma$ . . . . .                                         | 216  |
| XIX. HYPERBOLIC TANGENTS. $\tanh(\rho/\delta) = r/\gamma$ . . . . .                                          | 218  |
| XX. CORRECTING FACTOR. $\frac{\sinh \theta}{\theta}$ . . . . .                                               | 220  |
| XXI. CORRECTING FACTOR. $\frac{\tanh \theta}{\theta}$ . . . . .                                              | 222  |
| XXII. FUNCTIONS OF SEMI-IMAGINARIES. $f(\rho/45^\circ)$ . . . . .                                            | 224  |
| XXIII. HYPERBOLIC FUNCTION FORMULAS . . . . .                                                                | 226  |

TABLES OF COMPLEX HYPERBOLIC  
AND CIRCULAR FUNCTIONS

TABLE I. HYPERBOLIC SINES.  $\sinh(\rho/\delta) = r/\gamma$ 

|    | 0.1      |        | 0.2     |        | 0.3     |        | 0.4     |        | 0.5     |        |
|----|----------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| °  | °        |        | °       |        | °       |        | °       |        | °       |        |
| 45 | 0.10000  | 45.096 | 0.20000 | 45.383 | 0.30001 | 45.860 | 0.40005 | 46.532 | 0.50016 | 47.391 |
| 46 | 0.099993 | 46.095 | 0.19995 | 46.382 | 0.29985 | 46.858 | 0.39968 | 47.529 | 0.49944 | 48.388 |
| 47 | 0.099987 | 47.095 | 0.19990 | 47.381 | 0.29969 | 47.856 | 0.39931 | 48.526 | 0.49872 | 49.385 |
| 48 | 0.099981 | 48.094 | 0.19986 | 48.380 | 0.29954 | 48.854 | 0.39893 | 49.520 | 0.49799 | 50.378 |
| 49 | 0.099975 | 49.094 | 0.19982 | 49.378 | 0.29939 | 49.852 | 0.39856 | 50.513 | 0.49727 | 51.368 |
| 50 | 0.099970 | 50.094 | 0.19976 | 50.376 | 0.29923 | 50.848 | 0.39820 | 51.506 | 0.49656 | 52.357 |
| 51 | 0.099965 | 51.093 | 0.19972 | 51.374 | 0.29907 | 51.842 | 0.39784 | 52.497 | 0.49585 | 53.342 |
| 52 | 0.099960 | 52.092 | 0.19968 | 52.371 | 0.29892 | 52.834 | 0.39748 | 53.486 | 0.49514 | 54.325 |
| 53 | 0.099955 | 53.091 | 0.19963 | 53.367 | 0.29877 | 53.826 | 0.39712 | 54.472 | 0.49444 | 55.304 |
| 54 | 0.099950 | 54.090 | 0.19959 | 54.362 | 0.29862 | 54.818 | 0.39676 | 55.458 | 0.49374 | 56.281 |
| 55 | 0.099944 | 55.089 | 0.19955 | 55.357 | 0.29847 | 55.809 | 0.39641 | 56.440 | 0.49305 | 57.254 |
| 56 | 0.099939 | 56.088 | 0.19951 | 56.352 | 0.29833 | 56.799 | 0.39607 | 57.421 | 0.49238 | 58.226 |
| 57 | 0.099933 | 57.086 | 0.19946 | 57.347 | 0.29819 | 57.787 | 0.39572 | 58.400 | 0.49172 | 59.195 |
| 58 | 0.099928 | 58.085 | 0.19941 | 58.342 | 0.29804 | 58.773 | 0.39538 | 59.378 | 0.49106 | 60.160 |
| 59 | 0.099922 | 59.083 | 0.19937 | 59.336 | 0.29790 | 59.760 | 0.39505 | 60.354 | 0.49041 | 61.123 |
| 60 | 0.099917 | 60.082 | 0.19934 | 60.331 | 0.29777 | 60.746 | 0.39473 | 61.330 | 0.48978 | 62.085 |
| 61 | 0.099912 | 61.081 | 0.19929 | 61.324 | 0.29764 | 61.731 | 0.39441 | 62.302 | 0.48916 | 63.042 |
| 62 | 0.099907 | 62.079 | 0.19925 | 62.317 | 0.29751 | 62.715 | 0.39409 | 63.273 | 0.48855 | 63.998 |
| 63 | 0.099902 | 63.077 | 0.19921 | 63.309 | 0.29738 | 63.698 | 0.39379 | 64.243 | 0.48795 | 64.950 |
| 64 | 0.099897 | 64.075 | 0.19918 | 64.301 | 0.29725 | 64.680 | 0.39349 | 65.212 | 0.48738 | 65.900 |
| 65 | 0.099893 | 65.073 | 0.19914 | 65.293 | 0.29712 | 65.661 | 0.39320 | 66.179 | 0.48681 | 66.848 |
| 66 | 0.099889 | 66.071 | 0.19911 | 66.284 | 0.29700 | 66.641 | 0.39293 | 67.144 | 0.48627 | 67.794 |
| 67 | 0.099885 | 67.069 | 0.19908 | 67.275 | 0.29689 | 67.621 | 0.39266 | 68.108 | 0.48575 | 68.738 |
| 68 | 0.099881 | 68.066 | 0.19904 | 68.265 | 0.29678 | 68.599 | 0.39240 | 69.070 | 0.48523 | 69.679 |
| 69 | 0.099877 | 69.064 | 0.19901 | 69.255 | 0.29667 | 69.577 | 0.39214 | 70.030 | 0.48473 | 70.617 |
| 70 | 0.099873 | 70.062 | 0.19898 | 70.245 | 0.29657 | 70.555 | 0.39190 | 70.990 | 0.48426 | 71.554 |
| 71 | 0.099869 | 71.059 | 0.19895 | 71.235 | 0.29647 | 71.532 | 0.39167 | 71.948 | 0.48380 | 72.489 |
| 72 | 0.099865 | 72.057 | 0.19892 | 72.225 | 0.29637 | 72.508 | 0.39145 | 72.905 | 0.48338 | 73.422 |
| 73 | 0.099861 | 73.054 | 0.19889 | 73.214 | 0.29628 | 73.483 | 0.39123 | 73.861 | 0.48296 | 74.354 |
| 74 | 0.099858 | 74.051 | 0.19887 | 74.203 | 0.29620 | 74.458 | 0.39104 | 74.817 | 0.48257 | 75.284 |
| 75 | 0.099855 | 75.048 | 0.19885 | 75.192 | 0.29612 | 75.432 | 0.39084 | 75.771 | 0.48220 | 76.212 |
| 76 | 0.099852 | 76.045 | 0.19883 | 76.180 | 0.29604 | 76.406 | 0.39066 | 76.724 | 0.48184 | 77.138 |
| 77 | 0.099850 | 77.042 | 0.19881 | 77.168 | 0.29596 | 77.379 | 0.39050 | 77.676 | 0.48152 | 78.062 |
| 78 | 0.099847 | 78.039 | 0.19878 | 78.156 | 0.29590 | 78.351 | 0.39034 | 78.628 | 0.48121 | 78.986 |
| 79 | 0.099845 | 79.036 | 0.19876 | 79.144 | 0.28584 | 79.322 | 0.39018 | 79.578 | 0.48093 | 79.909 |
| 80 | 0.099843 | 80.033 | 0.19875 | 80.131 | 0.29578 | 80.294 | 0.39004 | 80.528 | 0.48067 | 80.830 |
| 81 | 0.099841 | 81.030 | 0.19873 | 81.118 | 0.29573 | 81.266 | 0.38993 | 81.477 | 0.48044 | 81.750 |
| 82 | 0.099839 | 82.026 | 0.19872 | 82.106 | 0.29569 | 82.238 | 0.38983 | 82.425 | 0.48023 | 82.669 |
| 83 | 0.099838 | 83.023 | 0.19871 | 83.093 | 0.29566 | 83.209 | 0.38973 | 83.373 | 0.48004 | 83.587 |
| 84 | 0.099837 | 84.020 | 0.19870 | 84.080 | 0.29563 | 84.180 | 0.38965 | 84.321 | 0.47987 | 84.505 |
| 85 | 0.099836 | 85.017 | 0.19869 | 85.067 | 0.29561 | 85.150 | 0.38958 | 85.268 | 0.47972 | 85.422 |
| 86 | 0.099835 | 86.014 | 0.19868 | 86.054 | 0.29559 | 86.120 | 0.38952 | 86.215 | 0.47960 | 86.338 |
| 87 | 0.099834 | 87.011 | 0.19868 | 87.041 | 0.29557 | 87.090 | 0.38948 | 87.162 | 0.47952 | 87.254 |
| 88 | 0.099833 | 88.008 | 0.19867 | 88.028 | 0.29555 | 88.060 | 0.38946 | 88.108 | 0.47947 | 88.170 |
| 89 | 0.099832 | 89.004 | 0.19867 | 89.014 | 0.29553 | 89.030 | 0.38944 | 89.054 | 0.47945 | 89.085 |
| 90 | 0.099831 | 90.000 | 0.19867 | 90.000 | 0.29552 | 90.000 | 0.38942 | 90.000 | 0.47943 | 90.000 |

Note.  $\sinh(o/\delta) = o/\delta$ .Examples.  $\sinh(0.3/65^\circ) = 0.29712/65^\circ.661 = 0.29712/65^\circ.39'.40''$ .  
 $\sinh^{-1}(0.39018/79^\circ.578) = 0.4/79^\circ$ .

TABLE I. HYPERBOLIC SINES.  $\sinh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 0.6     |        | 0.7     |        | 0.8     |        | 0.9     |        | 1.0     |        |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| °  | °       | °      | °       | °      | °       | °      | °       | °      | °       | °      |
| 45 | 0.60042 | 48.440 | 0.70094 | 49.676 | 0.80184 | 51.108 | 0.90327 | 52.728 | 1.00553 | 54.531 |
| 46 | 0.59918 | 49.437 | 0.69894 | 50.679 | 0.79885 | 52.112 | 0.89904 | 53.735 | 0.99975 | 55.540 |
| 47 | 0.59793 | 50.434 | 0.69695 | 51.676 | 0.79587 | 53.109 | 0.89482 | 54.734 | 0.99394 | 56.550 |
| 48 | 0.59667 | 51.426 | 0.69497 | 52.666 | 0.79291 | 54.099 | 0.89060 | 55.725 | 0.98816 | 57.543 |
| 49 | 0.59542 | 52.414 | 0.69299 | 53.652 | 0.78996 | 55.082 | 0.88640 | 56.707 | 0.98242 | 58.525 |
| 50 | 0.59418 | 53.398 | 0.69102 | 54.632 | 0.78703 | 56.058 | 0.88224 | 57.679 | 0.97672 | 59.495 |
| 51 | 0.59295 | 54.379 | 0.68907 | 55.606 | 0.78412 | 57.026 | 0.87810 | 58.642 | 0.97105 | 60.453 |
| 52 | 0.59174 | 55.355 | 0.68713 | 56.574 | 0.78124 | 57.987 | 0.87400 | 59.595 | 0.96543 | 61.399 |
| 53 | 0.59053 | 56.326 | 0.68521 | 57.537 | 0.77838 | 58.940 | 0.86993 | 60.538 | 0.95986 | 62.333 |
| 54 | 0.58932 | 57.293 | 0.68331 | 58.493 | 0.77555 | 59.886 | 0.86590 | 61.472 | 0.95435 | 63.255 |
| 55 | 0.58814 | 58.256 | 0.68144 | 59.445 | 0.77275 | 60.824 | 0.86192 | 62.396 | 0.94890 | 64.166 |
| 56 | 0.58698 | 59.215 | 0.67959 | 60.391 | 0.76999 | 61.755 | 0.85800 | 63.312 | 0.94353 | 65.065 |
| 57 | 0.58583 | 60.171 | 0.67776 | 61.331 | 0.76727 | 62.678 | 0.85414 | 64.218 | 0.93825 | 65.952 |
| 58 | 0.58469 | 61.122 | 0.67595 | 62.265 | 0.76459 | 63.593 | 0.85034 | 65.114 | 0.93305 | 66.827 |
| 59 | 0.58357 | 62.069 | 0.67419 | 63.193 | 0.76195 | 64.502 | 0.84660 | 66.000 | 0.92795 | 67.691 |
| 60 | 0.58249 | 63.013 | 0.67247 | 64.117 | 0.75938 | 65.405 | 0.84295 | 66.878 | 0.92295 | 68.544 |
| 61 | 0.58142 | 63.953 | 0.67078 | 65.036 | 0.75686 | 66.300 | 0.83937 | 67.747 | 0.91805 | 69.385 |
| 62 | 0.58037 | 64.889 | 0.66912 | 65.951 | 0.75439 | 67.189 | 0.83587 | 68.607 | 0.91325 | 70.215 |
| 63 | 0.57934 | 65.821 | 0.66749 | 66.859 | 0.75197 | 68.070 | 0.83244 | 69.458 | 0.90856 | 71.033 |
| 64 | 0.57834 | 66.749 | 0.66591 | 67.762 | 0.74962 | 68.944 | 0.82909 | 70.300 | 0.90400 | 71.841 |
| 65 | 0.57737 | 67.674 | 0.66437 | 68.661 | 0.74733 | 69.812 | 0.82585 | 71.136 | 0.89957 | 72.637 |
| 66 | 0.57643 | 68.596 | 0.66288 | 69.554 | 0.74512 | 70.674 | 0.82270 | 71.962 | 0.89527 | 73.424 |
| 67 | 0.57553 | 69.515 | 0.66145 | 70.444 | 0.74298 | 71.529 | 0.81967 | 72.780 | 0.89111 | 74.201 |
| 68 | 0.57465 | 70.430 | 0.66005 | 71.329 | 0.74091 | 72.379 | 0.81672 | 73.590 | 0.88708 | 74.968 |
| 69 | 0.57379 | 71.342 | 0.65870 | 72.209 | 0.73891 | 73.223 | 0.81387 | 74.392 | 0.88320 | 75.723 |
| 70 | 0.57297 | 72.251 | 0.65740 | 73.085 | 0.73698 | 74.061 | 0.81114 | 75.187 | 0.87947 | 76.469 |
| 71 | 0.57219 | 73.157 | 0.65616 | 73.957 | 0.73513 | 74.894 | 0.80853 | 75.975 | 0.87589 | 77.207 |
| 72 | 0.57145 | 74.061 | 0.65498 | 74.825 | 0.73337 | 75.722 | 0.80602 | 76.756 | 0.87247 | 77.936 |
| 73 | 0.57074 | 74.962 | 0.65385 | 75.689 | 0.73169 | 76.544 | 0.80363 | 77.530 | 0.86921 | 78.656 |
| 74 | 0.57006 | 75.860 | 0.65278 | 76.550 | 0.73009 | 77.361 | 0.80137 | 78.298 | 0.86612 | 79.368 |
| 75 | 0.56941 | 76.756 | 0.65176 | 77.408 | 0.72858 | 78.174 | 0.79924 | 79.059 | 0.86320 | 80.072 |
| 76 | 0.56881 | 77.649 | 0.65081 | 78.263 | 0.72716 | 78.982 | 0.79723 | 79.815 | 0.86045 | 80.769 |
| 77 | 0.56824 | 78.540 | 0.64992 | 79.114 | 0.72583 | 79.787 | 0.79535 | 80.566 | 0.85788 | 81.458 |
| 78 | 0.56772 | 79.429 | 0.64909 | 79.962 | 0.72459 | 80.588 | 0.79359 | 81.312 | 0.85549 | 82.141 |
| 79 | 0.56724 | 80.317 | 0.64832 | 80.808 | 0.72345 | 81.385 | 0.79197 | 82.053 | 0.85328 | 82.818 |
| 80 | 0.56679 | 81.203 | 0.64761 | 81.652 | 0.72241 | 82.179 | 0.79048 | 82.789 | 0.85125 | 83.489 |
| 81 | 0.56638 | 82.087 | 0.64697 | 82.493 | 0.72146 | 82.969 | 0.78913 | 83.522 | 0.84940 | 84.156 |
| 82 | 0.56602 | 82.970 | 0.64640 | 83.332 | 0.72061 | 83.757 | 0.78792 | 84.251 | 0.84774 | 84.817 |
| 83 | 0.56570 | 83.852 | 0.64589 | 84.169 | 0.71985 | 84.543 | 0.78685 | 84.976 | 0.84628 | 85.474 |
| 84 | 0.56542 | 84.732 | 0.64545 | 85.005 | 0.71919 | 85.327 | 0.78592 | 85.699 | 0.84501 | 86.128 |
| 85 | 0.56518 | 85.612 | 0.64507 | 85.839 | 0.71863 | 86.109 | 0.78513 | 86.420 | 0.84393 | 86.779 |
| 86 | 0.56498 | 86.490 | 0.64476 | 86.673 | 0.71817 | 86.889 | 0.78448 | 87.138 | 0.84305 | 87.426 |
| 87 | 0.56483 | 87.368 | 0.64452 | 87.506 | 0.71781 | 87.668 | 0.78397 | 87.855 | 0.84236 | 88.071 |
| 88 | 0.56473 | 88.246 | 0.64435 | 88.338 | 0.71755 | 88.446 | 0.78361 | 88.571 | 0.84186 | 88.715 |
| 89 | 0.56466 | 89.123 | 0.64425 | 89.169 | 0.71740 | 89.223 | 0.78339 | 89.286 | 0.84156 | 89.358 |
| 90 | 0.56464 | 90.000 | 0.64422 | 90.000 | 0.71736 | 90.000 | 0.78333 | 90.000 | 0.84147 | 90.000 |

Examples.  $\sinh(1.0/90^\circ) = 0.84147/90^\circ$ .  
 $\sinh^{-1}(0.87947/76^\circ.469) = 1.0/70^\circ$ .

TABLE I. HYPERBOLIC SINES.  $\sinh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 1.1     |        | 1.2     |        | 1.3     |        | 1.4     |        | 1.5     |        |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| °  | °       | °      | °       | °      | °       | °      | °       | °      | °       | °      |
| 45 | 1.1089  | 56.519 | 1.2138  | 58.692 | 1.3205  | 61.034 | 1.4297  | 63.568 | 1.5418  | 66.262 |
| 46 | 1.1012  | 57.543 | 1.2037  | 59.726 | 1.3078  | 62.092 | 1.4138  | 64.639 | 1.5222  | 67.355 |
| 47 | 1.0935  | 58.555 | 1.1937  | 60.748 | 1.2951  | 63.128 | 1.3979  | 65.689 | 1.5027  | 68.426 |
| 48 | 1.0858  | 59.553 | 1.1838  | 61.753 | 1.2824  | 64.142 | 1.3822  | 66.717 | 1.4834  | 69.474 |
| 49 | 1.0782  | 60.536 | 1.1739  | 62.741 | 1.2699  | 65.137 | 1.3665  | 67.723 | 1.4642  | 70.496 |
| 50 | 1.0706  | 61.506 | 1.1641  | 63.712 | 1.2574  | 66.113 | 1.3509  | 68.707 | 1.4451  | 71.492 |
| 51 | 1.0630  | 62.461 | 1.1543  | 64.666 | 1.2450  | 67.068 | 1.3355  | 69.668 | 1.4262  | 72.462 |
| 52 | 1.0556  | 63.401 | 1.1446  | 65.603 | 1.2327  | 68.004 | 1.3202  | 70.605 | 1.4075  | 73.405 |
| 53 | 1.0482  | 64.327 | 1.1350  | 66.523 | 1.2206  | 68.919 | 1.3051  | 71.519 | 1.3889  | 74.321 |
| 54 | 1.0409  | 65.239 | 1.1256  | 67.425 | 1.2086  | 69.814 | 1.2902  | 72.409 | 1.3706  | 75.210 |
| 55 | 1.0336  | 66.136 | 1.1162  | 68.310 | 1.1967  | 70.688 | 1.2754  | 73.275 | 1.3525  | 76.072 |
| 56 | 1.0265  | 67.019 | 1.1070  | 69.178 | 1.1850  | 71.542 | 1.2609  | 74.117 | 1.3347  | 76.907 |
| 57 | 1.0195  | 67.888 | 1.0979  | 70.028 | 1.1735  | 72.376 | 1.2466  | 74.936 | 1.3172  | 77.713 |
| 58 | 1.0126  | 68.742 | 1.0890  | 70.860 | 1.1622  | 73.188 | 1.2325  | 75.730 | 1.3000  | 78.491 |
| 59 | 1.0058  | 69.581 | 1.0802  | 71.675 | 1.1511  | 73.980 | 1.2187  | 76.500 | 1.2831  | 79.240 |
| 60 | 0.99920 | 70.406 | 1.0716  | 72.474 | 1.1403  | 74.752 | 1.2052  | 77.246 | 1.2665  | 79.964 |
| 61 | 0.99269 | 71.218 | 1.0632  | 73.255 | 1.1296  | 75.502 | 1.1919  | 77.967 | 1.2503  | 80.657 |
| 62 | 0.98633 | 72.016 | 1.0550  | 74.019 | 1.1192  | 76.232 | 1.1790  | 78.663 | 1.2345  | 81.321 |
| 63 | 0.98013 | 72.800 | 1.0470  | 74.767 | 1.1091  | 76.942 | 1.1664  | 79.335 | 1.2191  | 81.956 |
| 64 | 0.97409 | 73.570 | 1.0392  | 75.497 | 1.0992  | 77.632 | 1.1541  | 79.983 | 1.2040  | 82.562 |
| 65 | 0.96821 | 74.327 | 1.0316  | 76.211 | 1.0895  | 78.301 | 1.1421  | 80.607 | 1.1894  | 83.140 |
| 66 | 0.96251 | 75.071 | 1.0242  | 76.909 | 1.0802  | 78.950 | 1.1305  | 81.207 | 1.1752  | 83.690 |
| 67 | 0.95698 | 75.801 | 1.0171  | 77.590 | 1.0712  | 79.580 | 1.1193  | 81.783 | 1.1615  | 84.211 |
| 68 | 0.95165 | 76.519 | 1.0102  | 78.257 | 1.0625  | 80.191 | 1.1084  | 82.335 | 1.1482  | 84.704 |
| 69 | 0.94650 | 77.225 | 1.0035  | 78.907 | 1.0540  | 80.783 | 1.0979  | 82.865 | 1.1354  | 85.169 |
| 70 | 0.94156 | 77.918 | 0.99712 | 79.543 | 1.0459  | 81.357 | 1.0879  | 83.373 | 1.1231  | 85.607 |
| 71 | 0.93682 | 78.600 | 0.99099 | 80.164 | 1.0382  | 81.912 | 1.0783  | 83.858 | 1.1113  | 86.018 |
| 72 | 0.93229 | 79.271 | 0.98514 | 80.771 | 1.0308  | 82.450 | 1.0691  | 84.322 | 1.1000  | 86.402 |
| 73 | 0.92798 | 79.931 | 0.97957 | 81.365 | 1.0237  | 82.971 | 1.0603  | 84.765 | 1.0893  | 86.761 |
| 74 | 0.92388 | 80.580 | 0.97428 | 81.946 | 1.0170  | 83.476 | 1.0520  | 85.188 | 1.0791  | 87.096 |
| 75 | 0.92001 | 81.220 | 0.96927 | 82.514 | 1.0107  | 83.966 | 1.0441  | 85.591 | 1.0694  | 87.406 |
| 76 | 0.91637 | 81.850 | 0.96456 | 83.069 | 1.0047  | 84.440 | 1.0367  | 85.975 | 1.0603  | 87.693 |
| 77 | 0.91296 | 82.471 | 0.96015 | 83.614 | 0.99916 | 84.900 | 1.0298  | 86.342 | 1.0518  | 87.958 |
| 78 | 0.90978 | 83.084 | 0.95605 | 84.148 | 0.99397 | 85.346 | 1.0233  | 86.692 | 1.0439  | 88.201 |
| 79 | 0.90685 | 83.689 | 0.95226 | 84.672 | 0.98917 | 85.780 | 1.0173  | 87.026 | 1.0366  | 88.425 |
| 80 | 0.90416 | 84.286 | 0.94878 | 85.187 | 0.98477 | 86.202 | 1.0119  | 87.346 | 1.0299  | 88.630 |
| 81 | 0.90172 | 84.877 | 0.94562 | 85.693 | 0.98077 | 86.614 | 1.0069  | 87.651 | 1.0238  | 88.818 |
| 82 | 0.89953 | 85.462 | 0.94279 | 86.191 | 0.97715 | 87.016 | 1.0024  | 87.944 | 1.0183  | 88.991 |
| 83 | 0.89759 | 86.041 | 0.94029 | 86.683 | 0.97400 | 87.408 | 0.99845 | 88.227 | 1.0135  | 89.149 |
| 84 | 0.89590 | 86.616 | 0.93811 | 87.169 | 0.97124 | 87.793 | 0.99502 | 88.499 | 1.0093  | 89.294 |
| 85 | 0.89447 | 87.186 | 0.93626 | 87.649 | 0.96890 | 88.171 | 0.99210 | 88.762 | 1.0057  | 89.429 |
| 86 | 0.89330 | 87.753 | 0.93474 | 88.124 | 0.96698 | 88.544 | 0.98971 | 89.018 | 1.0027  | 89.554 |
| 87 | 0.89238 | 88.317 | 0.93356 | 88.596 | 0.96548 | 88.912 | 0.98785 | 89.269 | 1.0004  | 89.673 |
| 88 | 0.89173 | 88.879 | 0.93272 | 89.065 | 0.96441 | 89.277 | 0.98652 | 89.515 | 0.99880 | 89.785 |
| 89 | 0.89134 | 89.440 | 0.93221 | 89.533 | 0.96377 | 89.639 | 0.98572 | 89.758 | 0.99782 | 89.893 |
| 90 | 0.89121 | 90.000 | 0.93204 | 90.000 | 0.96356 | 90.000 | 0.98545 | 90.000 | 0.99749 | 90.000 |

Examples.  $\sinh(1.1/45^\circ) = 1.1089/56^\circ.519 = 1.1089/56^\circ.31'.08''$ .  
 $\sinh^{-1}(1.1084/82^\circ.335) = 1.4/68^\circ$ .

TABLE I. HYPERBOLIC SINES.  $\sinh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 1.6     |        | 1.7     |        | 1.8     |        | 1.9     |        | 2.0     |         |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|---------|
| °  | °       | °      | °       | °      | °       | °      | °       | °      | °       | °       |
| 45 | 1.6575  | 69.117 | 1.7776  | 72.133 | 1.9029  | 75.292 | 2.0343  | 78.590 | 2.1726  | 82.016  |
| 46 | 1.6338  | 70.241 | 1.7493  | 73.288 | 1.8693  | 76.486 | 1.9947  | 79.829 | 2.1266  | 83.304  |
| 47 | 1.6103  | 71.339 | 1.7210  | 74.418 | 1.8359  | 77.651 | 1.9554  | 81.037 | 2.0809  | 84.560  |
| 48 | 1.5868  | 72.409 | 1.6929  | 75.515 | 1.8027  | 78.784 | 1.9165  | 82.210 | 2.0356  | 85.778  |
| 49 | 1.5635  | 73.451 | 1.6651  | 76.582 | 1.7697  | 79.883 | 1.8779  | 83.346 | 1.9907  | 86.958  |
| 50 | 1.5404  | 74.465 | 1.6375  | 77.618 | 1.7370  | 80.946 | 1.8396  | 84.444 | 1.9462  | 88.098  |
| 51 | 1.5175  | 75.449 | 1.6102  | 78.620 | 1.7046  | 81.974 | 1.8016  | 85.503 | 1.9022  | 89.196  |
| 52 | 1.4949  | 76.402 | 1.5831  | 79.590 | 1.6726  | 82.966 | 1.7643  | 86.524 | 1.8588  | 90.253  |
| 53 | 1.4725  | 77.325 | 1.5563  | 80.527 | 1.6410  | 83.921 | 1.7273  | 87.505 | 1.8160  | 91.267  |
| 54 | 1.4504  | 78.218 | 1.5299  | 81.429 | 1.6098  | 84.839 | 1.6909  | 88.444 | 1.7737  | 92.236  |
| 55 | 1.4285  | 79.079 | 1.5039  | 82.296 | 1.5791  | 85.718 | 1.6550  | 89.342 | 1.7320  | 93.160  |
| 56 | 1.4070  | 79.910 | 1.4782  | 83.128 | 1.5488  | 86.558 | 1.6196  | 90.197 | 1.6911  | 94.039  |
| 57 | 1.3859  | 80.709 | 1.4530  | 83.924 | 1.5190  | 87.358 | 1.5848  | 91.008 | 1.6509  | 94.869  |
| 58 | 1.3651  | 81.475 | 1.4282  | 84.683 | 1.4898  | 88.116 | 1.5506  | 91.773 | 1.6114  | 95.650  |
| 59 | 1.3447  | 82.209 | 1.4039  | 85.406 | 1.4611  | 88.834 | 1.5171  | 92.493 | 1.5726  | 96.382  |
| 60 | 1.3247  | 82.910 | 1.3800  | 86.091 | 1.4330  | 89.510 | 1.4843  | 93.167 | 1.5347  | 97.062  |
| 61 | 1.3051  | 83.578 | 1.3567  | 86.738 | 1.4055  | 90.142 | 1.4523  | 93.792 | 1.4976  | 97.688  |
| 62 | 1.2860  | 84.212 | 1.3339  | 87.347 | 1.3787  | 90.730 | 1.4210  | 94.368 | 1.4614  | 98.261  |
| 63 | 1.2674  | 84.813 | 1.3117  | 87.917 | 1.3525  | 91.274 | 1.3904  | 94.893 | 1.4260  | 98.778  |
| 64 | 1.2492  | 85.380 | 1.2901  | 88.447 | 1.3270  | 91.774 | 1.3606  | 95.368 | 1.3915  | 99.238  |
| 65 | 1.2316  | 85.913 | 1.2690  | 88.938 | 1.3022  | 92.228 | 1.3316  | 95.792 | 1.3580  | 99.639  |
| 66 | 1.2145  | 86.413 | 1.2486  | 89.390 | 1.2781  | 92.636 | 1.3035  | 96.162 | 1.3255  | 99.980  |
| 67 | 1.1979  | 86.879 | 1.2288  | 89.802 | 1.2548  | 92.997 | 1.2762  | 96.478 | 1.2940  | 100.261 |
| 68 | 1.1819  | 87.311 | 1.2097  | 90.175 | 1.2322  | 93.312 | 1.2499  | 96.741 | 1.2635  | 100.479 |
| 69 | 1.1664  | 87.710 | 1.1913  | 90.508 | 1.2104  | 93.580 | 1.2244  | 96.949 | 1.2340  | 100.634 |
| 70 | 1.1516  | 88.076 | 1.1736  | 90.801 | 1.1895  | 93.802 | 1.1999  | 97.102 | 1.2056  | 100.725 |
| 71 | 1.1373  | 88.410 | 1.1566  | 91.055 | 1.1694  | 93.976 | 1.1764  | 97.199 | 1.1783  | 100.750 |
| 72 | 1.1237  | 88.712 | 1.1403  | 91.271 | 1.1502  | 94.104 | 1.1539  | 97.241 | 1.1521  | 100.709 |
| 73 | 1.1107  | 88.982 | 1.1248  | 91.448 | 1.1318  | 94.187 | 1.1324  | 97.227 | 1.1271  | 100.602 |
| 74 | 1.0984  | 89.221 | 1.1101  | 91.588 | 1.1144  | 94.224 | 1.1119  | 97.158 | 1.1033  | 100.428 |
| 75 | 1.0867  | 89.431 | 1.0961  | 91.692 | 1.0979  | 94.215 | 1.0925  | 97.034 | 1.0807  | 100.187 |
| 76 | 1.0757  | 89.613 | 1.0829  | 91.760 | 1.0823  | 94.162 | 1.0743  | 96.856 | 1.0594  | 99.880  |
| 77 | 1.0654  | 89.767 | 1.0706  | 91.794 | 1.0677  | 94.068 | 1.0571  | 96.625 | 1.0394  | 99.507  |
| 78 | 1.0553  | 89.894 | 1.0591  | 91.795 | 1.0541  | 93.933 | 1.0411  | 96.343 | 1.0207  | 99.070  |
| 79 | 1.0469  | 89.997 | 1.0485  | 91.765 | 1.0415  | 93.758 | 1.0262  | 96.012 | 1.0033  | 98.571  |
| 80 | 1.0388  | 90.076 | 1.0388  | 91.705 | 1.0299  | 93.545 | 1.0126  | 95.633 | 0.98729 | 98.011  |
| 81 | 1.0314  | 90.134 | 1.0299  | 91.618 | 1.0194  | 93.298 | 1.0002  | 95.209 | 0.97271 | 97.394  |
| 82 | 1.0248  | 90.171 | 1.0219  | 91.505 | 1.0099  | 93.019 | 0.98899 | 94.744 | 0.95958 | 96.722  |
| 83 | 1.0189  | 90.191 | 1.0149  | 91.370 | 1.0015  | 92.711 | 0.97907 | 94.240 | 0.94792 | 96.000  |
| 84 | 1.0138  | 90.194 | 1.0088  | 91.214 | 0.99422 | 92.374 | 0.97043 | 93.702 | 0.93775 | 95.233  |
| 85 | 1.0095  | 90.184 | 1.0036  | 91.039 | 0.98802 | 92.015 | 0.96309 | 93.133 | 0.92911 | 94.426  |
| 86 | 1.0060  | 90.161 | 0.99929 | 90.850 | 0.98203 | 91.636 | 0.95706 | 92.539 | 0.92201 | 93.584  |
| 87 | 1.0032  | 90.129 | 0.99596 | 90.649 | 0.97806 | 91.241 | 0.95236 | 91.923 | 0.91646 | 92.713  |
| 88 | 1.0012  | 90.090 | 0.99357 | 90.438 | 0.97612 | 90.834 | 0.94899 | 91.291 | 0.91248 | 91.821  |
| 89 | 1.0000  | 90.046 | 0.99214 | 90.220 | 0.97442 | 90.419 | 0.94697 | 90.648 | 0.91009 | 90.914  |
| 90 | 0.99957 | 90.000 | 0.99166 | 90.000 | 0.97385 | 90.000 | 0.94630 | 90.000 | 0.90930 | 90.000  |

Examples.  $\sinh(1.6/60^\circ) = 1.3247/82^\circ.910 = 1.3247/82^\circ.54'.36''$ .  
 $\sinh^{-1}(1.1999/97^\circ.102) = 1.9/70^\circ$ .

TABLE I. HYPERBOLIC SINES.  $\sinh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 2.1     |         | 2.2     |         | 2.3     |         | 2.4     |         | 2.5     |         |
|----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| °  | °       | °       | °       | °       | °       | °       | °       | °       | °       | °       |
| 45 | 2.3190  | 85.558  | 2.4745  | 89.205  | 2.6404  | 92.946  | 2.8177  | 96.769  | 3.0079  | 100.661 |
| 46 | 2.2658  | 86.905  | 2.4135  | 90.613  | 2.5707  | 94.419  | 2.7386  | 98.312  | 2.9185  | 102.278 |
| 47 | 2.2131  | 88.213  | 2.3530  | 91.981  | 2.5017  | 95.851  | 2.6603  | 99.813  | 2.8301  | 103.852 |
| 48 | 2.1608  | 89.482  | 2.2936  | 93.307  | 2.4334  | 97.241  | 2.5829  | 101.271 | 2.7429  | 105.383 |
| 49 | 2.1090  | 90.711  | 2.2337  | 94.592  | 2.3659  | 98.588  | 2.5065  | 102.685 | 2.6568  | 106.869 |
| 50 | 2.0577  | 91.898  | 2.1750  | 95.834  | 2.2992  | 99.890  | 2.4311  | 104.053 | 2.5720  | 108.309 |
| 51 | 2.0071  | 93.042  | 2.1171  | 97.031  | 2.2334  | 101.146 | 2.3568  | 105.375 | 2.4885  | 109.701 |
| 52 | 1.9571  | 94.142  | 2.0600  | 98.181  | 2.1685  | 102.354 | 2.2836  | 106.648 | 2.4064  | 111.044 |
| 53 | 1.9078  | 95.197  | 2.0037  | 99.284  | 2.1046  | 103.514 | 2.2117  | 107.871 | 2.3257  | 112.337 |
| 54 | 1.8592  | 96.205  | 1.9483  | 100.338 | 2.0418  | 104.623 | 2.1410  | 109.042 | 2.2465  | 113.578 |
| 55 | 1.8114  | 97.165  | 1.8938  | 101.342 | 1.9801  | 105.680 | 2.0744  | 110.160 | 2.1687  | 114.765 |
| 56 | 1.7644  | 98.076  | 1.8402  | 102.294 | 1.9195  | 106.683 | 2.0032  | 111.223 | 2.0925  | 115.896 |
| 57 | 1.7182  | 98.935  | 1.7876  | 103.193 | 1.8600  | 107.629 | 1.9364  | 112.228 | 2.0178  | 116.970 |
| 58 | 1.6729  | 99.742  | 1.7360  | 104.035 | 1.8016  | 108.518 | 1.8709  | 113.174 | 1.9447  | 117.984 |
| 59 | 1.6284  | 100.494 | 1.6854  | 104.820 | 1.7445  | 109.347 | 1.8068  | 114.059 | 1.8733  | 118.936 |
| 60 | 1.5849  | 101.191 | 1.6359  | 105.546 | 1.6886  | 110.114 | 1.7441  | 114.880 | 1.8035  | 119.823 |
| 61 | 1.5424  | 101.830 | 1.5875  | 106.210 | 1.6340  | 110.816 | 1.6829  | 115.634 | 1.7353  | 120.642 |
| 62 | 1.5008  | 102.410 | 1.5402  | 106.811 | 1.5807  | 111.451 | 1.6232  | 116.319 | 1.6688  | 121.391 |
| 63 | 1.4603  | 102.929 | 1.4941  | 107.345 | 1.5287  | 112.016 | 1.5649  | 116.931 | 1.6040  | 122.067 |
| 64 | 1.4208  | 103.386 | 1.4492  | 107.811 | 1.4780  | 112.509 | 1.5081  | 117.467 | 1.5409  | 122.665 |
| 65 | 1.3824  | 103.777 | 1.4055  | 108.207 | 1.4286  | 112.926 | 1.4528  | 117.924 | 1.4794  | 123.182 |
| 66 | 1.3451  | 104.101 | 1.3630  | 108.529 | 1.3806  | 113.264 | 1.3991  | 118.297 | 1.4196  | 123.613 |
| 67 | 1.3089  | 104.357 | 1.3218  | 108.775 | 1.3341  | 113.519 | 1.3469  | 118.583 | 1.3616  | 123.954 |
| 68 | 1.2738  | 104.542 | 1.2819  | 108.943 | 1.2890  | 113.688 | 1.2963  | 118.777 | 1.3052  | 124.198 |
| 69 | 1.2399  | 104.655 | 1.2433  | 109.029 | 1.2453  | 113.767 | 1.2473  | 118.874 | 1.2506  | 124.341 |
| 70 | 1.2072  | 104.694 | 1.2060  | 109.030 | 1.2031  | 113.752 | 1.1999  | 118.868 | 1.1977  | 124.376 |
| 71 | 1.1758  | 104.656 | 1.1701  | 108.944 | 1.1624  | 113.638 | 1.1541  | 118.754 | 1.1466  | 124.296 |
| 72 | 1.1457  | 104.541 | 1.1356  | 108.769 | 1.1232  | 113.422 | 1.1099  | 118.526 | 1.0972  | 124.092 |
| 73 | 1.1168  | 104.348 | 1.1026  | 108.501 | 1.0857  | 113.100 | 1.0675  | 118.177 | 1.0497  | 123.755 |
| 74 | 1.0893  | 104.074 | 1.0711  | 108.138 | 1.0498  | 112.667 | 1.0268  | 117.701 | 1.0040  | 123.277 |
| 75 | 1.0632  | 103.719 | 1.0411  | 107.678 | 1.0155  | 112.118 | 0.98795 | 117.091 | 0.96019 | 122.648 |
| 76 | 1.0385  | 103.283 | 1.0126  | 107.119 | 0.98292 | 111.449 | 0.95090 | 116.341 | 0.91831 | 121.857 |
| 77 | 1.0153  | 102.766 | 0.98581 | 106.459 | 0.95214 | 110.659 | 0.91576 | 115.443 | 0.87843 | 120.891 |
| 78 | 0.99353 | 102.168 | 0.96065 | 105.699 | 0.92320 | 109.744 | 0.88261 | 114.390 | 0.84063 | 119.740 |
| 79 | 0.97332 | 101.491 | 0.93722 | 104.839 | 0.89615 | 108.701 | 0.85152 | 113.178 | 0.80503 | 118.392 |
| 80 | 0.95468 | 100.736 | 0.91557 | 103.879 | 0.87109 | 107.531 | 0.82256 | 111.803 | 0.77173 | 116.836 |
| 81 | 0.93765 | 99.907  | 0.89575 | 102.822 | 0.84808 | 106.233 | 0.79589 | 110.261 | 0.74083 | 115.063 |
| 82 | 0.92229 | 99.008  | 0.87784 | 101.671 | 0.82719 | 104.810 | 0.77158 | 108.550 | 0.71251 | 113.065 |
| 83 | 0.90862 | 98.043  | 0.86187 | 100.432 | 0.80853 | 103.267 | 0.74974 | 106.676 | 0.68692 | 110.841 |
| 84 | 0.89668 | 97.015  | 0.84789 | 99.111  | 0.79215 | 101.611 | 0.73051 | 104.643 | 0.66422 | 108.390 |
| 85 | 0.88652 | 95.933  | 0.83596 | 97.715  | 0.77813 | 99.851  | 0.71396 | 102.462 | 0.64461 | 105.723 |
| 86 | 0.87817 | 94.806  | 0.82613 | 96.254  | 0.76655 | 98.000  | 0.70026 | 100.147 | 0.62827 | 102.855 |
| 87 | 0.87164 | 93.639  | 0.81845 | 94.740  | 0.75747 | 96.071  | 0.68948 | 97.717  | 0.61536 | 99.811  |
| 88 | 0.86696 | 92.443  | 0.81293 | 93.183  | 0.75095 | 94.082  | 0.68171 | 95.197  | 0.60602 | 96.624  |
| 89 | 0.86414 | 91.226  | 0.80961 | 91.599  | 0.74702 | 92.051  | 0.67703 | 92.615  | 0.60036 | 93.338  |
| 90 | 0.86321 | 90.000  | 0.80850 | 90.000  | 0.74571 | 90.000  | 0.67546 | 90.000  | 0.59847 | 90.000  |

Examples.  $\sinh(2.5/75^\circ) = 0.96019/122^\circ.648 = 0.96019/122^\circ.38'.53''$ .  
 $\sinh^{-1}(0.60036/93^\circ.338) = 2.5/89^\circ$ .

TABLE I. HYPERBOLIC SINES.  $\sinh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 2.6     |         | 2.7     |         | 2.8     |         | 2.9     |         | 3.0     |         |
|----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| °  | °       | °       | °       | °       | °       | °       | °       | °       | °       | °       |
| 45 | 3.2121  | 104.613 | 3.4318  | 108.614 | 3.6685  | 112.653 | 3.9236  | 116.721 | 4.1986  | 120.814 |
| 46 | 3.1115  | 106.307 | 3.3191  | 110.386 | 3.5426  | 114.506 | 3.7832  | 118.658 | 4.0426  | 122.832 |
| 47 | 3.0123  | 107.957 | 3.2079  | 112.116 | 3.4186  | 116.317 | 3.6453  | 120.551 | 3.8895  | 124.808 |
| 48 | 2.9144  | 109.564 | 3.0985  | 113.801 | 3.2966  | 118.084 | 3.5098  | 122.400 | 3.7394  | 126.741 |
| 49 | 2.8179  | 111.126 | 2.9908  | 115.442 | 3.1767  | 119.806 | 3.3768  | 124.205 | 3.5923  | 128.630 |
| 50 | 2.7229  | 112.641 | 2.8849  | 117.037 | 3.0590  | 121.483 | 3.2465  | 125.065 | 3.4483  | 130.474 |
| 51 | 2.6295  | 114.109 | 2.7809  | 118.585 | 2.9436  | 123.113 | 3.1189  | 127.679 | 3.3076  | 132.272 |
| 52 | 2.5378  | 115.528 | 2.6789  | 120.084 | 2.8306  | 124.694 | 2.9941  | 129.345 | 3.1701  | 134.024 |
| 53 | 2.4478  | 116.897 | 2.5789  | 121.532 | 2.7200  | 126.226 | 2.8721  | 130.963 | 3.0359  | 135.729 |
| 54 | 2.3595  | 118.214 | 2.4809  | 122.929 | 2.6118  | 127.708 | 2.7529  | 132.532 | 2.9051  | 137.386 |
| 55 | 2.2729  | 119.476 | 2.3850  | 124.274 | 2.5060  | 129.138 | 2.6366  | 134.050 | 2.7777  | 138.994 |
| 56 | 2.1882  | 120.684 | 2.2913  | 125.563 | 2.4027  | 130.515 | 2.5232  | 135.517 | 2.6536  | 140.553 |
| 57 | 2.1053  | 121.834 | 2.1996  | 126.797 | 2.3019  | 131.837 | 2.4127  | 136.932 | 2.5330  | 142.062 |
| 58 | 2.0242  | 122.925 | 2.1101  | 127.973 | 2.2036  | 133.103 | 2.3051  | 138.293 | 2.4157  | 143.519 |
| 59 | 1.9450  | 123.954 | 2.0228  | 129.088 | 2.1077  | 134.311 | 2.2004  | 139.598 | 2.3018  | 144.924 |
| 60 | 1.8677  | 124.918 | 1.9377  | 130.140 | 2.0144  | 135.459 | 2.0986  | 140.847 | 2.1912  | 146.276 |
| 61 | 1.7923  | 125.816 | 1.8547  | 131.128 | 1.9236  | 136.544 | 1.9997  | 142.037 | 2.0839  | 147.574 |
| 62 | 1.7187  | 126.644 | 1.7739  | 132.047 | 1.8352  | 137.565 | 1.9036  | 143.167 | 1.9798  | 148.817 |
| 63 | 1.6470  | 127.399 | 1.6952  | 132.895 | 1.7493  | 138.519 | 1.8103  | 144.235 | 1.8789  | 150.004 |
| 64 | 1.5772  | 128.077 | 1.6187  | 133.669 | 1.6658  | 139.403 | 1.7198  | 145.239 | 1.7812  | 151.133 |
| 65 | 1.5094  | 128.674 | 1.5442  | 134.364 | 1.5847  | 140.214 | 1.6319  | 146.176 | 1.6865  | 152.204 |
| 66 | 1.4435  | 129.185 | 1.4719  | 134.978 | 1.5060  | 140.948 | 1.5467  | 147.044 | 1.5948  | 153.214 |
| 67 | 1.3794  | 129.606 | 1.4016  | 135.504 | 1.4295  | 141.599 | 1.4641  | 147.838 | 1.5061  | 154.161 |
| 68 | 1.3171  | 129.930 | 1.3334  | 135.935 | 1.3553  | 142.164 | 1.3840  | 148.555 | 1.4202  | 155.043 |
| 69 | 1.2568  | 130.151 | 1.2673  | 136.266 | 1.2833  | 142.634 | 1.3063  | 149.190 | 1.3370  | 155.858 |
| 70 | 1.1983  | 130.262 | 1.2031  | 136.489 | 1.2136  | 143.005 | 1.2310  | 149.738 | 1.2565  | 156.602 |
| 71 | 1.1417  | 130.254 | 1.1409  | 136.596 | 1.1460  | 143.267 | 1.1581  | 150.191 | 1.1785  | 157.271 |
| 72 | 1.0870  | 130.117 | 1.0807  | 136.576 | 1.0804  | 143.411 | 1.0874  | 150.541 | 1.1030  | 157.860 |
| 73 | 1.0341  | 129.841 | 1.0226  | 136.415 | 1.0169  | 143.424 | 1.0189  | 150.780 | 1.0298  | 158.363 |
| 74 | 0.98316 | 129.414 | 0.96631 | 136.101 | 0.95550 | 143.292 | 0.95251 | 150.894 | 0.95893 | 158.772 |
| 75 | 0.93420 | 128.822 | 0.91207 | 135.617 | 0.89606 | 142.996 | 0.88819 | 150.867 | 0.89022 | 159.077 |
| 76 | 0.88720 | 128.049 | 0.85985 | 134.943 | 0.83862 | 142.516 | 0.82587 | 150.680 | 0.82356 | 159.265 |
| 77 | 0.84227 | 127.079 | 0.80965 | 134.058 | 0.78319 | 141.827 | 0.76551 | 150.309 | 0.75887 | 159.319 |
| 78 | 0.79948 | 125.893 | 0.76157 | 132.935 | 0.72979 | 140.897 | 0.70706 | 149.721 | 0.69605 | 159.216 |
| 79 | 0.75891 | 124.471 | 0.71568 | 131.542 | 0.67847 | 139.686 | 0.65054 | 148.876 | 0.63503 | 158.925 |
| 80 | 0.72068 | 122.793 | 0.67215 | 129.847 | 0.62934 | 138.145 | 0.59597 | 147.721 | 0.57573 | 158.404 |
| 81 | 0.68499 | 120.837 | 0.63109 | 127.812 | 0.58252 | 136.217 | 0.54340 | 146.183 | 0.51809 | 157.592 |
| 82 | 0.65203 | 118.583 | 0.59275 | 125.396 | 0.53822 | 133.833 | 0.49294 | 144.169 | 0.46213 | 156.404 |
| 83 | 0.62196 | 116.016 | 0.55739 | 122.554 | 0.49674 | 130.908 | 0.44484 | 141.553 | 0.40789 | 154.709 |
| 84 | 0.59507 | 113.126 | 0.52536 | 119.252 | 0.45849 | 127.351 | 0.39946 | 138.168 | 0.35548 | 152.313 |
| 85 | 0.57164 | 109.910 | 0.49705 | 115.458 | 0.42399 | 123.066 | 0.35738 | 133.795 | 0.30521 | 148.903 |
| 86 | 0.55191 | 106.382 | 0.47291 | 111.164 | 0.39394 | 117.968 | 0.31946 | 128.168 | 0.25775 | 143.974 |
| 87 | 0.53621 | 102.571 | 0.45344 | 106.390 | 0.36915 | 112.015 | 0.28691 | 121.005 | 0.21433 | 136.701 |
| 88 | 0.52479 | 98.524  | 0.43912 | 101.194 | 0.35055 | 105.244 | 0.26143 | 112.110 | 0.17731 | 125.843 |
| 89 | 0.51783 | 94.307  | 0.43033 | 95.682  | 0.33893 | 97.809  | 0.24496 | 101.576 | 0.15092 | 110.091 |
| 90 | 0.51550 | 90.000  | 0.42738 | 90.000  | 0.33499 | 90.000  | 0.23925 | 90.000  | 0.14112 | 90.000  |

Examples.  $\sinh(3.0/50^\circ) = 3.4483/130^\circ.474 = 3.4483/130^\circ.28'.26''$ .  
 $\sinh^{-1}(0.15092/110^\circ.091) = 3.0/89^\circ$ .



TABLE II. HYPERBOLIC COSINES.  $\cosh(\rho/\delta) = r/\gamma$

|    | 0.1     | 0.2   | 0.3     | 0.4   | 0.5     |       |         |       |         |       |
|----|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|
| 45 | 1.00001 | 0.287 | 1.00013 | 1.148 | 1.00067 | 2.578 | 1.00210 | 4.578 | 1.00519 | 7.141 |
| 46 | 0.99983 | 0.287 | 0.99943 | 1.146 | 0.99910 | 2.577 | 0.99933 | 4.584 | 1.00085 | 7.159 |
| 47 | 0.99966 | 0.286 | 0.99873 | 1.144 | 0.99753 | 2.576 | 0.99655 | 4.584 | 0.99649 | 7.166 |
| 48 | 0.99948 | 0.285 | 0.99804 | 1.141 | 0.99597 | 2.571 | 0.99378 | 4.578 | 0.99214 | 7.165 |
| 49 | 0.99931 | 0.283 | 0.99735 | 1.136 | 0.99441 | 2.562 | 0.99101 | 4.566 | 0.98780 | 7.155 |
| 50 | 0.99914 | 0.282 | 0.99666 | 1.131 | 0.99285 | 2.551 | 0.98824 | 4.551 | 0.98347 | 7.137 |
| 51 | 0.99897 | 0.280 | 0.99597 | 1.123 | 0.99131 | 2.536 | 0.98547 | 4.528 | 0.97917 | 7.109 |
| 52 | 0.99880 | 0.278 | 0.99529 | 1.115 | 0.98977 | 2.519 | 0.98274 | 4.500 | 0.97490 | 7.073 |
| 53 | 0.99863 | 0.275 | 0.99462 | 1.105 | 0.98825 | 2.498 | 0.98003 | 4.467 | 0.97065 | 7.028 |
| 54 | 0.99846 | 0.272 | 0.99395 | 1.094 | 0.98674 | 2.474 | 0.97734 | 4.427 | 0.96644 | 6.973 |
| 55 | 0.99830 | 0.269 | 0.99329 | 1.081 | 0.98525 | 2.447 | 0.97468 | 4.382 | 0.96226 | 6.910 |
| 56 | 0.99814 | 0.265 | 0.99263 | 1.067 | 0.98377 | 2.417 | 0.97205 | 4.332 | 0.95814 | 6.838 |
| 57 | 0.99798 | 0.261 | 0.99199 | 1.052 | 0.98232 | 2.383 | 0.96945 | 4.276 | 0.95406 | 6.756 |
| 58 | 0.99782 | 0.257 | 0.99135 | 1.036 | 0.98089 | 2.347 | 0.96690 | 4.214 | 0.95005 | 6.666 |
| 59 | 0.99766 | 0.253 | 0.99073 | 1.018 | 0.97948 | 2.308 | 0.96438 | 4.147 | 0.94609 | 6.567 |
| 60 | 0.99751 | 0.249 | 0.99012 | 0.999 | 0.97810 | 2.267 | 0.96191 | 4.075 | 0.94219 | 6.460 |
| 61 | 0.99736 | 0.244 | 0.98952 | 0.979 | 0.97674 | 2.221 | 0.95948 | 3.998 | 0.93838 | 6.343 |
| 62 | 0.99721 | 0.238 | 0.98893 | 0.957 | 0.97541 | 2.173 | 0.95711 | 3.914 | 0.93465 | 6.217 |
| 63 | 0.99707 | 0.232 | 0.98835 | 0.934 | 0.97411 | 2.123 | 0.95478 | 3.826 | 0.93099 | 6.083 |
| 64 | 0.99693 | 0.226 | 0.98780 | 0.910 | 0.97285 | 2.070 | 0.95251 | 3.733 | 0.92741 | 5.941 |
| 65 | 0.99679 | 0.220 | 0.98725 | 0.885 | 0.97163 | 2.014 | 0.95031 | 3.635 | 0.92393 | 5.789 |
| 66 | 0.99666 | 0.214 | 0.98672 | 0.859 | 0.97044 | 1.955 | 0.94816 | 3.532 | 0.92054 | 5.631 |
| 67 | 0.99653 | 0.207 | 0.98621 | 0.832 | 0.96927 | 1.894 | 0.94608 | 3.423 | 0.91725 | 5.463 |
| 68 | 0.99641 | 0.200 | 0.98571 | 0.804 | 0.96814 | 1.830 | 0.94407 | 3.310 | 0.91407 | 5.288 |
| 69 | 0.99629 | 0.193 | 0.98523 | 0.775 | 0.96706 | 1.764 | 0.94213 | 3.193 | 0.91100 | 5.106 |
| 70 | 0.99617 | 0.185 | 0.98477 | 0.744 | 0.96601 | 1.696 | 0.94026 | 3.072 | 0.90805 | 4.916 |
| 71 | 0.99606 | 0.177 | 0.98433 | 0.713 | 0.96500 | 1.626 | 0.93846 | 2.946 | 0.90521 | 4.718 |
| 72 | 0.99596 | 0.169 | 0.98391 | 0.681 | 0.96404 | 1.553 | 0.93674 | 2.816 | 0.90248 | 4.513 |
| 73 | 0.99586 | 0.161 | 0.98350 | 0.648 | 0.96313 | 1.479 | 0.93510 | 2.682 | 0.89989 | 4.302 |
| 74 | 0.99576 | 0.153 | 0.98312 | 0.614 | 0.96227 | 1.402 | 0.93354 | 2.544 | 0.89742 | 4.085 |
| 75 | 0.99567 | 0.144 | 0.98276 | 0.580 | 0.96145 | 1.324 | 0.93207 | 2.403 | 0.89508 | 3.861 |
| 76 | 0.99559 | 0.135 | 0.98242 | 0.545 | 0.96068 | 1.244 | 0.93069 | 2.258 | 0.89288 | 3.631 |
| 77 | 0.99551 | 0.126 | 0.98210 | 0.509 | 0.95996 | 1.162 | 0.92938 | 2.111 | 0.89081 | 3.396 |
| 78 | 0.99544 | 0.117 | 0.98181 | 0.472 | 0.95929 | 1.078 | 0.92816 | 1.960 | 0.88889 | 3.155 |
| 79 | 0.99537 | 0.108 | 0.98154 | 0.435 | 0.95866 | 0.993 | 0.92705 | 1.807 | 0.88711 | 2.910 |
| 80 | 0.99531 | 0.099 | 0.98128 | 0.397 | 0.95808 | 0.908 | 0.92603 | 1.652 | 0.88548 | 2.660 |
| 81 | 0.99525 | 0.090 | 0.98105 | 0.359 | 0.95756 | 0.821 | 0.92509 | 1.494 | 0.88399 | 2.406 |
| 82 | 0.99520 | 0.080 | 0.98085 | 0.320 | 0.95710 | 0.732 | 0.92425 | 1.333 | 0.88266 | 2.149 |
| 83 | 0.99515 | 0.070 | 0.98067 | 0.281 | 0.95670 | 0.643 | 0.92351 | 1.170 | 0.88147 | 1.888 |
| 84 | 0.99511 | 0.060 | 0.98051 | 0.242 | 0.95635 | 0.552 | 0.92287 | 1.006 | 0.88044 | 1.624 |
| 85 | 0.99508 | 0.050 | 0.98037 | 0.203 | 0.95605 | 0.461 | 0.92233 | 0.841 | 0.87957 | 1.357 |
| 86 | 0.99506 | 0.040 | 0.98026 | 0.163 | 0.95580 | 0.369 | 0.92189 | 0.675 | 0.87886 | 1.087 |
| 87 | 0.99504 | 0.030 | 0.98018 | 0.123 | 0.95560 | 0.277 | 0.92154 | 0.508 | 0.87830 | 0.816 |
| 88 | 0.99502 | 0.020 | 0.98012 | 0.082 | 0.95545 | 0.185 | 0.92128 | 0.340 | 0.87790 | 0.544 |
| 89 | 0.99501 | 0.010 | 0.98009 | 0.041 | 0.95537 | 0.093 | 0.92112 | 0.171 | 0.87766 | 0.272 |
| 90 | 0.99500 | 0.000 | 0.98007 | 0.000 | 0.95534 | 0.000 | 0.92106 | 0.000 | 0.87758 | 0.000 |

Note.  $\cosh(0/\delta) = 1.0/0^\circ$  for all values of  $\delta$ .

Examples.  $\cosh(0.5/81^\circ) = 0.88399/2^\circ.406 = 0.88399/2^\circ.24'.22''$ .  
 $\cosh^{-1}(0.97810/2^\circ.267) = 0.3/60^\circ$ .

TABLE II. HYPERBOLIC COSINES.  $\cosh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 0.6     |        | 0.7     |        | 0.8     |        | 0.9     |        | 1.0     |        |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| °  | °       | °      | °       | °      | °       | °      | °       | °      | °       | °      |
| 45 | 1.01070 | 10.254 | 1.01982 | 13.890 | 1.03360 | 18.010 | 1.05333 | 22.567 | 1.08031 | 27.487 |
| 46 | 1.00449 | 10.291 | 1.01133 | 13.960 | 1.02263 | 18.132 | 1.03959 | 22.755 | 1.06358 | 27.762 |
| 47 | 0.99825 | 10.315 | 1.00289 | 14.013 | 1.01164 | 18.231 | 1.02583 | 22.919 | 1.04680 | 28.011 |
| 48 | 0.99199 | 10.327 | 0.99441 | 14.050 | 1.00063 | 18.309 | 1.01203 | 23.059 | 1.03000 | 28.235 |
| 49 | 0.98575 | 10.326 | 0.98594 | 14.071 | 0.98963 | 18.368 | 0.99822 | 23.176 | 1.01315 | 28.433 |
| 50 | 0.97953 | 10.313 | 0.97748 | 14.075 | 0.97864 | 18.405 | 0.98443 | 23.267 | 0.99632 | 28.603 |
| 51 | 0.97333 | 10.287 | 0.96906 | 14.061 | 0.96768 | 18.421 | 0.97064 | 23.332 | 0.97950 | 28.743 |
| 52 | 0.96716 | 10.248 | 0.96066 | 14.031 | 0.95675 | 18.414 | 0.95690 | 23.372 | 0.96270 | 28.854 |
| 53 | 0.96103 | 10.196 | 0.95232 | 13.982 | 0.94587 | 18.384 | 0.94320 | 23.383 | 0.94596 | 28.933 |
| 54 | 0.95495 | 10.131 | 0.94404 | 13.916 | 0.93506 | 18.332 | 0.92957 | 23.367 | 0.92928 | 28.981 |
| 55 | 0.94893 | 10.053 | 0.93582 | 13.831 | 0.92432 | 18.256 | 0.91603 | 23.321 | 0.91267 | 28.994 |
| 56 | 0.94296 | 9.961  | 0.92768 | 13.728 | 0.91367 | 18.156 | 0.90257 | 23.246 | 0.89615 | 28.974 |
| 57 | 0.93706 | 9.856  | 0.91962 | 13.606 | 0.90312 | 18.031 | 0.88922 | 23.140 | 0.87976 | 28.917 |
| 58 | 0.93125 | 9.738  | 0.91166 | 13.465 | 0.89270 | 17.881 | 0.87602 | 23.003 | 0.86350 | 28.823 |
| 59 | 0.92552 | 9.606  | 0.90382 | 13.306 | 0.88240 | 17.705 | 0.86294 | 22.833 | 0.84739 | 28.689 |
| 60 | 0.91987 | 9.461  | 0.89607 | 13.127 | 0.87222 | 17.505 | 0.85001 | 22.631 | 0.83142 | 28.518 |
| 61 | 0.91433 | 9.303  | 0.88846 | 12.929 | 0.86221 | 17.278 | 0.83727 | 22.394 | 0.81564 | 28.303 |
| 62 | 0.90889 | 9.131  | 0.88100 | 12.713 | 0.85237 | 17.024 | 0.82471 | 22.122 | 0.80008 | 28.047 |
| 63 | 0.90357 | 8.945  | 0.87369 | 12.476 | 0.84271 | 16.743 | 0.81237 | 21.814 | 0.78475 | 27.745 |
| 64 | 0.89838 | 8.747  | 0.86653 | 12.220 | 0.83324 | 16.435 | 0.80025 | 21.471 | 0.76966 | 27.396 |
| 65 | 0.89332 | 8.536  | 0.85954 | 11.945 | 0.82398 | 16.100 | 0.78837 | 21.091 | 0.75484 | 26.999 |
| 66 | 0.88838 | 8.312  | 0.85272 | 11.652 | 0.81494 | 15.738 | 0.77675 | 20.673 | 0.74029 | 26.555 |
| 67 | 0.88358 | 8.076  | 0.84609 | 11.339 | 0.80613 | 15.349 | 0.76540 | 20.218 | 0.72603 | 26.061 |
| 68 | 0.87894 | 7.827  | 0.83966 | 11.007 | 0.79757 | 14.931 | 0.75435 | 19.722 | 0.71212 | 25.513 |
| 69 | 0.87445 | 7.565  | 0.83344 | 10.656 | 0.78927 | 14.486 | 0.74362 | 19.188 | 0.69857 | 24.911 |
| 70 | 0.87012 | 7.292  | 0.82744 | 10.287 | 0.78125 | 14.014 | 0.73322 | 18.615 | 0.68539 | 24.254 |
| 71 | 0.86596 | 7.007  | 0.82166 | 9.900  | 0.77352 | 13.515 | 0.72317 | 18.001 | 0.67261 | 23.541 |
| 72 | 0.86197 | 6.711  | 0.81611 | 9.496  | 0.76610 | 12.990 | 0.71347 | 17.348 | 0.66026 | 22.772 |
| 73 | 0.85816 | 6.404  | 0.81081 | 9.075  | 0.75899 | 12.438 | 0.70416 | 16.656 | 0.64836 | 21.946 |
| 74 | 0.85454 | 6.086  | 0.80576 | 8.637  | 0.75220 | 11.860 | 0.69527 | 15.926 | 0.63692 | 21.061 |
| 75 | 0.85111 | 5.758  | 0.80097 | 8.183  | 0.74575 | 11.257 | 0.68681 | 15.156 | 0.62599 | 20.115 |
| 76 | 0.84787 | 5.420  | 0.79646 | 7.713  | 0.73965 | 10.630 | 0.67878 | 14.347 | 0.61560 | 19.111 |
| 77 | 0.84484 | 5.073  | 0.79222 | 7.228  | 0.73392 | 9.979  | 0.67121 | 13.502 | 0.60577 | 18.049 |
| 78 | 0.84201 | 4.718  | 0.78826 | 6.729  | 0.72856 | 9.306  | 0.66412 | 12.621 | 0.59652 | 16.929 |
| 79 | 0.83939 | 4.355  | 0.78459 | 6.217  | 0.72359 | 8.612  | 0.65753 | 11.706 | 0.58790 | 15.753 |
| 80 | 0.83698 | 3.984  | 0.78121 | 5.694  | 0.71901 | 7.897  | 0.65145 | 10.755 | 0.57991 | 14.521 |
| 81 | 0.83479 | 3.606  | 0.77814 | 5.159  | 0.71484 | 7.164  | 0.64589 | 9.776  | 0.57259 | 13.237 |
| 82 | 0.83282 | 3.221  | 0.77538 | 4.613  | 0.71108 | 6.413  | 0.64087 | 8.768  | 0.56596 | 11.904 |
| 83 | 0.83108 | 2.831  | 0.77293 | 4.057  | 0.70774 | 5.647  | 0.63641 | 7.733  | 0.56005 | 10.526 |
| 84 | 0.82957 | 2.436  | 0.77079 | 3.493  | 0.70483 | 4.867  | 0.63252 | 6.674  | 0.55487 | 9.105  |
| 85 | 0.82828 | 2.037  | 0.76898 | 2.922  | 0.70236 | 4.075  | 0.62921 | 5.595  | 0.55046 | 7.647  |
| 86 | 0.82722 | 1.634  | 0.76750 | 2.345  | 0.70033 | 3.272  | 0.62648 | 4.498  | 0.54683 | 6.157  |
| 87 | 0.82640 | 1.228  | 0.76634 | 1.763  | 0.69875 | 2.462  | 0.62435 | 3.386  | 0.54399 | 4.642  |
| 88 | 0.82581 | 0.820  | 0.76551 | 1.177  | 0.69761 | 1.645  | 0.62283 | 2.263  | 0.54195 | 3.106  |
| 89 | 0.82546 | 0.410  | 0.76501 | 0.589  | 0.69693 | 0.823  | 0.62192 | 1.133  | 0.54072 | 1.556  |
| 90 | 0.82534 | 0.000  | 0.76484 | 0.000  | 0.69671 | 0.000  | 0.62161 | 0.000  | 0.54030 | 0.000  |

Examples.  $\cosh(0.7/71^\circ) = 0.82166/9^\circ.900 = 0.82166/9^\circ.54'.00''$ .  
 $\cosh^{-1}(0.69857/24^\circ.911) = 1.0/60^\circ$ .

TABLE II. HYPERBOLIC COSINES.  $\cosh (\rho / \delta) = r / \gamma$ . CONTINUED

|    | 1.1     |        | 1.2     |        | 1.3     |        | 1.4     |        | 1.5     |        |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| °  | °       | °      | °       | °      | °       | °      | °       | °      | °       | °      |
| 45 | 1.1157  | 32.686 | 1.1608  | 38.076 | 1.2163  | 43.570 | 1.2830  | 49.084 | 1.3616  | 54.550 |
| 46 | 1.0959  | 33.067 | 1.1376  | 38.582 | 1.1897  | 44.210 | 1.2529  | 49.864 | 1.3279  | 55.471 |
| 47 | 1.0759  | 33.424 | 1.1143  | 39.063 | 1.1630  | 44.827 | 1.2227  | 50.625 | 1.2941  | 56.378 |
| 48 | 1.0559  | 33.754 | 1.0910  | 39.517 | 1.1363  | 45.421 | 1.1926  | 51.365 | 1.2604  | 57.266 |
| 49 | 1.0359  | 34.056 | 1.0676  | 39.943 | 1.1095  | 45.989 | 1.1624  | 52.083 | 1.2267  | 58.134 |
| 50 | 1.0158  | 34.328 | 1.0443  | 40.341 | 1.0828  | 46.529 | 1.1322  | 52.777 | 1.1931  | 58.982 |
| 51 | 0.99578 | 34.570 | 1.0209  | 40.709 | 1.0561  | 47.041 | 1.1021  | 53.446 | 1.1596  | 59.810 |
| 52 | 0.97577 | 34.780 | 0.99757 | 41.045 | 1.0294  | 47.525 | 1.0720  | 54.090 | 1.1262  | 60.618 |
| 53 | 0.95580 | 34.957 | 0.97428 | 41.349 | 1.0027  | 47.978 | 1.0421  | 54.708 | 1.0929  | 61.407 |
| 54 | 0.93589 | 35.100 | 0.95104 | 41.618 | 0.97613 | 48.400 | 1.0122  | 55.299 | 1.0598  | 62.175 |
| 55 | 0.91605 | 35.206 | 0.92788 | 41.850 | 0.94964 | 48.788 | 0.98242 | 55.862 | 1.0268  | 62.921 |
| 56 | 0.89630 | 35.275 | 0.90481 | 42.045 | 0.92325 | 49.141 | 0.95277 | 56.396 | 0.99406 | 63.645 |
| 57 | 0.87666 | 35.305 | 0.88186 | 42.201 | 0.89697 | 49.458 | 0.92327 | 56.900 | 0.96149 | 64.347 |
| 58 | 0.85716 | 35.293 | 0.85902 | 42.314 | 0.87082 | 49.736 | 0.89393 | 57.372 | 0.92912 | 65.025 |
| 59 | 0.83781 | 35.238 | 0.83633 | 42.383 | 0.84482 | 49.973 | 0.86474 | 57.811 | 0.89696 | 65.681 |
| 60 | 0.81862 | 35.139 | 0.81379 | 42.406 | 0.81898 | 50.168 | 0.83573 | 58.214 | 0.86502 | 66.313 |
| 61 | 0.79961 | 34.992 | 0.79144 | 42.379 | 0.79331 | 50.315 | 0.80692 | 58.578 | 0.83332 | 66.919 |
| 62 | 0.78081 | 34.794 | 0.76929 | 42.299 | 0.76783 | 50.412 | 0.77831 | 58.902 | 0.80186 | 67.497 |
| 63 | 0.76223 | 34.545 | 0.74735 | 42.164 | 0.74256 | 50.457 | 0.74991 | 59.185 | 0.77065 | 68.048 |
| 64 | 0.74390 | 34.242 | 0.72505 | 41.970 | 0.71752 | 50.446 | 0.72173 | 59.421 | 0.73969 | 68.571 |
| 65 | 0.72584 | 33.881 | 0.70420 | 41.713 | 0.69271 | 50.373 | 0.69377 | 59.608 | 0.70899 | 69.064 |
| 66 | 0.70808 | 33.459 | 0.68304 | 41.389 | 0.66815 | 50.235 | 0.66605 | 59.742 | 0.67855 | 69.524 |
| 67 | 0.69063 | 32.974 | 0.66218 | 40.993 | 0.64385 | 50.026 | 0.63858 | 59.817 | 0.64836 | 69.950 |
| 68 | 0.67352 | 32.423 | 0.64164 | 40.520 | 0.61985 | 49.739 | 0.61136 | 59.828 | 0.61844 | 70.340 |
| 69 | 0.65677 | 31.804 | 0.62146 | 39.966 | 0.59618 | 49.368 | 0.58441 | 59.769 | 0.58877 | 70.689 |
| 70 | 0.64043 | 31.112 | 0.60166 | 39.324 | 0.57283 | 48.905 | 0.55775 | 59.633 | 0.55937 | 70.995 |
| 71 | 0.62452 | 30.344 | 0.58227 | 38.589 | 0.54984 | 48.342 | 0.53137 | 59.410 | 0.53022 | 71.253 |
| 72 | 0.60907 | 29.498 | 0.56334 | 37.754 | 0.52724 | 47.668 | 0.50529 | 59.090 | 0.50134 | 71.458 |
| 73 | 0.59410 | 28.570 | 0.54489 | 36.812 | 0.50506 | 46.870 | 0.47955 | 58.659 | 0.47271 | 71.602 |
| 74 | 0.57966 | 27.558 | 0.52697 | 35.756 | 0.48335 | 45.937 | 0.45414 | 58.104 | 0.44434 | 71.677 |
| 75 | 0.56579 | 26.458 | 0.50963 | 34.578 | 0.46215 | 44.856 | 0.42910 | 57.405 | 0.41622 | 71.672 |
| 76 | 0.55252 | 25.269 | 0.49292 | 33.271 | 0.44151 | 43.610 | 0.40446 | 56.542 | 0.38836 | 71.573 |
| 77 | 0.53990 | 23.989 | 0.47688 | 31.828 | 0.42149 | 42.180 | 0.38028 | 55.487 | 0.36075 | 71.361 |
| 78 | 0.52796 | 22.616 | 0.46159 | 30.242 | 0.40216 | 40.548 | 0.35658 | 54.206 | 0.33339 | 71.015 |
| 79 | 0.51675 | 21.151 | 0.44710 | 28.507 | 0.38360 | 38.692 | 0.33345 | 52.659 | 0.30630 | 70.501 |
| 80 | 0.50631 | 19.594 | 0.43349 | 26.616 | 0.36591 | 36.591 | 0.31099 | 50.800 | 0.27951 | 69.774 |
| 81 | 0.49669 | 17.946 | 0.42083 | 24.567 | 0.34921 | 34.223 | 0.28928 | 48.566 | 0.25304 | 68.774 |
| 82 | 0.48794 | 16.210 | 0.40919 | 22.359 | 0.33362 | 31.569 | 0.26848 | 45.888 | 0.22691 | 67.413 |
| 83 | 0.48009 | 14.391 | 0.39867 | 19.993 | 0.31928 | 28.608 | 0.24877 | 42.681 | 0.20122 | 65.561 |
| 84 | 0.47320 | 12.495 | 0.38935 | 17.475 | 0.30635 | 25.333 | 0.23039 | 38.853 | 0.17608 | 63.018 |
| 85 | 0.46730 | 10.529 | 0.38129 | 14.813 | 0.29499 | 21.739 | 0.21368 | 34.303 | 0.15169 | 59.474 |
| 86 | 0.46241 | 8.501  | 0.37457 | 12.022 | 0.28538 | 17.838 | 0.19902 | 28.949 | 0.12853 | 54.420 |
| 87 | 0.45857 | 6.422  | 0.36927 | 9.121  | 0.27770 | 13.659 | 0.18684 | 22.746 | 0.10710 | 47.029 |
| 88 | 0.45581 | 4.304  | 0.36545 | 6.132  | 0.27208 | 9.247  | 0.17765 | 15.729 | 0.08871 | 36.059 |
| 89 | 0.45415 | 2.159  | 0.36314 | 3.081  | 0.26865 | 4.668  | 0.17193 | 8.051  | 0.07570 | 20.198 |
| 90 | 0.45360 | 0.000  | 0.36236 | 0.000  | 0.26750 | 0.000  | 0.16997 | 0.000  | 0.07074 | 0.000  |

Examples.  $\cosh (1.3 / 73^\circ) = 0.50506 / 46^\circ.870 = 0.50506 / 46^\circ.52'.12''$ .  
 $\cosh^{-1} (0.07074 / 0^\circ) = 1.5 / 90^\circ$ .

TABLE II. HYPERBOLIC COSINES.  $\cosh(p/\delta) = r/\gamma$ . CONTINUED

|    | 1.6     |         | 1.7     |         | 1.8     |         | 1.9     |         | 2.0     |         |
|----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| °  | °       | °       | °       | °       | °       | °       | °       | °       | °       | °       |
| 45 | 1.4524  | 59.916  | 1.5556  | 65.149  | 1.6714  | 70.229  | 1.7999  | 75.152  | 1.9413  | 79.922  |
| 46 | 1.4149  | 60.974  | 1.5141  | 66.336  | 1.6257  | 71.536  | 1.7496  | 76.569  | 1.8861  | 81.437  |
| 47 | 1.3774  | 62.021  | 1.4727  | 67.516  | 1.5802  | 72.837  | 1.6997  | 77.979  | 1.8313  | 82.947  |
| 48 | 1.3400  | 63.051  | 1.4316  | 68.681  | 1.5350  | 74.125  | 1.6502  | 79.378  | 1.7771  | 84.445  |
| 49 | 1.3028  | 64.066  | 1.3906  | 69.833  | 1.4901  | 75.403  | 1.6012  | 80.768  | 1.7235  | 85.935  |
| 50 | 1.2657  | 65.065  | 1.3499  | 70.973  | 1.4456  | 76.671  | 1.5526  | 82.149  | 1.6706  | 87.417  |
| 51 | 1.2287  | 66.049  | 1.3095  | 72.100  | 1.4015  | 77.929  | 1.5046  | 83.522  | 1.6184  | 88.890  |
| 52 | 1.1919  | 67.018  | 1.2693  | 73.217  | 1.3579  | 79.179  | 1.4572  | 84.890  | 1.5669  | 90.356  |
| 53 | 1.1554  | 67.971  | 1.2295  | 74.324  | 1.3147  | 80.422  | 1.4104  | 86.252  | 1.5162  | 91.817  |
| 54 | 1.1191  | 68.909  | 1.1900  | 75.421  | 1.2719  | 81.659  | 1.3643  | 87.609  | 1.4664  | 93.275  |
| 55 | 1.0831  | 69.833  | 1.1509  | 76.508  | 1.2297  | 82.892  | 1.3188  | 88.963  | 1.4174  | 94.731  |
| 56 | 1.0473  | 70.744  | 1.1121  | 77.589  | 1.1879  | 84.122  | 1.2740  | 90.318  | 1.3693  | 96.188  |
| 57 | 1.0118  | 71.640  | 1.0737  | 78.663  | 1.1467  | 85.350  | 1.2299  | 91.674  | 1.3221  | 97.646  |
| 58 | 0.97653 | 72.522  | 1.0358  | 79.730  | 1.1061  | 86.576  | 1.1865  | 93.032  | 1.2758  | 99.108  |
| 59 | 0.94160 | 73.392  | 0.99826 | 80.795  | 1.0661  | 87.808  | 1.1439  | 94.398  | 1.2305  | 100.577 |
| 60 | 0.90699 | 74.249  | 0.96117 | 81.859  | 1.0266  | 89.045  | 1.1020  | 95.773  | 1.1861  | 102.057 |
| 61 | 0.87268 | 75.094  | 0.92451 | 82.919  | 0.98772 | 90.287  | 1.0609  | 97.158  | 1.1427  | 103.548 |
| 62 | 0.83871 | 75.926  | 0.88831 | 83.982  | 0.94946 | 91.540  | 1.0207  | 98.559  | 1.1003  | 105.054 |
| 63 | 0.80508 | 76.748  | 0.85256 | 85.048  | 0.91182 | 92.807  | 0.98119 | 99.978  | 1.0589  | 106.580 |
| 64 | 0.77177 | 77.561  | 0.81727 | 86.121  | 0.87480 | 94.092  | 0.94254 | 101.420 | 1.0186  | 108.129 |
| 65 | 0.73879 | 78.365  | 0.78245 | 87.204  | 0.83842 | 95.400  | 0.90472 | 102.891 | 0.97928 | 109.707 |
| 66 | 0.70615 | 79.161  | 0.74810 | 88.300  | 0.80268 | 96.734  | 0.86773 | 104.394 | 0.94100 | 111.318 |
| 67 | 0.67383 | 79.951  | 0.71420 | 89.415  | 0.76759 | 98.101  | 0.83160 | 105.937 | 0.90378 | 112.968 |
| 68 | 0.64184 | 80.736  | 0.68078 | 90.551  | 0.73314 | 99.508  | 0.79631 | 107.527 | 0.86764 | 114.663 |
| 69 | 0.61019 | 81.519  | 0.64782 | 91.717  | 0.69935 | 100.962 | 0.76190 | 109.170 | 0.83259 | 116.410 |
| 70 | 0.57887 | 82.300  | 0.61533 | 92.917  | 0.66624 | 102.473 | 0.72838 | 110.875 | 0.79865 | 118.215 |
| 71 | 0.54786 | 83.084  | 0.58331 | 94.161  | 0.63379 | 104.050 | 0.69576 | 112.653 | 0.76583 | 120.087 |
| 72 | 0.51715 | 83.872  | 0.55175 | 95.458  | 0.60202 | 105.706 | 0.66406 | 114.514 | 0.73414 | 122.035 |
| 73 | 0.48674 | 84.668  | 0.52065 | 96.819  | 0.57095 | 107.453 | 0.63320 | 116.471 | 0.70361 | 124.068 |
| 74 | 0.45663 | 85.477  | 0.49003 | 98.258  | 0.54061 | 109.308 | 0.60349 | 118.537 | 0.67428 | 126.197 |
| 75 | 0.42680 | 86.304  | 0.45988 | 99.793  | 0.51100 | 111.291 | 0.57470 | 120.726 | 0.64618 | 128.432 |
| 76 | 0.39724 | 87.156  | 0.43021 | 101.445 | 0.48217 | 113.423 | 0.54696 | 123.058 | 0.61934 | 130.785 |
| 77 | 0.36794 | 88.041  | 0.40104 | 103.241 | 0.45415 | 115.732 | 0.52030 | 125.550 | 0.59381 | 133.267 |
| 78 | 0.33888 | 88.972  | 0.37238 | 105.215 | 0.42701 | 118.248 | 0.49480 | 128.224 | 0.56965 | 135.890 |
| 79 | 0.31006 | 89.963  | 0.34428 | 107.409 | 0.40082 | 121.008 | 0.47055 | 131.100 | 0.54691 | 138.664 |
| 80 | 0.28146 | 91.036  | 0.31679 | 109.880 | 0.37567 | 124.055 | 0.44764 | 134.201 | 0.52568 | 141.601 |
| 81 | 0.25309 | 92.221  | 0.28997 | 112.701 | 0.35167 | 127.438 | 0.42617 | 137.553 | 0.50603 | 144.713 |
| 82 | 0.22494 | 93.561  | 0.26395 | 115.971 | 0.32897 | 131.210 | 0.40626 | 141.172 | 0.48805 | 148.003 |
| 83 | 0.19700 | 95.128  | 0.23887 | 119.816 | 0.30779 | 135.433 | 0.38806 | 145.082 | 0.47184 | 151.474 |
| 84 | 0.16926 | 97.033  | 0.21497 | 124.413 | 0.28838 | 140.167 | 0.37174 | 149.292 | 0.45750 | 155.127 |
| 85 | 0.14177 | 99.473  | 0.19261 | 129.984 | 0.27102 | 145.463 | 0.35749 | 153.806 | 0.44514 | 158.954 |
| 86 | 0.11467 | 102.829 | 0.17231 | 136.806 | 0.25603 | 151.356 | 0.34550 | 158.614 | 0.43486 | 162.943 |
| 87 | 0.08809 | 107.920 | 0.15476 | 145.176 | 0.24378 | 157.847 | 0.33593 | 163.691 | 0.42674 | 167.072 |
| 88 | 0.06261 | 116.850 | 0.14093 | 155.306 | 0.23469 | 164.877 | 0.32895 | 168.991 | 0.42088 | 171.315 |
| 89 | 0.04025 | 136.057 | 0.13195 | 167.116 | 0.22911 | 172.324 | 0.32472 | 174.453 | 0.41734 | 175.637 |
| 90 | 0.02920 | 180.000 | 0.12884 | 180.000 | 0.22720 | 180.000 | 0.32329 | 180.000 | 0.41615 | 180.000 |

Examples.  $\cosh(2.0/90^\circ) = 0.41615/180^\circ$ .  
 $\cosh^{-1}(0.54691/138^\circ.664) = 2.0/79^\circ$ .

TABLE II. HYPERBOLIC COSINES.  $\cosh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 2.1     |         | 2.2     |         | 2.3     |         | 2.4     |         | 2.5     |         |
|----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| °  | °       | °       | °       | °       | °       | °       | °       | °       | °       | °       |
| 45 | 2.0958  | 84.551  | 2.2636  | 89.050  | 2.4449  | 93.438  | 2.6403  | 97.730  | 2.8502  | 101.944 |
| 46 | 2.0350  | 86.159  | 2.1966  | 90.740  | 2.3711  | 95.202  | 2.5589  | 99.504  | 2.7603  | 103.842 |
| 47 | 1.9749  | 87.755  | 2.1306  | 92.417  | 2.2984  | 96.952  | 2.4788  | 101.380 | 2.6720  | 105.720 |
| 48 | 1.9155  | 89.341  | 2.0655  | 94.081  | 2.2269  | 98.687  | 2.4001  | 103.179 | 2.5853  | 107.578 |
| 49 | 1.8509  | 90.917  | 2.0013  | 95.734  | 2.1567  | 100.408 | 2.3229  | 104.962 | 2.5004  | 109.417 |
| 50 | 1.7992  | 92.484  | 1.9382  | 97.376  | 2.0877  | 102.116 | 2.2473  | 106.729 | 2.4173  | 111.238 |
| 51 | 1.7424  | 94.043  | 1.8763  | 99.009  | 2.0200  | 103.812 | 2.1732  | 108.482 | 2.3361  | 113.041 |
| 52 | 1.6865  | 95.594  | 1.8155  | 100.633 | 1.9537  | 105.498 | 2.1008  | 110.220 | 2.2568  | 114.826 |
| 53 | 1.6316  | 97.140  | 1.7559  | 102.249 | 1.8889  | 107.173 | 2.0301  | 111.944 | 2.1795  | 116.593 |
| 54 | 1.5777  | 98.681  | 1.6976  | 103.858 | 1.8255  | 108.838 | 1.9612  | 113.656 | 2.1042  | 118.344 |
| 55 | 1.5249  | 100.220 | 1.6406  | 105.462 | 1.7637  | 110.495 | 1.8940  | 115.356 | 2.0310  | 120.079 |
| 56 | 1.4732  | 101.758 | 1.5848  | 107.063 | 1.7034  | 112.145 | 1.8286  | 117.045 | 1.9599  | 121.799 |
| 57 | 1.4226  | 103.296 | 1.5303  | 108.662 | 1.6447  | 113.789 | 1.7651  | 118.724 | 1.8909  | 123.504 |
| 58 | 1.3731  | 104.836 | 1.4772  | 110.261 | 1.5876  | 115.430 | 1.7035  | 120.394 | 1.8241  | 125.195 |
| 59 | 1.3248  | 106.382 | 1.4256  | 111.862 | 1.5322  | 117.069 | 1.6438  | 122.057 | 1.7594  | 126.874 |
| 60 | 1.2776  | 107.936 | 1.3754  | 113.467 | 1.4784  | 118.708 | 1.5859  | 123.715 | 1.6969  | 128.541 |
| 61 | 1.2317  | 109.501 | 1.3266  | 115.079 | 1.4262  | 120.347 | 1.5299  | 125.369 | 1.6366  | 130.198 |
| 62 | 1.1870  | 111.079 | 1.2792  | 116.700 | 1.3758  | 121.991 | 1.4758  | 127.020 | 1.5785  | 131.846 |
| 63 | 1.1435  | 112.673 | 1.2332  | 118.333 | 1.3270  | 123.641 | 1.4237  | 128.671 | 1.5226  | 133.486 |
| 64 | 1.1012  | 114.288 | 1.1887  | 119.982 | 1.2799  | 125.300 | 1.3736  | 130.322 | 1.4689  | 135.120 |
| 65 | 1.0602  | 115.928 | 1.1457  | 121.650 | 1.2345  | 126.970 | 1.3254  | 131.977 | 1.4173  | 136.749 |
| 66 | 1.0204  | 117.597 | 1.1042  | 123.339 | 1.1908  | 128.654 | 1.2791  | 133.638 | 1.3679  | 138.374 |
| 67 | 0.98193 | 119.299 | 1.0642  | 125.055 | 1.1488  | 130.355 | 1.2347  | 135.306 | 1.3207  | 139.998 |
| 68 | 0.94474 | 121.041 | 1.0256  | 126.801 | 1.1086  | 132.077 | 1.1923  | 136.985 | 1.2756  | 141.622 |
| 69 | 0.90886 | 122.827 | 0.98860 | 128.581 | 1.0700  | 133.823 | 1.1518  | 138.676 | 1.2327  | 143.247 |
| 70 | 0.87429 | 124.662 | 0.95308 | 130.400 | 1.0332  | 135.595 | 1.1132  | 140.382 | 1.1919  | 144.876 |
| 71 | 0.84105 | 126.554 | 0.91908 | 132.263 | 0.99804 | 137.397 | 1.0765  | 142.106 | 1.1532  | 146.511 |
| 72 | 0.80915 | 128.509 | 0.88663 | 134.174 | 0.96464 | 139.233 | 1.0417  | 143.850 | 1.1166  | 148.153 |
| 73 | 0.77861 | 130.533 | 0.85573 | 136.137 | 0.93297 | 141.106 | 1.0088  | 145.616 | 1.0821  | 149.803 |
| 74 | 0.74945 | 132.634 | 0.82638 | 138.158 | 0.90303 | 143.019 | 0.97785 | 147.407 | 1.0497  | 151.464 |
| 75 | 0.72171 | 134.818 | 0.79862 | 140.242 | 0.87481 | 144.976 | 0.94877 | 149.225 | 1.0193  | 153.137 |
| 76 | 0.69541 | 137.094 | 0.77246 | 142.393 | 0.84833 | 146.980 | 0.92156 | 151.071 | 0.99093 | 154.823 |
| 77 | 0.67060 | 139.468 | 0.74789 | 144.615 | 0.82360 | 149.033 | 0.89622 | 152.948 | 0.96459 | 156.524 |
| 78 | 0.64732 | 141.946 | 0.72497 | 146.911 | 0.80062 | 151.136 | 0.87276 | 154.857 | 0.94026 | 158.241 |
| 79 | 0.62558 | 144.534 | 0.70372 | 149.285 | 0.77940 | 153.291 | 0.85117 | 156.798 | 0.91793 | 159.974 |
| 80 | 0.60544 | 147.237 | 0.68416 | 151.738 | 0.75996 | 155.499 | 0.83145 | 158.771 | 0.89757 | 161.723 |
| 81 | 0.58697 | 150.058 | 0.66632 | 154.271 | 0.74231 | 157.760 | 0.81361 | 160.778 | 0.87918 | 163.490 |
| 82 | 0.57021 | 152.997 | 0.65024 | 156.883 | 0.72646 | 160.074 | 0.79764 | 162.817 | 0.86276 | 165.274 |
| 83 | 0.55523 | 156.054 | 0.63595 | 159.572 | 0.71244 | 162.438 | 0.78354 | 164.888 | 0.84829 | 167.074 |
| 84 | 0.54209 | 159.225 | 0.62349 | 162.334 | 0.70025 | 164.848 | 0.77131 | 166.987 | 0.83575 | 168.889 |
| 85 | 0.53085 | 162.503 | 0.61287 | 165.164 | 0.68990 | 167.301 | 0.76095 | 169.112 | 0.82518 | 170.717 |
| 86 | 0.52156 | 165.876 | 0.60413 | 168.054 | 0.68141 | 169.793 | 0.75247 | 171.260 | 0.81650 | 172.558 |
| 87 | 0.51428 | 169.329 | 0.59731 | 170.994 | 0.67479 | 172.316 | 0.74587 | 173.427 | 0.80979 | 174.409 |
| 88 | 0.50905 | 172.849 | 0.59242 | 173.973 | 0.67006 | 174.863 | 0.74116 | 175.610 | 0.80499 | 176.269 |
| 89 | 0.50590 | 176.413 | 0.58948 | 176.980 | 0.66722 | 177.427 | 0.73834 | 177.803 | 0.80207 | 178.134 |
| 90 | 0.50485 | 180.000 | 0.58850 | 180.000 | 0.66628 | 180.000 | 0.73740 | 180.000 | 0.80114 | 180.000 |

Examples.  $\cosh(2.2/45^\circ) = 2.2636/89.050 = 2.2636/89.03'.00''$ .  
 $\cosh^{-1}(1.0821/149.803) = 2.5/73^\circ$ .

TABLE II. HYPERBOLIC COSINES.  $\cosh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 2.6     |         | 2.7     |         | 2.8     |         | 2.9     |         | 3.0     |         |
|----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| °  | °       | °       | °       | °       | °       | °       | °       | °       | °       | °       |
| 45 | 3.0753  | 106.093 | 3.3163  | 110.190 | 3.5741  | 114.248 | 3.8497  | 118.275 | 4.1443  | 122.282 |
| 46 | 2.9758  | 108.051 | 3.2062  | 112.207 | 3.4523  | 116.322 | 3.7146  | 120.406 | 3.9945  | 124.469 |
| 47 | 2.8783  | 109.087 | 3.0984  | 114.199 | 3.3329  | 118.368 | 3.5826  | 122.506 | 3.8483  | 126.623 |
| 48 | 2.7827  | 111.901 | 2.9928  | 116.166 | 3.2162  | 120.337 | 3.4536  | 124.575 | 3.7057  | 128.743 |
| 49 | 2.6891  | 113.793 | 2.8896  | 118.108 | 3.1023  | 122.378 | 3.3278  | 126.614 | 3.5667  | 130.828 |
| 50 | 2.5977  | 115.663 | 2.7880  | 120.025 | 2.9913  | 124.340 | 3.2053  | 128.621 | 3.4315  | 132.879 |
| 51 | 2.5085  | 117.512 | 2.6908  | 121.917 | 2.8832  | 126.273 | 3.0861  | 130.595 | 3.3001  | 134.894 |
| 52 | 2.4216  | 119.340 | 2.5952  | 123.784 | 2.7780  | 128.177 | 2.9703  | 132.536 | 3.1726  | 136.873 |
| 53 | 2.3369  | 121.147 | 2.5023  | 125.626 | 2.6759  | 130.052 | 2.8580  | 134.444 | 3.0490  | 138.815 |
| 54 | 2.2545  | 122.931 | 2.4120  | 127.442 | 2.5769  | 131.898 | 2.7492  | 136.320 | 2.9294  | 140.720 |
| 55 | 2.1745  | 124.695 | 2.3245  | 129.233 | 2.4809  | 133.715 | 2.6439  | 138.162 | 2.8137  | 142.587 |
| 56 | 2.0970  | 126.440 | 2.2397  | 131.000 | 2.3880  | 135.502 | 2.5421  | 139.969 | 2.7020  | 144.415 |
| 57 | 2.0219  | 128.165 | 2.1577  | 132.742 | 2.2983  | 137.259 | 2.4439  | 141.742 | 2.5943  | 146.203 |
| 58 | 1.9492  | 129.872 | 2.0785  | 134.460 | 2.2118  | 138.987 | 2.3492  | 143.479 | 2.4905  | 147.951 |
| 59 | 1.8790  | 131.560 | 2.0021  | 136.154 | 2.1284  | 140.685 | 2.2580  | 145.180 | 2.3907  | 149.659 |
| 60 | 1.8113  | 133.231 | 1.9284  | 137.824 | 2.0481  | 142.354 | 2.1703  | 146.846 | 2.2949  | 151.325 |
| 61 | 1.7460  | 134.885 | 1.8575  | 139.471 | 1.9709  | 143.993 | 2.0861  | 148.476 | 2.2030  | 152.949 |
| 62 | 1.6832  | 136.523 | 1.7894  | 141.095 | 1.8968  | 145.601 | 2.0053  | 150.071 | 2.1148  | 154.530 |
| 63 | 1.6229  | 138.146 | 1.7241  | 142.696 | 1.8259  | 147.179 | 1.9280  | 151.627 | 2.0304  | 156.067 |
| 64 | 1.5650  | 139.754 | 1.6615  | 144.276 | 1.7580  | 148.728 | 1.8541  | 153.146 | 1.9498  | 157.559 |
| 65 | 1.5096  | 141.349 | 1.6016  | 145.834 | 1.6931  | 150.247 | 1.7835  | 154.628 | 1.8729  | 159.005 |
| 66 | 1.4566  | 142.932 | 1.5445  | 147.371 | 1.6311  | 151.737 | 1.7162  | 156.071 | 1.7997  | 160.406 |
| 67 | 1.4060  | 144.504 | 1.4900  | 148.887 | 1.5721  | 153.198 | 1.6522  | 157.477 | 1.7301  | 161.759 |
| 68 | 1.3578  | 146.066 | 1.4382  | 150.384 | 1.5161  | 154.628 | 1.5914  | 158.844 | 1.6640  | 163.064 |
| 69 | 1.3120  | 147.620 | 1.3889  | 151.863 | 1.4629  | 156.030 | 1.5338  | 160.172 | 1.6013  | 164.321 |
| 70 | 1.2685  | 149.164 | 1.3422  | 153.322 | 1.4126  | 157.404 | 1.4793  | 161.460 | 1.5420  | 165.528 |
| 71 | 1.2273  | 150.705 | 1.2981  | 154.764 | 1.3650  | 158.749 | 1.4278  | 162.709 | 1.4861  | 166.685 |
| 72 | 1.1885  | 152.239 | 1.2565  | 156.189 | 1.3202  | 160.066 | 1.3793  | 163.919 | 1.4335  | 167.792 |
| 73 | 1.1519  | 153.770 | 1.2174  | 157.599 | 1.2782  | 161.355 | 1.3338  | 165.090 | 1.3840  | 168.848 |
| 74 | 1.1176  | 155.298 | 1.1807  | 158.994 | 1.2388  | 162.617 | 1.2912  | 166.222 | 1.3377  | 169.853 |
| 75 | 1.0855  | 156.825 | 1.1465  | 160.374 | 1.2020  | 163.853 | 1.2514  | 167.315 | 1.2946  | 170.806 |
| 76 | 1.0556  | 158.351 | 1.1147  | 161.742 | 1.1677  | 165.063 | 1.2144  | 168.370 | 1.2545  | 171.708 |
| 77 | 1.0278  | 159.878 | 1.0852  | 163.096 | 1.1360  | 166.248 | 1.1802  | 169.387 | 1.2174  | 172.560 |
| 78 | 1.0022  | 161.406 | 1.0580  | 164.440 | 1.1069  | 167.409 | 1.1486  | 170.368 | 1.1832  | 173.362 |
| 79 | 0.97880 | 162.937 | 1.0331  | 165.773 | 1.0803  | 168.547 | 1.1200  | 171.313 | 1.1519  | 174.116 |
| 80 | 0.95745 | 164.471 | 1.0105  | 167.096 | 1.0561  | 169.664 | 1.0938  | 172.225 | 1.1235  | 174.823 |
| 81 | 0.93821 | 166.008 | 0.99006 | 168.411 | 1.0342  | 170.760 | 1.0702  | 173.104 | 1.0979  | 175.485 |
| 82 | 0.92106 | 167.549 | 0.97189 | 169.718 | 1.0147  | 171.837 | 1.0492  | 173.953 | 1.0751  | 176.104 |
| 83 | 0.90596 | 169.094 | 0.95592 | 171.018 | 0.99765 | 172.897 | 1.0307  | 174.774 | 1.0550  | 176.685 |
| 84 | 0.89290 | 170.643 | 0.94211 | 172.312 | 0.98287 | 173.942 | 1.0148  | 175.571 | 1.0377  | 177.220 |
| 85 | 0.88187 | 172.196 | 0.93045 | 173.601 | 0.97041 | 174.973 | 1.0014  | 176.344 | 1.0231  | 177.742 |
| 86 | 0.87286 | 173.752 | 0.92093 | 174.886 | 0.96025 | 175.993 | 0.99042 | 177.099 | 1.0112  | 178.228 |
| 87 | 0.86587 | 175.311 | 0.91354 | 176.168 | 0.95235 | 177.003 | 0.98190 | 177.838 | 1.0019  | 178.691 |
| 88 | 0.86089 | 176.873 | 0.90827 | 177.447 | 0.94671 | 178.006 | 0.97582 | 178.566 | 0.99528 | 179.137 |
| 89 | 0.85789 | 178.436 | 0.90512 | 178.724 | 0.94334 | 179.004 | 0.97217 | 179.285 | 0.99130 | 179.571 |
| 90 | 0.85689 | 180.000 | 0.90407 | 180.000 | 0.94222 | 180.000 | 0.97096 | 180.000 | 0.98999 | 180.000 |

Examples.  $\cosh(2.8/85^\circ) = 0.97041/174^\circ.973 = 0.97041/174^\circ.58'.23''$ .  
 $\cosh^{-1}(1.5420/165^\circ.528) = 3.0/70^\circ$ .

TABLE III. HYPERBOLIC TANGENTS.  $\tanh (\rho / \delta) = r / \gamma$

|    | 0.1            | 0.2            | 0.3            | 0.4            | 0.5            |
|----|----------------|----------------|----------------|----------------|----------------|
| 45 | 0.10000 44.812 | 0.19997 44.235 | 0.29981 43.282 | 0.39921 41.954 | 0.49757 40.250 |
| 46 | 0.10001 45.812 | 0.20006 45.236 | 0.30012 44.281 | 0.39995 42.945 | 0.49902 41.229 |
| 47 | 0.10002 46.812 | 0.20015 46.237 | 0.30043 45.280 | 0.40069 43.942 | 0.50047 42.219 |
| 48 | 0.10003 47.812 | 0.20024 47.239 | 0.30075 46.283 | 0.40143 44.942 | 0.50192 43.213 |
| 49 | 0.10004 48.813 | 0.20034 48.242 | 0.30107 47.290 | 0.40217 45.947 | 0.50340 44.213 |
| 50 | 0.10006 49.813 | 0.20043 49.245 | 0.30138 48.297 | 0.40293 46.955 | 0.50490 45.220 |
| 51 | 0.10007 50.813 | 0.20053 50.251 | 0.30169 49.306 | 0.40370 47.960 | 0.50639 46.233 |
| 52 | 0.10008 51.814 | 0.20062 51.256 | 0.30201 50.315 | 0.40446 48.986 | 0.50789 47.252 |
| 53 | 0.10009 52.816 | 0.20071 52.262 | 0.30232 51.328 | 0.40521 50.005 | 0.50939 48.276 |
| 54 | 0.10010 53.818 | 0.20081 53.268 | 0.30263 52.344 | 0.40596 51.031 | 0.51089 49.308 |
| 55 | 0.10011 54.820 | 0.20090 54.276 | 0.30294 53.362 | 0.40671 52.058 | 0.51239 50.344 |
| 56 | 0.10013 55.823 | 0.20099 55.285 | 0.30325 54.382 | 0.40746 53.089 | 0.51389 51.388 |
| 57 | 0.10014 56.825 | 0.20107 56.295 | 0.30355 55.404 | 0.40820 54.124 | 0.51538 52.439 |
| 58 | 0.10015 57.828 | 0.20115 57.306 | 0.30385 56.426 | 0.40892 55.164 | 0.51687 53.494 |
| 59 | 0.10016 58.830 | 0.20124 58.318 | 0.30414 57.452 | 0.40964 56.207 | 0.51835 54.556 |
| 60 | 0.10017 59.833 | 0.20132 59.332 | 0.30444 58.479 | 0.41037 57.255 | 0.51983 55.625 |
| 61 | 0.10018 60.837 | 0.20140 60.345 | 0.30473 59.510 | 0.41107 58.305 | 0.52128 56.700 |
| 62 | 0.10019 61.841 | 0.20148 61.360 | 0.30501 60.542 | 0.41176 59.359 | 0.52271 57.781 |
| 63 | 0.10020 62.845 | 0.20156 62.375 | 0.30528 61.575 | 0.41244 60.417 | 0.52412 58.867 |
| 64 | 0.10020 63.849 | 0.20164 63.391 | 0.30555 62.610 | 0.41311 61.479 | 0.52552 59.959 |
| 65 | 0.10021 64.853 | 0.20171 64.408 | 0.30580 63.647 | 0.41376 62.544 | 0.52689 61.058 |
| 66 | 0.10022 65.857 | 0.20179 65.425 | 0.30605 64.686 | 0.41441 63.612 | 0.52824 62.164 |
| 67 | 0.10023 66.862 | 0.20186 66.443 | 0.30630 65.727 | 0.41504 64.685 | 0.52957 63.276 |
| 68 | 0.10024 67.866 | 0.20193 67.461 | 0.30655 66.769 | 0.41565 65.760 | 0.53085 64.391 |
| 69 | 0.10025 68.871 | 0.20199 68.480 | 0.30678 67.813 | 0.41623 66.837 | 0.53209 65.511 |
| 70 | 0.10026 69.877 | 0.20206 69.501 | 0.30701 68.859 | 0.41680 67.918 | 0.53330 66.638 |
| 71 | 0.10026 70.882 | 0.20212 70.522 | 0.30722 69.906 | 0.41735 69.002 | 0.53446 67.771 |
| 72 | 0.10027 71.888 | 0.20217 71.544 | 0.30742 70.955 | 0.41788 70.089 | 0.53560 68.909 |
| 73 | 0.10028 72.893 | 0.20223 72.566 | 0.30762 72.005 | 0.41838 71.179 | 0.53669 70.052 |
| 74 | 0.10028 73.898 | 0.20228 73.589 | 0.30781 73.056 | 0.41886 72.272 | 0.53773 71.200 |
| 75 | 0.10029 74.904 | 0.20234 74.612 | 0.30799 74.108 | 0.41932 73.368 | 0.53872 72.351 |
| 76 | 0.10029 75.910 | 0.20239 75.635 | 0.30816 75.162 | 0.41975 74.466 | 0.53965 73.507 |
| 77 | 0.10030 76.916 | 0.20243 76.659 | 0.30831 76.217 | 0.42016 75.566 | 0.54054 74.667 |
| 78 | 0.10030 77.922 | 0.20246 77.684 | 0.30846 77.273 | 0.42054 76.668 | 0.54138 75.831 |
| 79 | 0.10031 78.928 | 0.20250 78.709 | 0.30860 78.329 | 0.42088 77.771 | 0.54215 76.999 |
| 80 | 0.10031 79.934 | 0.20254 79.734 | 0.30872 79.386 | 0.42120 78.876 | 0.54285 78.170 |
| 81 | 0.10032 80.940 | 0.20257 80.759 | 0.30884 80.445 | 0.42150 79.983 | 0.54349 79.344 |
| 82 | 0.10032 81.946 | 0.20260 81.785 | 0.30894 81.506 | 0.42177 81.092 | 0.54407 80.520 |
| 83 | 0.10032 82.953 | 0.20263 82.812 | 0.30904 82.567 | 0.42201 82.203 | 0.54459 81.699 |
| 84 | 0.10033 83.960 | 0.20265 83.838 | 0.30912 83.628 | 0.42222 83.315 | 0.54503 82.881 |
| 85 | 0.10033 84.967 | 0.20267 84.864 | 0.30920 84.689 | 0.42239 84.427 | 0.54540 84.065 |
| 86 | 0.10033 85.974 | 0.20268 85.891 | 0.30926 85.751 | 0.42252 85.540 | 0.54571 85.251 |
| 87 | 0.10033 86.981 | 0.20270 86.918 | 0.30930 86.813 | 0.42264 86.654 | 0.54596 86.438 |
| 88 | 0.10033 87.988 | 0.20270 87.946 | 0.30933 87.875 | 0.42274 87.768 | 0.54616 87.626 |
| 89 | 0.10033 88.994 | 0.20271 88.973 | 0.30934 88.937 | 0.42279 88.883 | 0.54628 88.813 |
| 90 | 0.10033 90.000 | 0.20271 90.000 | 0.30934 90.000 | 0.42280 90.000 | 0.54631 90.000 |

Note.  $\tanh (0 / \delta) = 0 / \gamma$ .

Examples.  $\tanh (0.5 / 60^{\circ}) = 0.51983 / 55^{\circ}.625 = 0.51983 / 55^{\circ}.37'.30''$ .  
 $\tanh ^{-1}(0.54628 / 88^{\circ}.813) = 0.5 / 89^{\circ}$ .

TABLE III. HYPERBOLIC TANGENTS.  $\tanh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 0.6            | 0.7            | 0.8            | 0.9            | 1.0            |
|----|----------------|----------------|----------------|----------------|----------------|
| 45 | 0.59406 38.183 | 0.68732 35.786 | 0.77577 33.098 | 0.85756 30.161 | 0.93077 27.044 |
| 46 | 0.59650 39.146 | 0.69109 36.719 | 0.78117 33.980 | 0.86480 30.980 | 0.93909 27.784 |
| 47 | 0.59898 40.119 | 0.69495 37.663 | 0.78671 34.878 | 0.87229 31.815 | 0.94950 28.539 |
| 48 | 0.60149 41.099 | 0.69888 38.617 | 0.79240 35.790 | 0.88001 32.666 | 0.95938 29.308 |
| 49 | 0.60403 42.088 | 0.70287 39.581 | 0.79824 36.715 | 0.88799 33.531 | 0.96966 30.092 |
| 50 | 0.60660 43.085 | 0.70694 40.557 | 0.80421 37.653 | 0.89620 34.412 | 0.98032 30.892 |
| 51 | 0.60920 44.092 | 0.71107 41.545 | 0.81031 38.605 | 0.90466 35.310 | 0.99136 31.710 |
| 52 | 0.61182 45.107 | 0.71527 42.543 | 0.81655 39.573 | 0.91337 36.223 | 1.00282 32.545 |
| 53 | 0.61447 46.130 | 0.71952 43.555 | 0.82291 40.556 | 0.92231 37.155 | 1.01469 33.400 |
| 54 | 0.61713 47.162 | 0.72382 44.577 | 0.82940 41.554 | 0.93150 38.105 | 1.02697 34.274 |
| 55 | 0.61980 48.203 | 0.72817 45.612 | 0.83601 42.568 | 0.94094 39.075 | 1.03970 35.172 |
| 56 | 0.62248 49.254 | 0.73257 46.662 | 0.84274 43.599 | 0.95063 40.066 | 1.05287 36.091 |
| 57 | 0.62517 50.315 | 0.73700 47.725 | 0.84957 44.647 | 0.96056 41.078 | 1.06648 37.035 |
| 58 | 0.62785 51.384 | 0.74145 48.800 | 0.85649 45.712 | 0.97069 42.111 | 1.08054 38.004 |
| 59 | 0.63053 52.463 | 0.74593 49.888 | 0.86351 46.797 | 0.98106 43.167 | 1.09506 39.002 |
| 60 | 0.63322 53.552 | 0.75047 50.990 | 0.87063 47.900 | 0.99168 44.247 | 1.11009 40.026 |
| 61 | 0.63588 54.650 | 0.75499 52.107 | 0.87781 49.022 | 1.00251 45.353 | 1.12555 41.082 |
| 62 | 0.63855 55.758 | 0.75950 53.238 | 0.88504 50.165 | 1.01353 46.486 | 1.14144 42.168 |
| 63 | 0.64115 56.876 | 0.76400 54.383 | 0.89232 51.327 | 1.02471 47.645 | 1.15777 43.289 |
| 64 | 0.64376 58.002 | 0.76848 55.542 | 0.89965 52.509 | 1.03604 48.831 | 1.17454 44.445 |
| 65 | 0.64633 59.138 | 0.77294 56.716 | 0.90698 53.712 | 1.04753 50.044 | 1.19173 45.638 |
| 66 | 0.64886 60.284 | 0.77737 57.902 | 0.91433 54.936 | 1.05916 51.289 | 1.20935 46.869 |
| 67 | 0.65135 61.439 | 0.78177 59.105 | 0.92166 56.180 | 1.07090 52.562 | 1.22737 48.140 |
| 68 | 0.65380 62.603 | 0.78609 60.322 | 0.92894 57.448 | 1.08268 53.868 | 1.24569 49.455 |
| 69 | 0.65618 63.777 | 0.79033 61.553 | 0.93616 58.737 | 1.09447 55.204 | 1.26429 50.812 |
| 70 | 0.65850 64.959 | 0.79450 62.798 | 0.94332 60.047 | 1.10627 56.572 | 1.28316 52.215 |
| 71 | 0.66075 66.150 | 0.79858 64.057 | 0.95037 61.379 | 1.11803 57.974 | 1.30221 53.666 |
| 72 | 0.66294 67.350 | 0.80256 65.329 | 0.95727 62.732 | 1.12972 59.408 | 1.32140 55.164 |
| 73 | 0.66505 68.559 | 0.80641 66.614 | 0.96402 64.106 | 1.14126 60.874 | 1.34063 56.710 |
| 74 | 0.66708 69.775 | 0.81014 67.913 | 0.97060 65.501 | 1.15260 62.372 | 1.35986 58.307 |
| 75 | 0.66902 70.998 | 0.81371 69.225 | 0.97697 66.917 | 1.16370 63.903 | 1.37894 59.957 |
| 76 | 0.67086 72.228 | 0.81713 70.550 | 0.98311 68.352 | 1.17450 65.468 | 1.39775 61.658 |
| 77 | 0.67261 73.466 | 0.82038 71.886 | 0.98898 69.808 | 1.18493 67.064 | 1.41620 63.409 |
| 78 | 0.67425 74.711 | 0.82345 73.233 | 0.99455 71.282 | 1.19495 68.691 | 1.43412 65.212 |
| 79 | 0.67577 75.962 | 0.82632 74.591 | 0.99981 72.773 | 1.20447 70.347 | 1.45141 67.065 |
| 80 | 0.67718 77.219 | 0.82899 75.958 | 1.00473 74.282 | 1.21344 72.034 | 1.46790 68.968 |
| 81 | 0.67847 78.481 | 0.83144 77.334 | 1.00926 75.805 | 1.22179 73.746 | 1.48345 70.919 |
| 82 | 0.67964 79.749 | 0.83366 78.719 | 1.01339 77.344 | 1.22940 75.483 | 1.49790 72.913 |
| 83 | 0.68068 81.021 | 0.83564 80.112 | 1.01710 78.896 | 1.23640 77.243 | 1.51110 74.948 |
| 84 | 0.68159 82.296 | 0.83738 81.512 | 1.02036 80.460 | 1.24253 79.025 | 1.52289 77.023 |
| 85 | 0.68236 83.575 | 0.83887 82.917 | 1.02316 82.034 | 1.24781 80.825 | 1.53314 79.132 |
| 86 | 0.68299 84.856 | 0.84009 84.328 | 1.02548 83.617 | 1.25219 82.640 | 1.54170 81.269 |
| 87 | 0.68349 86.140 | 0.84105 85.743 | 1.02730 85.206 | 1.25566 84.468 | 1.54848 83.429 |
| 88 | 0.68385 87.426 | 0.84173 87.161 | 1.02860 86.801 | 1.25814 86.308 | 1.55339 85.609 |
| 89 | 0.68406 88.713 | 0.84214 88.580 | 1.02937 88.400 | 1.25963 88.153 | 1.55637 87.802 |
| 90 | 0.68413 90.000 | 0.84229 90.000 | 1.02960 90.000 | 1.26015 90.000 | 1.55740 90.000 |

Examples.  $\tanh(0.9/77^\circ) = 1.18493/67^\circ.064 = 1.18493/67^\circ.03'.50''$ .  
 $\tanh^{-1}(0.66708/60^\circ.775) = 0.6/74^\circ$ .



TABLE III. HYPERBOLIC TANGENTS.  $\tanh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | I.1     |        | I.2    |        | I.3    |        | I.4    |        | I.5    |        |
|----|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| °  | °       | °      | °      | °      | °      | °      | °      | °      | °      | °      |
| 45 | 0.99389 | 23.833 | 1.0457 | 20.616 | 1.0857 | 17.464 | 1.1143 | 14.484 | 1.1323 | 11.712 |
| 46 | 1.0049  | 24.476 | 1.0582 | 21.144 | 1.0993 | 17.882 | 1.1284 | 14.775 | 1.1464 | 11.884 |
| 47 | 1.0164  | 25.131 | 1.0713 | 21.685 | 1.1136 | 18.301 | 1.1433 | 15.064 | 1.1613 | 12.048 |
| 48 | 1.0283  | 25.799 | 1.0851 | 22.236 | 1.1286 | 18.721 | 1.1590 | 15.352 | 1.1770 | 12.208 |
| 49 | 1.0408  | 26.480 | 1.0995 | 22.798 | 1.1445 | 19.148 | 1.1756 | 15.640 | 1.1936 | 12.362 |
| 50 | 1.0539  | 27.178 | 1.1147 | 23.371 | 1.1613 | 19.584 | 1.1932 | 15.930 | 1.2112 | 12.510 |
| 51 | 1.0676  | 27.891 | 1.1307 | 23.957 | 1.1789 | 20.027 | 1.2118 | 16.222 | 1.2299 | 12.652 |
| 52 | 1.0818  | 28.621 | 1.1474 | 24.558 | 1.1976 | 20.479 | 1.2315 | 16.515 | 1.2498 | 12.787 |
| 53 | 1.0967  | 29.370 | 1.1650 | 25.174 | 1.2173 | 20.941 | 1.2524 | 16.811 | 1.2709 | 12.914 |
| 54 | 1.1122  | 30.139 | 1.1835 | 25.807 | 1.2381 | 21.414 | 1.2746 | 17.110 | 1.2933 | 13.035 |
| 55 | 1.1284  | 30.930 | 1.2030 | 26.460 | 1.2602 | 21.900 | 1.2982 | 17.413 | 1.3172 | 13.151 |
| 56 | 1.1453  | 31.744 | 1.2235 | 27.133 | 1.2836 | 22.401 | 1.3234 | 17.721 | 1.3427 | 13.262 |
| 57 | 1.1629  | 32.583 | 1.2450 | 27.827 | 1.3084 | 22.918 | 1.3502 | 18.036 | 1.3700 | 13.366 |
| 58 | 1.1813  | 33.449 | 1.2677 | 28.546 | 1.3346 | 23.452 | 1.3787 | 18.358 | 1.3992 | 13.466 |
| 59 | 1.2005  | 34.343 | 1.2916 | 29.292 | 1.3626 | 24.007 | 1.4093 | 18.689 | 1.4305 | 13.559 |
| 60 | 1.2206  | 35.267 | 1.3168 | 30.068 | 1.3923 | 24.584 | 1.4421 | 19.032 | 1.4642 | 13.651 |
| 61 | 1.2415  | 36.226 | 1.3434 | 30.876 | 1.4239 | 25.187 | 1.4772 | 19.389 | 1.5005 | 13.738 |
| 62 | 1.2632  | 37.222 | 1.3714 | 31.720 | 1.4576 | 25.820 | 1.5148 | 19.761 | 1.5396 | 13.824 |
| 63 | 1.2858  | 38.255 | 1.4009 | 32.603 | 1.4935 | 26.485 | 1.5553 | 20.150 | 1.5819 | 13.908 |
| 64 | 1.3094  | 39.328 | 1.4321 | 33.527 | 1.5319 | 27.186 | 1.5990 | 20.562 | 1.6277 | 13.991 |
| 65 | 1.3339  | 40.446 | 1.4649 | 34.498 | 1.5729 | 27.928 | 1.6463 | 20.999 | 1.6776 | 14.076 |
| 66 | 1.3593  | 41.612 | 1.4995 | 35.520 | 1.6167 | 28.715 | 1.6974 | 21.465 | 1.7319 | 14.166 |
| 67 | 1.3856  | 42.827 | 1.5359 | 36.597 | 1.6637 | 29.554 | 1.7528 | 21.966 | 1.7914 | 14.261 |
| 68 | 1.4129  | 44.096 | 1.5743 | 37.737 | 1.7140 | 30.452 | 1.8131 | 22.507 | 1.8566 | 14.364 |
| 69 | 1.4411  | 45.421 | 1.6148 | 38.941 | 1.7680 | 31.415 | 1.8788 | 23.096 | 1.9285 | 14.480 |
| 70 | 1.4702  | 46.806 | 1.6573 | 40.219 | 1.8260 | 32.452 | 1.9506 | 23.740 | 2.0078 | 14.612 |
| 71 | 1.5001  | 48.256 | 1.7020 | 41.575 | 1.8882 | 33.570 | 2.0293 | 24.448 | 2.0959 | 14.765 |
| 72 | 1.5307  | 49.773 | 1.7488 | 43.017 | 1.9551 | 34.782 | 2.1158 | 25.232 | 2.1942 | 14.944 |
| 73 | 1.5620  | 51.361 | 1.7977 | 44.553 | 2.0269 | 36.101 | 2.2111 | 26.106 | 2.3043 | 15.159 |
| 74 | 1.5938  | 53.022 | 1.8488 | 46.190 | 2.1041 | 37.539 | 2.3164 | 27.084 | 2.4285 | 15.419 |
| 75 | 1.6261  | 54.762 | 1.9019 | 47.936 | 2.1869 | 39.110 | 2.4332 | 28.186 | 2.5694 | 15.734 |
| 76 | 1.6585  | 56.581 | 1.9568 | 49.798 | 2.2757 | 40.830 | 2.5631 | 29.433 | 2.7303 | 16.120 |
| 77 | 1.6910  | 58.482 | 2.0134 | 51.786 | 2.3706 | 42.720 | 2.7078 | 30.855 | 2.9157 | 16.597 |
| 78 | 1.7232  | 60.468 | 2.0713 | 53.906 | 2.4716 | 44.798 | 2.8697 | 32.486 | 3.1312 | 17.186 |
| 79 | 1.7550  | 62.538 | 2.1299 | 56.165 | 2.5786 | 47.088 | 3.0509 | 34.367 | 3.3843 | 17.924 |
| 80 | 1.7858  | 64.692 | 2.1887 | 58.571 | 2.6912 | 49.611 | 3.2536 | 36.546 | 3.6845 | 18.856 |
| 81 | 1.8155  | 66.931 | 2.2471 | 61.126 | 2.8085 | 52.391 | 3.4807 | 39.085 | 4.0459 | 20.044 |
| 82 | 1.8435  | 69.252 | 2.3040 | 63.832 | 2.9290 | 55.447 | 3.7337 | 42.056 | 4.4876 | 21.578 |
| 83 | 1.8696  | 71.650 | 2.3585 | 66.690 | 3.0506 | 58.800 | 4.0135 | 45.546 | 5.0365 | 23.588 |
| 84 | 1.8933  | 74.121 | 2.4094 | 69.694 | 3.1704 | 62.460 | 4.3188 | 49.646 | 5.7311 | 26.276 |
| 85 | 1.9142  | 76.657 | 2.4555 | 72.836 | 3.2845 | 66.432 | 4.6429 | 54.459 | 6.6297 | 29.955 |
| 86 | 1.9318  | 79.252 | 2.4955 | 76.102 | 3.3884 | 70.706 | 4.9729 | 60.069 | 7.8015 | 35.134 |
| 87 | 1.9460  | 81.895 | 2.5281 | 79.475 | 3.4767 | 75.253 | 5.2871 | 66.523 | 9.3413 | 42.644 |
| 88 | 1.9564  | 84.575 | 2.5523 | 82.933 | 3.5446 | 80.030 | 5.5531 | 73.786 | 11.259 | 53.726 |
| 89 | 1.9627  | 87.281 | 2.5671 | 86.452 | 3.5875 | 84.971 | 5.7333 | 81.707 | 13.182 | 69.695 |
| 90 | 1.9648  | 90.000 | 2.5721 | 90.000 | 3.6021 | 90.000 | 5.9978 | 90.000 | 14.101 | 90.000 |

Examples.  $\tanh(1.4/64^\circ) = 1.5990/20^\circ.562 = 1.5990/20^\circ.33'.43''$ .  
 $\tanh^{-1}(1.7550/62^\circ.538) = 1.1/79^\circ$ .

TABLE III. HYPERBOLIC TANGENTS.  $\tanh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 1.6    |        | 1.7    |        | 1.8    |        | 1.9    |        | 2.0    |        |
|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| °  | °      | °      | °      | °      | °      | °      | °      | °      | °      | °      |
| 45 | 1.1413 | 9.201  | 1.1428 | 6.984  | 1.1385 | 5.063  | 1.1302 | 3.438  | 1.1191 | 2.094  |
| 46 | 1.1548 | 9.267  | 1.1553 | 6.952  | 1.1499 | 4.950  | 1.1401 | 3.260  | 1.1275 | 1.867  |
| 47 | 1.1691 | 9.318  | 1.1686 | 6.902  | 1.1618 | 4.814  | 1.1505 | 3.058  | 1.1303 | 1.613  |
| 48 | 1.1842 | 9.358  | 1.1826 | 6.834  | 1.1743 | 4.659  | 1.1614 | 2.832  | 1.1455 | 1.333  |
| 49 | 1.2002 | 9.385  | 1.1974 | 6.749  | 1.1876 | 4.480  | 1.1728 | 2.578  | 1.1550 | 1.023  |
| 50 | 1.2171 | 9.400  | 1.2131 | 6.645  | 1.2016 | 4.275  | 1.1848 | 2.295  | 1.1650 | 0.681  |
| 51 | 1.2351 | 9.400  | 1.2296 | 6.520  | 1.2163 | 4.045  | 1.1975 | 1.981  | 1.1754 | 0.306  |
| 52 | 1.2541 | 9.384  | 1.2472 | 6.373  | 1.2318 | 3.787  | 1.2107 | 1.634  | 1.1863 | 0.103  |
| 53 | 1.2744 | 9.354  | 1.2659 | 6.203  | 1.2483 | 3.499  | 1.2247 | 1.253  | 1.1977 | 0.550  |
| 54 | 1.2960 | 9.309  | 1.2857 | 6.008  | 1.2657 | 3.180  | 1.2394 | 0.835  | 1.2096 | 1.039  |
| 55 | 1.3190 | 9.246  | 1.3067 | 5.788  | 1.2842 | 2.826  | 1.2549 | 0.379  | 1.2220 | 1.571  |
| 56 | 1.3435 | 9.166  | 1.3292 | 5.539  | 1.3038 | 2.436  | 1.2713 | 0.121  | 1.2350 | 2.149  |
| 57 | 1.3698 | 9.069  | 1.3532 | 5.261  | 1.3247 | 2.008  | 1.2886 | 0.666  | 1.2487 | 2.777  |
| 58 | 1.3979 | 8.953  | 1.3788 | 4.953  | 1.3469 | 1.540  | 1.3069 | 1.259  | 1.2630 | 3.458  |
| 59 | 1.4281 | 8.817  | 1.4063 | 4.611  | 1.3706 | 1.026  | 1.3263 | 1.905  | 1.2781 | 4.195  |
| 60 | 1.4606 | 8.661  | 1.4358 | 4.232  | 1.3959 | 0.465  | 1.3470 | 2.606  | 1.2939 | 4.995  |
| 61 | 1.4956 | 8.484  | 1.4675 | 3.819  | 1.4230 | 0.145  | 1.3689 | 3.366  | 1.3106 | 5.860  |
| 62 | 1.5334 | 8.286  | 1.5017 | 3.365  | 1.4521 | 0.810  | 1.3922 | 4.191  | 1.3281 | 6.793  |
| 63 | 1.5743 | 8.065  | 1.5386 | 2.869  | 1.4833 | 1.533  | 1.4170 | 5.085  | 1.3466 | 7.802  |
| 64 | 1.6187 | 7.819  | 1.5785 | 2.326  | 1.5169 | 2.318  | 1.4435 | 6.052  | 1.3662 | 8.891  |
| 65 | 1.6670 | 7.548  | 1.6218 | 1.734  | 1.5531 | 3.172  | 1.4718 | 7.099  | 1.3868 | 10.068 |
| 66 | 1.7198 | 7.252  | 1.6690 | 1.090  | 1.5923 | 4.098  | 1.5022 | 8.232  | 1.4086 | 11.338 |
| 67 | 1.7777 | 6.928  | 1.7206 | 0.387  | 1.6347 | 5.104  | 1.5347 | 9.459  | 1.4317 | 12.707 |
| 68 | 1.8413 | 6.575  | 1.7770 | 0.376  | 1.6807 | 6.196  | 1.5696 | 10.786 | 1.4562 | 14.184 |
| 69 | 1.9116 | 6.191  | 1.8389 | 1.209  | 1.7308 | 7.382  | 1.6070 | 12.221 | 1.4821 | 15.776 |
| 70 | 1.9893 | 5.776  | 1.9072 | 2.116  | 1.7854 | 8.671  | 1.6474 | 13.773 | 1.5095 | 17.490 |
| 71 | 2.0759 | 5.326  | 1.9828 | 3.106  | 1.8451 | 10.074 | 1.6908 | 15.454 | 1.5385 | 19.337 |
| 72 | 2.1728 | 4.840  | 2.0667 | 4.187  | 1.9105 | 11.602 | 1.7376 | 17.273 | 1.5693 | 21.326 |
| 73 | 2.2819 | 4.314  | 2.1603 | 5.371  | 1.9823 | 13.266 | 1.7881 | 19.244 | 1.6019 | 23.466 |
| 74 | 2.4053 | 3.744  | 2.2652 | 6.670  | 2.0613 | 15.084 | 1.8425 | 21.379 | 1.6363 | 25.769 |
| 75 | 2.5461 | 3.127  | 2.3834 | 8.101  | 2.1484 | 17.076 | 1.9011 | 23.692 | 1.6725 | 28.245 |
| 76 | 2.7079 | 2.457  | 2.5172 | 9.685  | 2.2446 | 19.261 | 1.9641 | 26.202 | 1.7105 | 30.905 |
| 77 | 2.8957 | 1.726  | 2.6696 | 11.447 | 2.3510 | 21.664 | 2.0317 | 28.925 | 1.7503 | 33.760 |
| 78 | 3.1157 | 0.922  | 2.8442 | 13.420 | 2.4685 | 24.315 | 2.1040 | 31.881 | 1.7917 | 36.820 |
| 79 | 3.3766 | 0.034  | 3.0455 | 15.644 | 2.5984 | 27.250 | 2.1809 | 35.088 | 1.8344 | 40.093 |
| 80 | 3.6909 | 0.960  | 3.2790 | 18.175 | 2.7416 | 30.510 | 2.2621 | 38.568 | 1.8781 | 43.590 |
| 81 | 4.0755 | 2.087  | 3.5519 | 21.083 | 2.8987 | 34.140 | 2.3469 | 42.344 | 1.9222 | 47.319 |
| 82 | 4.5558 | 3.390  | 3.8718 | 24.466 | 3.0700 | 38.191 | 2.4344 | 46.428 | 1.9661 | 51.281 |
| 83 | 5.1722 | 4.937  | 4.2487 | 28.446 | 3.2540 | 42.722 | 2.5230 | 50.842 | 2.0090 | 55.474 |
| 84 | 5.9896 | 6.839  | 4.6927 | 33.199 | 3.4477 | 47.793 | 2.6104 | 55.590 | 2.0497 | 59.894 |
| 85 | 7.1203 | 9.289  | 5.2102 | 38.945 | 3.6459 | 53.448 | 2.6940 | 60.673 | 2.0872 | 64.528 |
| 86 | 8.7720 | 12.668 | 5.7994 | 45.956 | 3.8392 | 59.720 | 2.7701 | 66.075 | 2.1202 | 69.359 |
| 87 | 11.388 | 17.791 | 6.4356 | 54.527 | 4.0157 | 66.606 | 2.8350 | 71.768 | 2.1476 | 74.359 |
| 88 | 15.990 | 26.760 | 7.0503 | 64.868 | 4.1592 | 74.043 | 2.8849 | 77.700 | 2.1680 | 79.494 |
| 89 | 24.844 | 46.011 | 7.5193 | 76.896 | 4.2531 | 81.905 | 2.9163 | 83.805 | 2.1807 | 84.723 |
| 90 | 34.232 | 90.000 | 7.6968 | 90.000 | 4.2863 | 90.000 | 2.9271 | 90.000 | 2.1850 | 90.000 |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(1.6/\sqrt{54}) = 1.2960/\sqrt{9 \cdot 309} = 1.2960/\sqrt{9 \cdot 18' \cdot 32''}$   
 $\tanh(2.0/\sqrt{64}) = 1.3662/\sqrt{8 \cdot 891} = 1.3662/\sqrt{8 \cdot 53' \cdot 28''}$   
 $\tanh^{-1}(1.4718/\sqrt{7 \cdot 099}) = 1.9/\sqrt{65''}$

TABLE III. HYPERBOLIC TANGENTS.  $\tanh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 2.1    |        | 2.2    |        | 2.3    |        | 2.4     |        | 2.5     |
|----|--------|--------|--------|--------|--------|--------|---------|--------|---------|
| °  | °      | °      | °      | °      | °      | °      | °       | °      | °       |
| 45 | 1.1065 | 1.007  | 1.0932 | 0.155  | 1.0799 | 0.492  | 1.0672  | 0.961  | 1.0553  |
| 46 | 1.1134 | 0.746  | 1.0987 | 0.127  | 1.0842 | 0.783  | 1.0702  | 1.252  | 1.0573  |
| 47 | 1.1206 | 0.458  | 1.1044 | 0.436  | 1.0884 | 1.101  | 1.0732  | 1.567  | 1.0592  |
| 48 | 1.1280 | 0.141  | 1.1102 | 0.774  | 1.0927 | 1.446  | 1.0762  | 1.908  | 1.0610  |
| 49 | 1.1357 | 0.206  | 1.1161 | 1.142  | 1.0970 | 1.820  | 1.0790  | 2.277  | 1.0626  |
| 50 | 1.1437 | 0.586  | 1.1222 | 1.542  | 1.1013 | 2.226  | 1.0818  | 2.676  | 1.0640  |
| 51 | 1.1519 | 1.001  | 1.1284 | 1.978  | 1.1056 | 2.666  | 1.0845  | 3.107  | 1.0653  |
| 52 | 1.1604 | 1.452  | 1.1347 | 2.452  | 1.1099 | 3.144  | 1.0870  | 3.572  | 1.0663  |
| 53 | 1.1693 | 1.943  | 1.1411 | 2.965  | 1.1142 | 3.659  | 1.0894  | 4.073  | 1.0671  |
| 54 | 1.1784 | 2.476  | 1.1477 | 3.520  | 1.1185 | 4.215  | 1.0917  | 4.614  | 1.0676  |
| 55 | 1.1878 | 3.055  | 1.1544 | 4.120  | 1.1227 | 4.815  | 1.0937  | 5.196  | 1.0678  |
| 56 | 1.1976 | 3.682  | 1.1612 | 4.769  | 1.1268 | 5.462  | 1.0955  | 5.822  | 1.0677  |
| 57 | 1.2078 | 4.361  | 1.1681 | 5.469  | 1.1309 | 6.160  | 1.0971  | 6.496  | 1.0672  |
| 58 | 1.2183 | 5.094  | 1.1751 | 6.226  | 1.1348 | 6.912  | 1.0984  | 7.220  | 1.0662  |
| 59 | 1.2292 | 5.888  | 1.1822 | 7.042  | 1.1386 | 7.722  | 1.0993  | 7.998  | 1.0647  |
| 60 | 1.2405 | 6.745  | 1.1894 | 7.921  | 1.1422 | 8.594  | 1.0999  | 8.835  | 1.0628  |
| 61 | 1.2522 | 7.671  | 1.1967 | 8.869  | 1.1457 | 9.531  | 1.1001  | 9.735  | 1.0603  |
| 62 | 1.2644 | 8.669  | 1.2041 | 9.889  | 1.1490 | 10.540 | 1.0999  | 10.701 | 1.0572  |
| 63 | 1.2771 | 9.744  | 1.2116 | 10.988 | 1.1520 | 11.625 | 1.0992  | 11.740 | 1.0535  |
| 64 | 1.2903 | 10.902 | 1.2192 | 12.171 | 1.1548 | 12.791 | 1.0980  | 12.855 | 1.0490  |
| 65 | 1.3039 | 12.151 | 1.2268 | 13.443 | 1.1573 | 14.044 | 1.0962  | 14.053 | 1.0438  |
| 66 | 1.3181 | 13.496 | 1.2344 | 14.810 | 1.1595 | 15.390 | 1.0939  | 15.341 | 1.0378  |
| 67 | 1.3329 | 14.942 | 1.2421 | 16.280 | 1.1613 | 16.836 | 1.0909  | 16.723 | 1.0310  |
| 68 | 1.3483 | 16.499 | 1.2499 | 17.858 | 1.1628 | 18.389 | 1.0872  | 18.208 | 1.0232  |
| 69 | 1.3642 | 18.172 | 1.2576 | 19.552 | 1.1639 | 20.056 | 1.0829  | 19.802 | 1.0145  |
| 70 | 1.3808 | 19.968 | 1.2654 | 21.370 | 1.1645 | 21.843 | 1.0779  | 21.514 | 1.0049  |
| 71 | 1.3980 | 21.898 | 1.2732 | 23.319 | 1.1647 | 23.759 | 1.0721  | 23.352 | 0.99427 |
| 72 | 1.4159 | 23.968 | 1.2809 | 25.405 | 1.1644 | 25.811 | 1.0655  | 25.324 | 0.98265 |
| 73 | 1.4344 | 26.185 | 1.2885 | 27.636 | 1.1637 | 28.006 | 1.0582  | 27.439 | 0.97006 |
| 74 | 1.4535 | 28.560 | 1.2961 | 30.020 | 1.1625 | 30.352 | 1.0501  | 29.706 | 0.95649 |
| 75 | 1.4732 | 31.099 | 1.3036 | 32.564 | 1.1608 | 32.858 | 1.0413  | 32.134 | 0.94202 |
| 76 | 1.4934 | 33.811 | 1.3109 | 35.274 | 1.1587 | 35.531 | 1.0318  | 34.730 | 0.92671 |
| 77 | 1.5140 | 36.702 | 1.3181 | 38.156 | 1.1561 | 38.374 | 1.0218  | 37.505 | 0.91068 |
| 78 | 1.5348 | 39.778 | 1.3251 | 41.212 | 1.1531 | 41.392 | 1.0113  | 40.467 | 0.89405 |
| 79 | 1.5559 | 43.043 | 1.3318 | 44.446 | 1.1498 | 44.590 | 1.0004  | 43.620 | 0.87701 |
| 80 | 1.5768 | 46.501 | 1.3383 | 47.859 | 1.1462 | 47.968 | 0.98931 | 46.968 | 0.85979 |
| 81 | 1.5974 | 50.151 | 1.3444 | 51.449 | 1.1425 | 51.527 | 0.97822 | 50.517 | 0.84264 |
| 82 | 1.6174 | 53.989 | 1.3500 | 55.212 | 1.1387 | 55.264 | 0.96733 | 54.267 | 0.82584 |
| 83 | 1.6364 | 58.011 | 1.3552 | 59.140 | 1.1349 | 59.171 | 0.95686 | 58.212 | 0.80977 |
| 84 | 1.6541 | 62.210 | 1.3599 | 63.223 | 1.1312 | 63.237 | 0.94710 | 62.344 | 0.79476 |
| 85 | 1.6700 | 66.570 | 1.3640 | 67.449 | 1.1279 | 67.450 | 0.93826 | 66.650 | 0.78120 |
| 86 | 1.6837 | 71.070 | 1.3674 | 71.800 | 1.1250 | 71.793 | 0.93061 | 71.113 | 0.76946 |
| 87 | 1.6949 | 75.690 | 1.3702 | 76.254 | 1.1225 | 76.245 | 0.92439 | 75.710 | 0.75991 |
| 88 | 1.7031 | 80.406 | 1.3722 | 80.790 | 1.1207 | 80.781 | 0.91979 | 80.413 | 0.75283 |
| 89 | 1.7081 | 85.187 | 1.3734 | 85.381 | 1.1196 | 85.376 | 0.91606 | 85.188 | 0.74849 |
| 90 | 1.7099 | 90.000 | 1.3738 | 90.000 | 1.1192 | 90.000 | 0.91601 | 90.000 | 0.74702 |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(2.1/48^\circ) = 1.1280/0.141 = 1.1280/0.08'.28''$ . $\tanh(2.1/49^\circ) = 1.1357/0.206 = 1.1357/0.12'.22''$ . $\tanh^{-1}(1.0318/\sqrt{34^\circ.730}) = 2.4/76^\circ$ .

TABLE III. HYPERBOLIC TANGENTS.  $\tanh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 2.6     |        | 2.7     |        | 2.8     |        | 2.9     |        | 3.0     |        |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| °  | °       | °      | °       | °      | °       | °      | °       | °      | °       | °      |
| 45 | 1.0445  | 1.480  | 1.0348  | 1.576  | 1.0264  | 1.595  | 1.0192  | 1.554  | 1.0131  | 1.468  |
| 46 | 1.0456  | 1.744  | 1.0352  | 1.821  | 1.0262  | 1.816  | 1.0185  | 1.748  | 1.0120  | 1.637  |
| 47 | 1.0465  | 2.030  | 1.0354  | 2.083  | 1.0257  | 2.051  | 1.0175  | 1.955  | 1.0107  | 1.815  |
| 48 | 1.0473  | 2.337  | 1.0353  | 2.365  | 1.0250  | 2.303  | 1.0163  | 2.175  | 1.0091  | 2.002  |
| 49 | 1.0479  | 2.667  | 1.0350  | 2.666  | 1.0240  | 2.572  | 1.0148  | 2.409  | 1.0072  | 2.198  |
| 50 | 1.0483  | 3.022  | 1.0344  | 2.988  | 1.0227  | 2.857  | 1.0129  | 2.656  | 1.0049  | 2.405  |
| 51 | 1.0483  | 3.403  | 1.0335  | 3.332  | 1.0210  | 3.160  | 1.0106  | 2.916  | 1.0023  | 2.622  |
| 52 | 1.0480  | 3.812  | 1.0323  | 3.700  | 1.0189  | 3.483  | 1.0080  | 3.191  | 0.99922 | 2.849  |
| 53 | 1.0475  | 4.250  | 1.0307  | 4.094  | 1.0164  | 3.826  | 1.0049  | 3.481  | 0.99572 | 3.086  |
| 54 | 1.0466  | 4.717  | 1.0286  | 4.513  | 1.0135  | 4.190  | 1.0013  | 3.788  | 0.99174 | 3.334  |
| 55 | 1.0452  | 5.219  | 1.0260  | 4.959  | 1.0101  | 4.577  | 0.99722 | 4.112  | 0.98722 | 3.593  |
| 56 | 1.0434  | 5.756  | 1.0230  | 5.437  | 1.0061  | 4.987  | 0.99254 | 4.452  | 0.98211 | 3.862  |
| 57 | 1.0412  | 6.331  | 1.0194  | 5.945  | 1.0015  | 5.422  | 0.98724 | 4.810  | 0.97637 | 4.141  |
| 58 | 1.0385  | 6.947  | 1.0152  | 6.487  | 0.99625 | 5.884  | 0.98126 | 5.186  | 0.96995 | 4.432  |
| 59 | 1.0351  | 7.606  | 1.0104  | 7.066  | 0.99027 | 6.374  | 0.97454 | 5.582  | 0.96279 | 4.735  |
| 60 | 1.0311  | 8.313  | 1.0048  | 7.684  | 0.98353 | 6.895  | 0.96701 | 5.999  | 0.95481 | 5.049  |
| 61 | 1.0265  | 9.069  | 0.99847 | 8.343  | 0.97597 | 7.449  | 0.95862 | 6.439  | 0.94597 | 5.375  |
| 62 | 1.0211  | 9.879  | 0.99129 | 9.048  | 0.96751 | 8.036  | 0.94930 | 6.904  | 0.93619 | 5.713  |
| 63 | 1.0149  | 10.747 | 0.98322 | 9.801  | 0.95807 | 8.660  | 0.93877 | 7.392  | 0.92540 | 6.063  |
| 64 | 1.0079  | 11.677 | 0.97420 | 10.607 | 0.94760 | 9.325  | 0.92756 | 7.907  | 0.91352 | 6.426  |
| 65 | 0.99990 | 12.675 | 0.96414 | 11.470 | 0.93603 | 10.033 | 0.91500 | 8.452  | 0.90047 | 6.801  |
| 66 | 0.99099 | 13.747 | 0.95300 | 12.393 | 0.92328 | 10.789 | 0.90121 | 9.027  | 0.88618 | 7.192  |
| 67 | 0.98106 | 14.898 | 0.94070 | 13.383 | 0.90927 | 11.599 | 0.88612 | 9.639  | 0.87055 | 7.598  |
| 68 | 0.97006 | 16.136 | 0.92720 | 14.449 | 0.89396 | 12.464 | 0.86964 | 10.289 | 0.85350 | 8.021  |
| 69 | 0.95794 | 17.469 | 0.91243 | 15.597 | 0.87726 | 13.396 | 0.85169 | 10.982 | 0.83495 | 8.463  |
| 70 | 0.94467 | 18.902 | 0.89635 | 16.833 | 0.85912 | 14.399 | 0.83221 | 11.722 | 0.81482 | 8.926  |
| 71 | 0.93022 | 20.451 | 0.87893 | 18.168 | 0.83950 | 15.482 | 0.81113 | 12.518 | 0.79302 | 9.414  |
| 72 | 0.91458 | 22.122 | 0.86013 | 19.613 | 0.81834 | 16.655 | 0.78839 | 13.378 | 0.76946 | 9.932  |
| 73 | 0.89775 | 23.929 | 0.83994 | 21.184 | 0.79562 | 17.931 | 0.76394 | 14.310 | 0.74408 | 10.485 |
| 74 | 0.87976 | 25.884 | 0.81839 | 22.893 | 0.77134 | 19.325 | 0.73774 | 15.328 | 0.71683 | 11.081 |
| 75 | 0.86066 | 28.003 | 0.79552 | 24.757 | 0.74551 | 20.857 | 0.70977 | 16.448 | 0.68765 | 11.729 |
| 76 | 0.84053 | 30.302 | 0.77140 | 26.799 | 0.71815 | 22.547 | 0.68005 | 17.690 | 0.65650 | 12.443 |
| 77 | 0.81949 | 32.799 | 0.74612 | 29.038 | 0.68938 | 24.421 | 0.64861 | 19.078 | 0.62338 | 13.241 |
| 78 | 0.79769 | 35.513 | 0.71985 | 31.505 | 0.65930 | 26.512 | 0.61551 | 20.647 | 0.58829 | 14.146 |
| 79 | 0.77535 | 38.466 | 0.69278 | 34.231 | 0.62807 | 28.861 | 0.58087 | 22.437 | 0.55129 | 15.191 |
| 80 | 0.75271 | 41.678 | 0.66520 | 37.249 | 0.59596 | 31.519 | 0.54486 | 24.504 | 0.51245 | 16.419 |
| 81 | 0.73010 | 45.171 | 0.63743 | 40.599 | 0.56326 | 34.543 | 0.50773 | 26.921 | 0.47190 | 17.893 |
| 82 | 0.70790 | 48.966 | 0.60990 | 44.322 | 0.53039 | 38.004 | 0.46981 | 29.784 | 0.42986 | 19.700 |
| 83 | 0.68652 | 53.078 | 0.58311 | 48.464 | 0.49791 | 41.989 | 0.43156 | 33.221 | 0.38661 | 21.976 |
| 84 | 0.66645 | 57.517 | 0.55765 | 53.060 | 0.46648 | 46.591 | 0.39363 | 37.403 | 0.34257 | 24.916 |
| 85 | 0.64820 | 62.286 | 0.53421 | 58.143 | 0.43691 | 51.907 | 0.35689 | 42.549 | 0.29833 | 28.839 |
| 86 | 0.63230 | 67.370 | 0.51353 | 63.722 | 0.41024 | 58.025 | 0.32255 | 48.931 | 0.25491 | 34.254 |
| 87 | 0.61928 | 72.740 | 0.49636 | 69.778 | 0.38762 | 64.988 | 0.29220 | 56.833 | 0.21393 | 41.990 |
| 88 | 0.60959 | 78.349 | 0.48347 | 76.253 | 0.37028 | 72.762 | 0.26791 | 66.456 | 0.17815 | 53.294 |
| 89 | 0.60361 | 84.129 | 0.47544 | 83.042 | 0.35929 | 81.195 | 0.25198 | 77.709 | 0.15225 | 69.480 |
| 90 | 0.60160 | 90.000 | 0.47273 | 90.000 | 0.35553 | 90.000 | 0.24641 | 90.000 | 0.14255 | 90.000 |

Note. Negative quantities are in heavy type.

Examples  $\tanh(2.6/\underline{65^\circ}) = 0.99990/\underline{12^\circ.675} = 0.99990/\underline{12^\circ.40'.30''}$ .  
 $\tanh^{-1}(0.88618/\sqrt{7^\circ.192}) = 3.0/\underline{66^\circ}$ .

TABLE IV. CORRECTING FACTOR.  $\frac{\sinh \theta}{\theta} = r / \gamma$

|    | 0.1     |       | 0.2     |       | 0.3     |       | 0.4     |       | 0.5     |       |
|----|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|
| °  | °       | °     | °       | °     | °       | °     | °       | °     | °       | °     |
| 45 | 1.00000 | 0.096 | 1.00000 | 0.383 | 1.00000 | 0.860 | 1.00013 | 1.532 | 1.00032 | 2.391 |
| 46 | 0.99993 | 0.095 | 0.99975 | 0.382 | 0.99950 | 0.858 | 0.99920 | 1.529 | 0.99888 | 2.388 |
| 47 | 0.99987 | 0.095 | 0.99950 | 0.381 | 0.99897 | 0.856 | 0.99828 | 1.526 | 0.99744 | 2.385 |
| 48 | 0.99981 | 0.094 | 0.99930 | 0.380 | 0.99847 | 0.854 | 0.99733 | 1.520 | 0.99598 | 2.378 |
| 49 | 0.99975 | 0.094 | 0.99910 | 0.378 | 0.99797 | 0.852 | 0.99640 | 1.513 | 0.99454 | 2.368 |
| 50 | 0.99970 | 0.094 | 0.99880 | 0.376 | 0.99743 | 0.848 | 0.99550 | 1.506 | 0.99312 | 2.357 |
| 51 | 0.99965 | 0.093 | 0.99860 | 0.374 | 0.99690 | 0.842 | 0.99460 | 1.497 | 0.99170 | 2.342 |
| 52 | 0.99960 | 0.092 | 0.99840 | 0.371 | 0.99640 | 0.834 | 0.99370 | 1.486 | 0.99028 | 2.325 |
| 53 | 0.99955 | 0.091 | 0.99815 | 0.367 | 0.99590 | 0.826 | 0.99280 | 1.472 | 0.98888 | 2.304 |
| 54 | 0.99950 | 0.090 | 0.99795 | 0.362 | 0.99540 | 0.818 | 0.99190 | 1.458 | 0.98748 | 2.281 |
| 55 | 0.99944 | 0.089 | 0.99775 | 0.357 | 0.99490 | 0.809 | 0.99103 | 1.440 | 0.98610 | 2.254 |
| 56 | 0.99939 | 0.088 | 0.99755 | 0.352 | 0.99443 | 0.799 | 0.99018 | 1.421 | 0.98476 | 2.226 |
| 57 | 0.99933 | 0.086 | 0.99730 | 0.347 | 0.99397 | 0.787 | 0.98930 | 1.400 | 0.98344 | 2.195 |
| 58 | 0.99928 | 0.085 | 0.99705 | 0.342 | 0.99347 | 0.773 | 0.98845 | 1.378 | 0.98212 | 2.160 |
| 59 | 0.99922 | 0.083 | 0.99685 | 0.336 | 0.99300 | 0.760 | 0.98763 | 1.354 | 0.98082 | 2.123 |
| 60 | 0.99917 | 0.082 | 0.99670 | 0.331 | 0.99257 | 0.746 | 0.98685 | 1.330 | 0.97956 | 2.085 |
| 61 | 0.99912 | 0.081 | 0.99645 | 0.324 | 0.99213 | 0.731 | 0.98603 | 1.302 | 0.97832 | 2.042 |
| 62 | 0.99907 | 0.079 | 0.99625 | 0.317 | 0.99170 | 0.715 | 0.98523 | 1.273 | 0.97710 | 1.998 |
| 63 | 0.99902 | 0.077 | 0.99605 | 0.309 | 0.99127 | 0.698 | 0.98448 | 1.243 | 0.97590 | 1.950 |
| 64 | 0.99897 | 0.075 | 0.99585 | 0.301 | 0.99083 | 0.680 | 0.98373 | 1.212 | 0.97476 | 1.900 |
| 65 | 0.99893 | 0.073 | 0.99570 | 0.293 | 0.99040 | 0.661 | 0.98300 | 1.179 | 0.97364 | 1.848 |
| 66 | 0.99889 | 0.071 | 0.99555 | 0.284 | 0.99000 | 0.641 | 0.98232 | 1.144 | 0.97254 | 1.794 |
| 67 | 0.99885 | 0.069 | 0.99540 | 0.275 | 0.98963 | 0.621 | 0.98165 | 1.108 | 0.97150 | 1.738 |
| 68 | 0.99881 | 0.066 | 0.99520 | 0.265 | 0.98927 | 0.599 | 0.98100 | 1.070 | 0.97046 | 1.679 |
| 69 | 0.99877 | 0.064 | 0.99505 | 0.255 | 0.98891 | 0.577 | 0.98035 | 1.030 | 0.96946 | 1.617 |
| 70 | 0.99873 | 0.062 | 0.99490 | 0.245 | 0.98857 | 0.555 | 0.97975 | 0.990 | 0.96852 | 1.554 |
| 71 | 0.99869 | 0.059 | 0.99475 | 0.235 | 0.98823 | 0.532 | 0.97918 | 0.948 | 0.96760 | 1.489 |
| 72 | 0.99865 | 0.057 | 0.99460 | 0.225 | 0.98790 | 0.508 | 0.97863 | 0.905 | 0.96676 | 1.422 |
| 73 | 0.99861 | 0.054 | 0.99445 | 0.214 | 0.98760 | 0.483 | 0.97808 | 0.861 | 0.96592 | 1.354 |
| 74 | 0.99858 | 0.051 | 0.99435 | 0.203 | 0.98733 | 0.458 | 0.97758 | 0.817 | 0.96514 | 1.284 |
| 75 | 0.99855 | 0.048 | 0.99425 | 0.192 | 0.98707 | 0.432 | 0.97710 | 0.771 | 0.96440 | 1.212 |
| 76 | 0.99852 | 0.045 | 0.99415 | 0.180 | 0.98680 | 0.406 | 0.97665 | 0.724 | 0.96368 | 1.138 |
| 77 | 0.99850 | 0.042 | 0.99405 | 0.168 | 0.98653 | 0.379 | 0.97625 | 0.676 | 0.96304 | 1.062 |
| 78 | 0.99847 | 0.039 | 0.99395 | 0.156 | 0.98633 | 0.351 | 0.97585 | 0.628 | 0.96246 | 0.986 |
| 79 | 0.99845 | 0.036 | 0.99385 | 0.144 | 0.98613 | 0.322 | 0.97545 | 0.578 | 0.96190 | 0.909 |
| 80 | 0.99843 | 0.033 | 0.99375 | 0.131 | 0.98593 | 0.294 | 0.97512 | 0.528 | 0.96134 | 0.830 |
| 81 | 0.99841 | 0.030 | 0.99365 | 0.118 | 0.98577 | 0.266 | 0.97483 | 0.477 | 0.96088 | 0.750 |
| 82 | 0.99839 | 0.026 | 0.99360 | 0.106 | 0.98563 | 0.238 | 0.97458 | 0.425 | 0.96046 | 0.669 |
| 83 | 0.99838 | 0.023 | 0.99355 | 0.093 | 0.98553 | 0.209 | 0.97433 | 0.373 | 0.96008 | 0.587 |
| 84 | 0.99837 | 0.020 | 0.99350 | 0.080 | 0.98543 | 0.180 | 0.97413 | 0.321 | 0.95974 | 0.505 |
| 85 | 0.99836 | 0.017 | 0.99345 | 0.067 | 0.98537 | 0.150 | 0.97395 | 0.268 | 0.95944 | 0.422 |
| 86 | 0.99835 | 0.014 | 0.99340 | 0.054 | 0.98530 | 0.120 | 0.97380 | 0.215 | 0.95920 | 0.338 |
| 87 | 0.99834 | 0.011 | 0.99335 | 0.041 | 0.98523 | 0.090 | 0.97370 | 0.162 | 0.95904 | 0.254 |
| 88 | 0.99833 | 0.008 | 0.99335 | 0.028 | 0.98517 | 0.060 | 0.97365 | 0.108 | 0.95894 | 0.170 |
| 89 | 0.99832 | 0.004 | 0.99335 | 0.014 | 0.98510 | 0.030 | 0.97360 | 0.054 | 0.95890 | 0.085 |
| 90 | 0.99831 | 0.000 | 0.99335 | 0.000 | 0.98507 | 0.000 | 0.97355 | 0.000 | 0.95886 | 0.000 |

Note.  $\frac{\sinh \theta}{\theta} = 1.0$  when  $\theta = 0 / \delta$ .

Example.  $\frac{\sinh (0.3 / 69^\circ)}{0.3 / 69^\circ} = 0.98891 / 0^\circ.577 = 0.98891 / 0^\circ.34'.37''$ .

TABLE IV. CORRECTING FACTOR.  $\frac{\sinh \theta}{\theta} = r/\gamma$ . CONTINUED

|    | 0.6.    |       | 0.7     |       | 0.8     |       | 0.9     |       | 1.0     |       |
|----|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|
| °  | °       | °     | °       | °     | °       | °     | °       | °     | °       | °     |
| 45 | 1.00070 | 3.440 | 1.00134 | 4.676 | 1.00230 | 6.108 | 1.00363 | 7.728 | 1.00553 | 9.531 |
| 46 | 0.99863 | 3.437 | 0.99849 | 4.679 | 0.99856 | 6.112 | 0.99893 | 7.735 | 0.99975 | 9.546 |
| 47 | 0.99655 | 3.434 | 0.99564 | 4.676 | 0.99484 | 6.109 | 0.99425 | 7.734 | 0.99394 | 9.550 |
| 48 | 0.99445 | 3.426 | 0.99281 | 4.666 | 0.99114 | 6.099 | 0.98955 | 7.725 | 0.98816 | 9.543 |
| 49 | 0.99237 | 3.414 | 0.98999 | 4.652 | 0.98745 | 6.082 | 0.98488 | 7.707 | 0.98242 | 9.525 |
| 50 | 0.99030 | 3.398 | 0.98717 | 4.632 | 0.98379 | 6.058 | 0.98026 | 7.679 | 0.97672 | 9.495 |
| 51 | 0.98825 | 3.379 | 0.98439 | 4.606 | 0.98015 | 6.026 | 0.97567 | 7.642 | 0.97105 | 9.453 |
| 52 | 0.98623 | 3.355 | 0.98161 | 4.574 | 0.97655 | 5.987 | 0.97111 | 7.595 | 0.96543 | 9.399 |
| 53 | 0.98421 | 3.326 | 0.97887 | 4.537 | 0.97298 | 5.940 | 0.96659 | 7.538 | 0.95986 | 9.333 |
| 54 | 0.98220 | 3.293 | 0.97616 | 4.493 | 0.96944 | 5.886 | 0.96211 | 7.472 | 0.95435 | 9.255 |
| 55 | 0.98023 | 3.256 | 0.97349 | 4.445 | 0.96594 | 5.824 | 0.95769 | 7.396 | 0.94890 | 9.166 |
| 56 | 0.97830 | 3.215 | 0.97084 | 4.391 | 0.96249 | 5.755 | 0.95333 | 7.312 | 0.94353 | 9.065 |
| 57 | 0.97638 | 3.171 | 0.96823 | 4.331 | 0.95909 | 5.678 | 0.94904 | 7.218 | 0.93825 | 8.952 |
| 58 | 0.97448 | 3.122 | 0.96564 | 4.265 | 0.95574 | 5.593 | 0.94482 | 7.114 | 0.93305 | 8.827 |
| 59 | 0.97262 | 3.069 | 0.96313 | 4.193 | 0.95244 | 5.502 | 0.94067 | 7.000 | 0.92795 | 8.691 |
| 60 | 0.97081 | 3.013 | 0.96067 | 4.117 | 0.94923 | 5.405 | 0.93661 | 6.878 | 0.92295 | 8.544 |
| 61 | 0.96903 | 2.953 | 0.95826 | 4.036 | 0.94608 | 5.300 | 0.93263 | 6.747 | 0.91805 | 8.385 |
| 62 | 0.96728 | 2.889 | 0.95589 | 3.951 | 0.94299 | 5.189 | 0.92874 | 6.607 | 0.91325 | 8.215 |
| 63 | 0.96557 | 2.821 | 0.95356 | 3.859 | 0.93996 | 5.070 | 0.92493 | 6.458 | 0.90856 | 8.033 |
| 64 | 0.96390 | 2.749 | 0.95130 | 3.762 | 0.93703 | 4.944 | 0.92121 | 6.300 | 0.90400 | 7.841 |
| 65 | 0.96228 | 2.674 | 0.94911 | 3.661 | 0.93416 | 4.812 | 0.91761 | 6.136 | 0.89957 | 7.637 |
| 66 | 0.96072 | 2.596 | 0.94697 | 3.554 | 0.93140 | 4.674 | 0.91411 | 5.962 | 0.89527 | 7.424 |
| 67 | 0.95922 | 2.515 | 0.94493 | 3.444 | 0.92873 | 4.529 | 0.91074 | 5.780 | 0.89111 | 7.201 |
| 68 | 0.95775 | 2.430 | 0.94293 | 3.329 | 0.92614 | 4.379 | 0.90747 | 5.590 | 0.88708 | 6.968 |
| 69 | 0.95632 | 2.342 | 0.94100 | 3.209 | 0.92364 | 4.223 | 0.90430 | 5.392 | 0.88320 | 6.723 |
| 70 | 0.95495 | 2.251 | 0.93914 | 3.085 | 0.92123 | 4.061 | 0.90127 | 5.187 | 0.87947 | 6.469 |
| 71 | 0.95365 | 2.157 | 0.93737 | 2.957 | 0.91891 | 3.894 | 0.89837 | 4.975 | 0.87589 | 6.207 |
| 72 | 0.95242 | 2.061 | 0.93569 | 2.825 | 0.91671 | 3.722 | 0.89558 | 4.756 | 0.87247 | 5.936 |
| 73 | 0.95123 | 1.962 | 0.93407 | 2.689 | 0.91461 | 3.544 | 0.89292 | 4.530 | 0.86921 | 5.656 |
| 74 | 0.95010 | 1.860 | 0.93254 | 2.550 | 0.91261 | 3.361 | 0.89041 | 4.298 | 0.86612 | 5.368 |
| 75 | 0.94902 | 1.756 | 0.93109 | 2.408 | 0.91073 | 3.174 | 0.88804 | 4.059 | 0.86320 | 5.072 |
| 76 | 0.94802 | 1.649 | 0.92973 | 2.263 | 0.90895 | 2.982 | 0.88581 | 3.815 | 0.86045 | 4.769 |
| 77 | 0.94707 | 1.540 | 0.92846 | 2.114 | 0.90729 | 2.787 | 0.88372 | 3.566 | 0.85788 | 4.458 |
| 78 | 0.94620 | 1.429 | 0.92727 | 1.962 | 0.90574 | 2.588 | 0.88177 | 3.312 | 0.85549 | 4.141 |
| 79 | 0.94540 | 1.317 | 0.92617 | 1.808 | 0.90431 | 2.385 | 0.87997 | 3.053 | 0.85328 | 3.818 |
| 80 | 0.94465 | 1.203 | 0.92516 | 1.652 | 0.90301 | 2.179 | 0.87831 | 2.789 | 0.85125 | 3.489 |
| 81 | 0.94397 | 1.087 | 0.92424 | 1.493 | 0.90183 | 1.969 | 0.87681 | 2.522 | 0.84940 | 3.156 |
| 82 | 0.94337 | 0.970 | 0.92343 | 1.332 | 0.90076 | 1.757 | 0.87547 | 2.251 | 0.84774 | 2.817 |
| 83 | 0.94283 | 0.852 | 0.92270 | 1.169 | 0.89981 | 1.543 | 0.87428 | 1.976 | 0.84628 | 2.474 |
| 84 | 0.94237 | 0.732 | 0.92207 | 1.005 | 0.89899 | 1.327 | 0.87324 | 1.699 | 0.84501 | 2.128 |
| 85 | 0.94197 | 0.612 | 0.92153 | 0.839 | 0.89829 | 1.109 | 0.87237 | 1.420 | 0.84393 | 1.779 |
| 86 | 0.94163 | 0.490 | 0.92109 | 0.673 | 0.89771 | 0.889 | 0.87164 | 1.138 | 0.84305 | 1.426 |
| 87 | 0.94138 | 0.368 | 0.92074 | 0.506 | 0.89726 | 0.668 | 0.87108 | 0.855 | 0.84236 | 1.071 |
| 88 | 0.94122 | 0.246 | 0.92050 | 0.338 | 0.89694 | 0.445 | 0.87068 | 0.571 | 0.84186 | 0.715 |
| 89 | 0.94110 | 0.123 | 0.92036 | 0.169 | 0.89675 | 0.223 | 0.87043 | 0.286 | 0.84156 | 0.358 |
| 90 | 0.94107 | 0.000 | 0.92031 | 0.000 | 0.89670 | 0.000 | 0.87037 | 0.000 | 0.84147 | 0.000 |

Example.  $\frac{\sinh (1.0 / 85^{\circ})}{1.0 / 85^{\circ}} = 0.84393 / 1^{\circ}.779 = 0.84393 / 1^{\circ}.46'.44''$ .

TABLE IV. CORRECTING FACTOR.  $\frac{\sinh \theta}{\theta} = r / \gamma$ . CONTINUED

| 1.1 |         | 1.2    |         | 1.3    |         | 1.4    |         | 1.5    |         |
|-----|---------|--------|---------|--------|---------|--------|---------|--------|---------|
| °   | °       | °      | °       | °      | °       | °      | °       | °      | °       |
| 45  | 1.0081  | 11.519 | 1.0115  | 13.692 | 1.0158  | 16.034 | 1.0212  | 18.568 | 1.0279  |
| 46  | 1.0011  | 11.543 | 1.0031  | 13.726 | 1.0060  | 16.092 | 1.0099  | 18.639 | 1.0148  |
| 47  | 0.99409 | 11.555 | 0.99475 | 13.748 | 0.99623 | 16.128 | 0.99850 | 18.689 | 1.0018  |
| 48  | 0.98709 | 11.553 | 0.98650 | 13.753 | 0.98654 | 16.142 | 0.98729 | 18.717 | 0.98893 |
| 49  | 0.98018 | 11.536 | 0.97825 | 13.741 | 0.97685 | 16.137 | 0.97607 | 18.723 | 0.97613 |
| 50  | 0.97327 | 11.506 | 0.97008 | 13.712 | 0.96722 | 16.113 | 0.96493 | 18.707 | 0.96340 |
| 51  | 0.96636 | 11.461 | 0.96192 | 13.666 | 0.95770 | 16.068 | 0.95393 | 18.668 | 0.95080 |
| 52  | 0.95964 | 11.401 | 0.95383 | 13.603 | 0.94822 | 16.004 | 0.94300 | 18.605 | 0.93833 |
| 53  | 0.95291 | 11.327 | 0.94583 | 13.523 | 0.93892 | 15.919 | 0.93221 | 18.519 | 0.92593 |
| 54  | 0.94627 | 11.239 | 0.93800 | 13.425 | 0.92969 | 15.814 | 0.92157 | 18.409 | 0.91373 |
| 55  | 0.93963 | 11.136 | 0.93017 | 13.310 | 0.92054 | 15.688 | 0.91100 | 18.275 | 0.90167 |
| 56  | 0.93318 | 11.019 | 0.92250 | 13.178 | 0.91154 | 15.542 | 0.90064 | 18.117 | 0.88980 |
| 57  | 0.92682 | 10.888 | 0.91492 | 13.028 | 0.90269 | 15.376 | 0.89043 | 17.936 | 0.87813 |
| 58  | 0.92054 | 10.742 | 0.90750 | 12.860 | 0.89400 | 15.188 | 0.88036 | 17.730 | 0.86667 |
| 59  | 0.91436 | 10.581 | 0.90017 | 12.675 | 0.88546 | 14.980 | 0.87050 | 17.500 | 0.85540 |
| 60  | 0.90836 | 10.406 | 0.89300 | 12.474 | 0.87715 | 14.752 | 0.86086 | 17.246 | 0.84433 |
| 61  | 0.90244 | 10.218 | 0.88600 | 12.255 | 0.86892 | 14.502 | 0.85136 | 16.967 | 0.83353 |
| 62  | 0.89666 | 10.016 | 0.87918 | 12.019 | 0.86092 | 14.232 | 0.84214 | 16.663 | 0.82300 |
| 63  | 0.89103 | 9.800  | 0.87250 | 11.767 | 0.85315 | 13.942 | 0.83314 | 16.335 | 0.81273 |
| 64  | 0.88554 | 9.570  | 0.86600 | 11.497 | 0.84554 | 13.632 | 0.82436 | 15.983 | 0.80267 |
| 65  | 0.88019 | 9.327  | 0.85967 | 11.211 | 0.83808 | 13.301 | 0.81579 | 15.607 | 0.79293 |
| 66  | 0.87501 | 9.071  | 0.85350 | 10.909 | 0.83094 | 12.950 | 0.80750 | 15.207 | 0.78347 |
| 67  | 0.86998 | 8.801  | 0.84758 | 10.594 | 0.82400 | 12.580 | 0.79950 | 14.783 | 0.77433 |
| 68  | 0.86514 | 8.519  | 0.84183 | 10.257 | 0.81731 | 12.191 | 0.79171 | 14.335 | 0.76547 |
| 69  | 0.86045 | 8.225  | 0.83625 | 9.907  | 0.81077 | 11.783 | 0.78421 | 13.865 | 0.75693 |
| 70  | 0.85596 | 7.918  | 0.83093 | 9.543  | 0.80454 | 11.357 | 0.77707 | 13.373 | 0.74873 |
| 71  | 0.85165 | 7.600  | 0.82583 | 9.164  | 0.79862 | 10.912 | 0.77021 | 12.858 | 0.74087 |
| 72  | 0.84754 | 7.271  | 0.82095 | 8.771  | 0.79292 | 10.450 | 0.76364 | 12.322 | 0.73333 |
| 73  | 0.84362 | 6.931  | 0.81631 | 8.365  | 0.78746 | 9.971  | 0.75736 | 11.765 | 0.72620 |
| 74  | 0.83989 | 6.580  | 0.81190 | 7.946  | 0.78231 | 9.476  | 0.75143 | 11.188 | 0.71940 |
| 75  | 0.83637 | 6.220  | 0.80773 | 7.514  | 0.77746 | 8.966  | 0.74579 | 10.591 | 0.71293 |
| 76  | 0.83306 | 5.850  | 0.80380 | 7.069  | 0.77287 | 8.440  | 0.74050 | 9.975  | 0.70687 |
| 77  | 0.82996 | 5.471  | 0.80013 | 6.614  | 0.76858 | 7.900  | 0.73557 | 9.342  | 0.70120 |
| 78  | 0.82707 | 5.084  | 0.79671 | 6.148  | 0.76459 | 7.346  | 0.73093 | 8.692  | 0.69593 |
| 79  | 0.82441 | 4.689  | 0.79355 | 5.672  | 0.76090 | 6.780  | 0.72664 | 8.026  | 0.69107 |
| 80  | 0.82196 | 4.286  | 0.79065 | 5.187  | 0.75751 | 6.202  | 0.72279 | 7.346  | 0.68660 |
| 81  | 0.81975 | 3.877  | 0.78802 | 4.693  | 0.75444 | 5.614  | 0.71921 | 6.651  | 0.68253 |
| 82  | 0.81775 | 3.462  | 0.78566 | 4.191  | 0.75165 | 5.016  | 0.71600 | 5.944  | 0.67887 |
| 83  | 0.81599 | 3.041  | 0.78358 | 3.683  | 0.74923 | 4.408  | 0.71319 | 5.227  | 0.67567 |
| 84  | 0.81445 | 2.616  | 0.78175 | 3.169  | 0.74711 | 3.793  | 0.71073 | 4.499  | 0.67287 |
| 85  | 0.81315 | 2.186  | 0.78022 | 2.649  | 0.74531 | 3.171  | 0.70864 | 3.762  | 0.67047 |
| 86  | 0.81209 | 1.753  | 0.77895 | 2.124  | 0.74383 | 2.544  | 0.70694 | 3.018  | 0.66847 |
| 87  | 0.81125 | 1.317  | 0.77797 | 1.596  | 0.74268 | 1.912  | 0.70561 | 2.269  | 0.66693 |
| 88  | 0.81066 | 0.879  | 0.77727 | 1.065  | 0.74185 | 1.277  | 0.70466 | 1.515  | 0.66587 |
| 89  | 0.81031 | 0.440  | 0.77684 | 0.533  | 0.74136 | 0.639  | 0.70409 | 0.758  | 0.66521 |
| 90  | 0.81019 | 0.000  | 0.77670 | 0.000  | 0.74120 | 0.000  | 0.70389 | 0.000  | 0.66499 |

Example.  $\frac{\sinh (1.5 / 65^{\circ})}{1.5 / 65^{\circ}} = 0.79293 / 18^{\circ}.140 = 0.79293 / 18^{\circ}.08'.24''.$

TABLE IV. CORRECTING FACTOR.  $\frac{\sinh \theta}{\theta} = r / \gamma$ . CONTINUED

|    | 1.6     |        | 1.7     |        | 1.8     |        | 1.9     |        | 2.0     |        |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| °  | °       |        | °       |        | °       |        | °       |        | °       |        |
| 45 | 1.0359  | 24.117 | 1.0456  | 27.133 | 1.0572  | 30.292 | 1.0707  | 33.590 | 1.0863  | 37.016 |
| 46 | 1.0211  | 24.241 | 1.0290  | 27.288 | 1.0385  | 30.486 | 1.0499  | 33.829 | 1.0633  | 37.304 |
| 47 | 1.0064  | 24.339 | 1.0124  | 27.418 | 1.0199  | 30.651 | 1.0292  | 34.037 | 1.0405  | 37.560 |
| 48 | 0.99175 | 24.409 | 0.99582 | 27.515 | 1.0015  | 30.784 | 1.0087  | 34.210 | 1.0178  | 37.778 |
| 49 | 0.97719 | 24.451 | 0.97947 | 27.582 | 0.98317 | 30.883 | 0.98837 | 34.346 | 0.99535 | 37.958 |
| 50 | 0.96275 | 24.465 | 0.96323 | 27.618 | 0.96500 | 30.946 | 0.96821 | 34.444 | 0.97310 | 38.098 |
| 51 | 0.94844 | 24.449 | 0.94718 | 27.620 | 0.94700 | 30.974 | 0.94821 | 34.503 | 0.95110 | 38.196 |
| 52 | 0.93431 | 24.402 | 0.93123 | 27.590 | 0.92922 | 30.966 | 0.92858 | 34.524 | 0.92940 | 38.253 |
| 53 | 0.92031 | 24.325 | 0.91547 | 27.527 | 0.91167 | 30.921 | 0.90911 | 34.505 | 0.90800 | 38.267 |
| 54 | 0.90650 | 24.218 | 0.89994 | 27.429 | 0.89433 | 30.839 | 0.88995 | 34.444 | 0.88685 | 38.236 |
| 55 | 0.89281 | 24.079 | 0.88465 | 27.296 | 0.87728 | 30.718 | 0.87106 | 34.342 | 0.86600 | 38.160 |
| 56 | 0.87938 | 23.910 | 0.86953 | 27.128 | 0.86044 | 30.558 | 0.85242 | 34.197 | 0.84555 | 38.039 |
| 57 | 0.86619 | 23.709 | 0.85471 | 26.924 | 0.84389 | 30.358 | 0.83411 | 34.008 | 0.82545 | 37.860 |
| 58 | 0.85319 | 23.475 | 0.84012 | 26.683 | 0.82767 | 30.116 | 0.81611 | 33.773 | 0.80570 | 37.650 |
| 59 | 0.84044 | 23.209 | 0.82582 | 26.406 | 0.81172 | 29.834 | 0.79847 | 33.493 | 0.78630 | 37.382 |
| 60 | 0.82794 | 22.910 | 0.81176 | 26.091 | 0.79611 | 29.510 | 0.78121 | 33.167 | 0.76735 | 37.062 |
| 61 | 0.81569 | 22.578 | 0.79806 | 25.738 | 0.78083 | 29.142 | 0.76437 | 32.792 | 0.74880 | 36.688 |
| 62 | 0.80375 | 22.212 | 0.78464 | 25.347 | 0.76594 | 28.730 | 0.74795 | 32.368 | 0.73070 | 36.261 |
| 63 | 0.79213 | 21.813 | 0.77159 | 24.917 | 0.75139 | 28.274 | 0.73179 | 31.893 | 0.71300 | 35.778 |
| 64 | 0.78075 | 21.380 | 0.75888 | 24.447 | 0.73722 | 27.774 | 0.71611 | 31.368 | 0.69575 | 35.238 |
| 65 | 0.76975 | 20.913 | 0.74647 | 23.938 | 0.72344 | 27.228 | 0.70084 | 30.792 | 0.67900 | 34.639 |
| 66 | 0.75906 | 20.413 | 0.73447 | 23.390 | 0.71006 | 26.636 | 0.68605 | 30.162 | 0.66275 | 33.980 |
| 67 | 0.74869 | 19.879 | 0.72282 | 22.802 | 0.69711 | 25.997 | 0.67168 | 29.478 | 0.64700 | 33.261 |
| 68 | 0.73869 | 19.311 | 0.71159 | 22.175 | 0.68455 | 25.312 | 0.65779 | 28.741 | 0.63175 | 32.479 |
| 69 | 0.72900 | 18.710 | 0.70076 | 21.508 | 0.67244 | 24.580 | 0.64421 | 27.949 | 0.61700 | 31.634 |
| 70 | 0.71975 | 18.076 | 0.69035 | 20.801 | 0.66083 | 23.802 | 0.63153 | 27.102 | 0.60280 | 30.725 |
| 71 | 0.71081 | 17.410 | 0.68035 | 20.055 | 0.64967 | 22.976 | 0.61916 | 26.199 | 0.58915 | 29.750 |
| 72 | 0.70231 | 16.712 | 0.67076 | 19.271 | 0.63900 | 22.104 | 0.60931 | 25.241 | 0.57905 | 28.709 |
| 73 | 0.69419 | 15.982 | 0.66165 | 18.448 | 0.62878 | 21.187 | 0.59900 | 24.227 | 0.56935 | 27.602 |
| 74 | 0.68650 | 15.221 | 0.65300 | 17.588 | 0.61911 | 20.224 | 0.58826 | 23.158 | 0.55916 | 26.428 |
| 75 | 0.67919 | 14.431 | 0.64476 | 16.692 | 0.60994 | 19.215 | 0.57850 | 22.034 | 0.54935 | 25.187 |
| 76 | 0.67231 | 13.613 | 0.63700 | 15.760 | 0.60128 | 18.162 | 0.56842 | 20.856 | 0.52970 | 23.880 |
| 77 | 0.66588 | 12.767 | 0.62976 | 14.794 | 0.59317 | 17.068 | 0.55837 | 19.625 | 0.51970 | 22.507 |
| 78 | 0.65988 | 11.894 | 0.62300 | 13.795 | 0.58561 | 15.933 | 0.54959 | 18.343 | 0.51035 | 21.070 |
| 79 | 0.65431 | 10.997 | 0.61676 | 12.765 | 0.57861 | 14.758 | 0.54011 | 17.012 | 0.50165 | 19.571 |
| 80 | 0.64925 | 10.076 | 0.61106 | 11.705 | 0.57217 | 13.545 | 0.53295 | 15.633 | 0.49364 | 18.011 |
| 81 | 0.64463 | 9.134  | 0.60582 | 10.618 | 0.56633 | 12.298 | 0.52642 | 14.209 | 0.48636 | 16.391 |
| 82 | 0.64050 | 8.171  | 0.60112 | 9.505  | 0.56106 | 11.019 | 0.52052 | 12.744 | 0.47979 | 14.722 |
| 83 | 0.63681 | 7.191  | 0.59700 | 8.370  | 0.55639 | 9.711  | 0.51530 | 11.240 | 0.47396 | 13.000 |
| 84 | 0.63363 | 6.194  | 0.59341 | 7.214  | 0.55234 | 8.374  | 0.51075 | 9.702  | 0.46888 | 11.233 |
| 85 | 0.63094 | 5.184  | 0.59035 | 6.039  | 0.54890 | 7.015  | 0.50689 | 8.133  | 0.46455 | 9.426  |
| 86 | 0.62875 | 4.161  | 0.58782 | 4.850  | 0.54607 | 5.636  | 0.50372 | 6.539  | 0.46101 | 7.584  |
| 87 | 0.62700 | 3.129  | 0.58586 | 3.649  | 0.54387 | 4.241  | 0.50124 | 4.923  | 0.45823 | 5.713  |
| 88 | 0.62575 | 2.090  | 0.58445 | 2.438  | 0.54229 | 2.834  | 0.49947 | 3.291  | 0.45624 | 3.821  |
| 89 | 0.62500 | 1.046  | 0.58361 | 1.230  | 0.54135 | 1.419  | 0.49841 | 1.648  | 0.45505 | 1.914  |
| 90 | 0.62473 | 0.000  | 0.58333 | 0.000  | 0.54103 | 0.000  | 0.49805 | 0.000  | 0.45465 | 0.000  |

Example.  $\frac{\sinh (1.8 / 77^{\circ})}{1.8 / 77^{\circ}} = 0.59317 / 17^{\circ}.068 = 0.59317 / 17^{\circ}.4'.05''$ .



TABLE IV. CORRECTING FACTOR.  $\frac{\sinh \theta}{\theta} = r/\gamma$ . CONTINUED

|    | 2.1     |        | 2.2     |        | 2.3     |        | 2.4     |        | 2.5     |        |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| °  | °       | °      | °       | °      | °       | °      | °       | °      | °       | °      |
| 45 | 1.1043  | 40.558 | 1.1248  | 44.205 | 1.1480  | 47.946 | 1.1740  | 51.769 | 1.2032  | 55.661 |
| 46 | 1.0789  | 40.905 | 1.0970  | 44.613 | 1.1177  | 48.419 | 1.1411  | 52.312 | 1.1674  | 56.278 |
| 47 | 1.0538  | 41.213 | 1.0695  | 44.981 | 1.0877  | 48.851 | 1.1085  | 52.813 | 1.1320  | 56.852 |
| 48 | 1.0289  | 41.482 | 1.0423  | 45.307 | 1.0580  | 49.241 | 1.0762  | 53.271 | 1.0972  | 57.383 |
| 49 | 1.0043  | 41.711 | 1.0153  | 45.592 | 1.0286  | 49.588 | 1.0444  | 53.685 | 1.0627  | 57.869 |
| 50 | 0.97986 | 41.898 | 0.98864 | 45.834 | 0.99965 | 49.890 | 1.0130  | 54.053 | 1.0288  | 58.309 |
| 51 | 0.95576 | 42.042 | 0.96232 | 46.031 | 0.97104 | 50.146 | 0.98200 | 54.375 | 0.99540 | 58.701 |
| 52 | 0.93195 | 42.142 | 0.93636 | 46.181 | 0.94283 | 50.354 | 0.95150 | 54.648 | 0.96256 | 59.044 |
| 53 | 0.90848 | 42.197 | 0.91077 | 46.284 | 0.91504 | 50.514 | 0.92154 | 54.871 | 0.93028 | 59.337 |
| 54 | 0.88533 | 42.205 | 0.88559 | 46.338 | 0.88774 | 50.623 | 0.89208 | 55.042 | 0.89860 | 59.578 |
| 55 | 0.86257 | 42.165 | 0.86082 | 46.342 | 0.86091 | 50.680 | 0.86308 | 55.160 | 0.86748 | 59.765 |
| 56 | 0.84019 | 42.076 | 0.83646 | 46.294 | 0.83457 | 50.683 | 0.83467 | 55.223 | 0.83700 | 59.896 |
| 57 | 0.81819 | 41.935 | 0.81255 | 46.193 | 0.80869 | 50.629 | 0.80683 | 55.228 | 0.80712 | 59.970 |
| 58 | 0.79662 | 41.742 | 0.78909 | 46.035 | 0.78331 | 50.518 | 0.77954 | 55.174 | 0.77788 | 59.984 |
| 59 | 0.77543 | 41.494 | 0.76609 | 45.820 | 0.75848 | 50.347 | 0.75283 | 55.059 | 0.74932 | 59.936 |
| 60 | 0.75471 | 41.191 | 0.74359 | 45.546 | 0.73417 | 50.114 | 0.72671 | 54.880 | 0.72140 | 59.823 |
| 61 | 0.73447 | 40.830 | 0.72159 | 45.210 | 0.71043 | 49.816 | 0.70121 | 54.634 | 0.69412 | 59.642 |
| 62 | 0.71467 | 40.410 | 0.70009 | 44.811 | 0.68725 | 49.451 | 0.67633 | 54.319 | 0.66752 | 59.391 |
| 63 | 0.69538 | 39.929 | 0.67914 | 44.345 | 0.66465 | 49.016 | 0.65204 | 53.931 | 0.64160 | 59.067 |
| 64 | 0.67657 | 39.386 | 0.65873 | 43.811 | 0.64261 | 48.509 | 0.62838 | 53.467 | 0.61636 | 58.665 |
| 65 | 0.65829 | 38.777 | 0.63886 | 43.207 | 0.62113 | 47.926 | 0.60533 | 52.924 | 0.59176 | 58.182 |
| 66 | 0.64053 | 38.101 | 0.61955 | 42.529 | 0.60026 | 47.264 | 0.58296 | 52.297 | 0.56784 | 57.613 |
| 67 | 0.62329 | 37.357 | 0.60082 | 41.775 | 0.58004 | 46.519 | 0.56121 | 51.583 | 0.54464 | 56.954 |
| 68 | 0.60657 | 36.542 | 0.58268 | 40.943 | 0.56043 | 45.688 | 0.54013 | 50.777 | 0.52208 | 56.198 |
| 69 | 0.59043 | 35.655 | 0.56514 | 40.029 | 0.54143 | 44.767 | 0.51971 | 49.874 | 0.50024 | 55.341 |
| 70 | 0.57487 | 34.694 | 0.54818 | 39.030 | 0.52309 | 43.752 | 0.49996 | 48.868 | 0.47908 | 54.376 |
| 71 | 0.55990 | 33.656 | 0.53186 | 37.944 | 0.50539 | 42.638 | 0.48088 | 47.754 | 0.45864 | 53.206 |
| 72 | 0.54557 | 32.541 | 0.51618 | 36.769 | 0.48835 | 41.422 | 0.46246 | 46.526 | 0.43888 | 52.092 |
| 73 | 0.53181 | 31.348 | 0.50118 | 35.501 | 0.47204 | 40.100 | 0.44479 | 45.177 | 0.41988 | 50.755 |
| 74 | 0.51871 | 30.074 | 0.48682 | 34.138 | 0.45644 | 38.667 | 0.42783 | 43.701 | 0.40160 | 49.277 |
| 75 | 0.50629 | 28.719 | 0.47323 | 32.678 | 0.44152 | 37.118 | 0.41165 | 42.091 | 0.38408 | 47.648 |
| 76 | 0.49453 | 27.283 | 0.46027 | 31.119 | 0.42736 | 35.449 | 0.39621 | 40.341 | 0.36732 | 45.857 |
| 77 | 0.48347 | 25.766 | 0.44810 | 29.459 | 0.41397 | 33.659 | 0.38157 | 38.443 | 0.35137 | 43.891 |
| 78 | 0.47311 | 24.168 | 0.43666 | 27.609 | 0.40135 | 31.744 | 0.36775 | 36.390 | 0.33625 | 41.740 |
| 79 | 0.46349 | 22.491 | 0.42601 | 25.839 | 0.38963 | 29.701 | 0.35480 | 34.178 | 0.32201 | 39.392 |
| 80 | 0.45461 | 20.736 | 0.41617 | 23.879 | 0.37873 | 27.531 | 0.34273 | 31.803 | 0.30869 | 36.836 |
| 81 | 0.44650 | 18.907 | 0.40716 | 21.822 | 0.36873 | 25.233 | 0.33162 | 29.261 | 0.29633 | 34.063 |
| 82 | 0.43919 | 17.008 | 0.39902 | 19.671 | 0.35965 | 22.810 | 0.32149 | 26.550 | 0.28500 | 31.065 |
| 83 | 0.43268 | 15.043 | 0.39176 | 17.432 | 0.35153 | 20.267 | 0.31239 | 23.676 | 0.27477 | 27.841 |
| 84 | 0.42699 | 13.015 | 0.38540 | 15.111 | 0.34441 | 17.611 | 0.30438 | 20.643 | 0.26569 | 24.390 |
| 85 | 0.42215 | 10.933 | 0.37998 | 12.715 | 0.33831 | 14.851 | 0.29748 | 17.462 | 0.25784 | 20.723 |
| 86 | 0.41818 | 8.806  | 0.37551 | 10.254 | 0.33328 | 12.000 | 0.29178 | 14.147 | 0.25131 | 16.855 |
| 87 | 0.41507 | 6.639  | 0.37202 | 7.740  | 0.32933 | 9.071  | 0.28728 | 10.717 | 0.24614 | 12.811 |
| 88 | 0.41284 | 4.443  | 0.36951 | 5.183  | 0.32650 | 6.082  | 0.28405 | 7.197  | 0.24241 | 8.624  |
| 89 | 0.41150 | 2.226  | 0.36800 | 2.599  | 0.32479 | 3.051  | 0.28210 | 3.615  | 0.24014 | 4.338  |
| 90 | 0.41105 | 0.000  | 0.36750 | 0.000  | 0.32422 | 0.000  | 0.28144 | 0.000  | 0.23939 | 0.000  |

Example.  $\frac{\sinh (2.3 / 84^{\circ})}{2.3 / 84^{\circ}} = 0.34441 / 17^{\circ}.611 = 0.34441 / 17^{\circ}.36'.40''$ .

TABLE IV. CORRECTING FACTOR.  $\frac{\sinh \theta}{\theta} = r/\gamma$ . CONTINUED

|    | 2.6     |        | 2.7     |        | 2.8     |        | 2.9      |        | 3.0      |        |
|----|---------|--------|---------|--------|---------|--------|----------|--------|----------|--------|
| •  | °       | °      | °       | °      | °       | °      | °        | °      | °        | °      |
| 45 | 1.2354  | 59.613 | 1.2710  | 63.614 | 1.3102  | 67.653 | 1.3530   | 71.721 | 1.3995   | 75.814 |
| 46 | 1.1967  | 60.307 | 1.2293  | 64.386 | 1.2652  | 68.506 | 1.3046   | 72.658 | 1.3475   | 76.832 |
| 47 | 1.1586  | 60.957 | 1.1881  | 65.116 | 1.2209  | 69.317 | 1.2570   | 73.551 | 1.2965   | 77.808 |
| 48 | 1.1209  | 61.564 | 1.1476  | 65.801 | 1.1774  | 70.084 | 1.2103   | 74.400 | 1.2465   | 78.741 |
| 49 | 1.0838  | 62.126 | 1.1077  | 66.442 | 1.1345  | 70.806 | 1.1644   | 75.205 | 1.1974   | 79.630 |
| 50 | 1.0473  | 62.641 | 1.0685  | 67.037 | 1.0925  | 71.483 | 1.1195   | 75.965 | 1.1494   | 80.474 |
| 51 | 1.0113  | 63.109 | 1.0300  | 67.585 | 1.0513  | 72.113 | 1.0755   | 76.679 | 1.1025   | 81.272 |
| 52 | 0.97608 | 63.528 | 0.99219 | 68.084 | 1.0109  | 72.694 | 1.0324   | 77.345 | 1.0567   | 82.024 |
| 53 | 0.94145 | 63.897 | 0.95514 | 68.532 | 0.97143 | 73.226 | 0.99038  | 77.963 | 1.0120   | 82.729 |
| 54 | 0.90750 | 64.214 | 0.91881 | 68.929 | 0.93279 | 73.708 | 0.94928  | 78.532 | 0.96837  | 83.386 |
| 55 | 0.87419 | 64.476 | 0.88333 | 69.274 | 0.89500 | 74.138 | 0.90917  | 79.050 | 0.92590  | 83.994 |
| 56 | 0.84162 | 64.684 | 0.84863 | 69.563 | 0.85811 | 74.515 | 0.87007  | 79.517 | 0.88453  | 84.553 |
| 57 | 0.80973 | 64.834 | 0.81467 | 69.797 | 0.82211 | 74.837 | 0.83197  | 79.932 | 0.84433  | 85.062 |
| 58 | 0.77854 | 64.925 | 0.78152 | 69.973 | 0.78700 | 75.103 | 0.79487  | 80.293 | 0.80523  | 85.510 |
| 59 | 0.74808 | 64.954 | 0.74919 | 70.088 | 0.75275 | 75.311 | 0.75876  | 80.598 | 0.76727  | 85.924 |
| 60 | 0.71835 | 64.918 | 0.71767 | 70.140 | 0.71943 | 75.459 | 0.72366  | 80.847 | 0.73040  | 86.276 |
| 61 | 0.68935 | 64.816 | 0.68692 | 70.128 | 0.68700 | 75.544 | 0.68955  | 81.037 | 0.69463  | 86.574 |
| 62 | 0.66104 | 64.644 | 0.65700 | 70.047 | 0.65543 | 75.565 | 0.65641  | 81.167 | 0.65993  | 86.817 |
| 63 | 0.63346 | 64.399 | 0.62786 | 69.895 | 0.62475 | 75.519 | 0.62424  | 81.235 | 0.62630  | 87.004 |
| 64 | 0.60662 | 64.077 | 0.59952 | 69.669 | 0.59493 | 75.403 | 0.59303  | 81.239 | 0.59373  | 87.133 |
| 65 | 0.58054 | 63.674 | 0.57193 | 69.364 | 0.56596 | 75.214 | 0.56272  | 81.176 | 0.56217  | 87.204 |
| 66 | 0.55519 | 63.185 | 0.54514 | 68.978 | 0.53785 | 74.948 | 0.53334  | 81.044 | 0.53160  | 87.214 |
| 67 | 0.53054 | 62.606 | 0.51911 | 68.504 | 0.51054 | 74.599 | 0.50486  | 80.838 | 0.50203  | 87.161 |
| 68 | 0.50658 | 61.930 | 0.49386 | 67.935 | 0.48404 | 74.164 | 0.47724  | 80.555 | 0.47340  | 87.043 |
| 69 | 0.48338 | 61.151 | 0.46937 | 67.266 | 0.45832 | 73.634 | 0.45045  | 80.190 | 0.44567  | 86.858 |
| 70 | 0.46089 | 60.262 | 0.44559 | 66.489 | 0.43343 | 73.005 | 0.42448  | 79.738 | 0.41883  | 86.602 |
| 71 | 0.43911 | 59.254 | 0.42256 | 65.596 | 0.40928 | 72.267 | 0.39934  | 79.191 | 0.39283  | 86.271 |
| 72 | 0.41808 | 58.117 | 0.40026 | 64.576 | 0.38586 | 71.411 | 0.37497  | 78.541 | 0.36767  | 85.860 |
| 73 | 0.39773 | 56.841 | 0.37872 | 63.415 | 0.36321 | 70.424 | 0.35135  | 77.780 | 0.34327  | 85.363 |
| 74 | 0.37814 | 55.414 | 0.35789 | 62.101 | 0.34125 | 69.292 | 0.32845  | 76.894 | 0.31964  | 84.772 |
| 75 | 0.35930 | 53.822 | 0.33780 | 60.617 | 0.32002 | 67.996 | 0.30627  | 75.867 | 0.29674  | 84.077 |
| 76 | 0.34123 | 52.049 | 0.31846 | 58.943 | 0.29951 | 66.516 | 0.28478  | 74.680 | 0.27452  | 83.265 |
| 77 | 0.32395 | 50.079 | 0.29987 | 57.058 | 0.27971 | 64.827 | 0.26397  | 73.309 | 0.25296  | 82.319 |
| 78 | 0.30749 | 47.893 | 0.28206 | 54.935 | 0.26064 | 62.897 | 0.24379  | 71.721 | 0.23202  | 81.216 |
| 79 | 0.29189 | 45.471 | 0.26507 | 52.542 | 0.24231 | 60.686 | 0.22432  | 69.876 | 0.21168  | 79.925 |
| 80 | 0.27718 | 42.793 | 0.24894 | 49.847 | 0.22476 | 58.145 | 0.20551  | 67.721 | 0.19191  | 78.404 |
| 81 | 0.26346 | 39.837 | 0.23374 | 46.812 | 0.20804 | 55.217 | 0.18738  | 65.183 | 0.17270  | 76.592 |
| 82 | 0.25078 | 36.583 | 0.21954 | 43.396 | 0.19222 | 51.833 | 0.16998  | 62.169 | 0.15404  | 74.404 |
| 83 | 0.23922 | 33.016 | 0.20644 | 39.554 | 0.17741 | 47.908 | 0.15343  | 58.553 | 0.13596  | 71.709 |
| 84 | 0.22887 | 29.126 | 0.19458 | 35.252 | 0.16375 | 43.351 | 0.13774  | 54.168 | 0.11849  | 68.313 |
| 85 | 0.21986 | 24.910 | 0.18409 | 30.458 | 0.15142 | 38.066 | 0.12323  | 48.795 | 0.10174  | 63.903 |
| 86 | 0.21227 | 20.382 | 0.17515 | 25.164 | 0.14069 | 31.968 | 0.11016  | 42.168 | 0.085917 | 57.974 |
| 87 | 0.20623 | 15.571 | 0.16794 | 19.390 | 0.13184 | 25.015 | 0.098934 | 34.005 | 0.071443 | 49.701 |
| 88 | 0.20184 | 10.524 | 0.16264 | 13.194 | 0.12520 | 17.244 | 0.090148 | 24.110 | 0.059103 | 37.843 |
| 89 | 0.19917 | 5.307  | 0.15938 | 6.682  | 0.12105 | 8.809  | 0.084469 | 12.576 | 0.050307 | 21.091 |
| 90 | 0.19827 | 0.000  | 0.15829 | 0.000  | 0.11964 | 0.000  | 0.082845 | 0.000  | 0.047040 | 0.000  |

Example.  $\frac{\sinh (3.0 / 86^{\circ})}{3.0 / 86^{\circ}} = 0.085917 / 57^{\circ}.974 = 0.085917 / 59^{\circ}.58'.26''.$

TABLE V. CORRECTING FACTOR.  $\frac{\tanh \theta}{\theta} = r/\gamma$

|    | 0.1     |       | 0.2     |       | 0.3     |       | 0.4     |       | 0.5     |       |
|----|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|
| °  | °       | °     | °       | °     | °       | °     | °       | °     | °       | °     |
| 45 | 1.00000 | 0.188 | 0.99985 | 0.765 | 0.99937 | 1.718 | 0.99803 | 3.046 | 0.99514 | 4.750 |
| 46 | 1.00010 | 0.188 | 1.00030 | 0.764 | 1.00040 | 1.719 | 0.99988 | 3.055 | 0.99804 | 4.771 |
| 47 | 1.00021 | 0.188 | 1.00075 | 0.763 | 1.00143 | 1.720 | 1.00173 | 3.058 | 1.00094 | 4.781 |
| 48 | 1.00033 | 0.188 | 1.00120 | 0.761 | 1.00250 | 1.717 | 1.00358 | 3.058 | 1.00384 | 4.787 |
| 49 | 1.00044 | 0.187 | 1.00170 | 0.758 | 1.00357 | 1.710 | 1.00543 | 3.053 | 1.00680 | 4.787 |
| 50 | 1.00056 | 0.187 | 1.00215 | 0.755 | 1.00460 | 1.703 | 1.00733 | 3.045 | 1.00980 | 4.780 |
| 51 | 1.00068 | 0.187 | 1.00265 | 0.749 | 1.00563 | 1.694 | 1.00925 | 3.031 | 1.01278 | 4.767 |
| 52 | 1.00080 | 0.186 | 1.00310 | 0.744 | 1.00670 | 1.685 | 1.01115 | 3.014 | 1.01578 | 4.747 |
| 53 | 1.00092 | 0.184 | 1.00355 | 0.738 | 1.00773 | 1.672 | 1.01303 | 2.995 | 1.01878 | 4.724 |
| 54 | 1.00103 | 0.182 | 1.00405 | 0.732 | 1.00877 | 1.656 | 1.01490 | 2.969 | 1.02178 | 4.692 |
| 55 | 1.00114 | 0.180 | 1.00450 | 0.724 | 1.00980 | 1.638 | 1.01675 | 2.942 | 1.02478 | 4.656 |
| 56 | 1.00125 | 0.177 | 1.00495 | 0.715 | 1.01083 | 1.618 | 1.01865 | 2.911 | 1.02778 | 4.612 |
| 57 | 1.00135 | 0.175 | 1.00535 | 0.705 | 1.01183 | 1.596 | 1.02050 | 2.876 | 1.03076 | 4.561 |
| 58 | 1.00146 | 0.172 | 1.00575 | 0.694 | 1.01283 | 1.574 | 1.02230 | 2.836 | 1.03374 | 4.506 |
| 59 | 1.00156 | 0.170 | 1.00620 | 0.682 | 1.01380 | 1.548 | 1.02410 | 2.793 | 1.03670 | 4.444 |
| 60 | 1.0017  | 0.167 | 1.0067  | 0.668 | 1.0148  | 1.521 | 1.0259  | 2.745 | 1.0397  | 4.375 |
| 61 | 1.0018  | 0.163 | 1.0070  | 0.655 | 1.0158  | 1.490 | 1.0278  | 2.695 | 1.0425  | 4.300 |
| 62 | 1.0019  | 0.159 | 1.0074  | 0.640 | 1.0167  | 1.458 | 1.0294  | 2.641 | 1.0454  | 4.220 |
| 63 | 1.0020  | 0.155 | 1.0078  | 0.625 | 1.0176  | 1.425 | 1.0311  | 2.583 | 1.0482  | 4.133 |
| 64 | 1.0020  | 0.151 | 1.0082  | 0.609 | 1.0185  | 1.390 | 1.0328  | 2.521 | 1.0510  | 4.041 |
| 65 | 1.0021  | 0.147 | 1.0086  | 0.592 | 1.0193  | 1.353 | 1.0344  | 2.456 | 1.0538  | 3.942 |
| 66 | 1.0022  | 0.143 | 1.0090  | 0.575 | 1.0202  | 1.314 | 1.0360  | 2.388 | 1.0564  | 3.836 |
| 67 | 1.0023  | 0.138 | 1.0093  | 0.557 | 1.0210  | 1.273 | 1.0376  | 2.315 | 1.0591  | 3.724 |
| 68 | 1.0024  | 0.134 | 1.0097  | 0.539 | 1.0218  | 1.231 | 1.0391  | 2.240 | 1.0617  | 3.609 |
| 69 | 1.0025  | 0.129 | 1.0100  | 0.520 | 1.0226  | 1.187 | 1.0406  | 2.163 | 1.0642  | 3.489 |
| 70 | 1.0026  | 0.123 | 1.0103  | 0.499 | 1.0234  | 1.141 | 1.0420  | 2.082 | 1.0666  | 3.362 |
| 71 | 1.0026  | 0.118 | 1.0106  | 0.478 | 1.0241  | 1.094 | 1.0434  | 1.998 | 1.0689  | 3.229 |
| 72 | 1.0027  | 0.112 | 1.0109  | 0.456 | 1.0247  | 1.045 | 1.0447  | 1.911 | 1.0712  | 3.091 |
| 73 | 1.0028  | 0.107 | 1.0112  | 0.434 | 1.0254  | 0.995 | 1.0460  | 1.821 | 1.0734  | 2.948 |
| 74 | 1.0028  | 0.102 | 1.0114  | 0.411 | 1.0260  | 0.944 | 1.0472  | 1.728 | 1.0755  | 2.800 |
| 75 | 1.0029  | 0.096 | 1.0117  | 0.388 | 1.0266  | 0.892 | 1.0483  | 1.632 | 1.0774  | 2.649 |
| 76 | 1.0029  | 0.090 | 1.0120  | 0.365 | 1.0272  | 0.838 | 1.0494  | 1.534 | 1.0793  | 2.493 |
| 77 | 1.0030  | 0.084 | 1.0122  | 0.341 | 1.0277  | 0.783 | 1.0504  | 1.434 | 1.0811  | 2.333 |
| 78 | 1.0030  | 0.078 | 1.0123  | 0.316 | 1.0282  | 0.727 | 1.0513  | 1.332 | 1.0828  | 2.169 |
| 79 | 1.0031  | 0.072 | 1.0125  | 0.291 | 1.0287  | 0.671 | 1.0522  | 1.229 | 1.0843  | 2.001 |
| 80 | 1.0031  | 0.066 | 1.0127  | 0.266 | 1.0291  | 0.614 | 1.0530  | 1.124 | 1.0857  | 1.830 |
| 81 | 1.0032  | 0.060 | 1.0129  | 0.241 | 1.0295  | 0.555 | 1.0538  | 1.017 | 1.0870  | 1.656 |
| 82 | 1.0032  | 0.054 | 1.0130  | 0.215 | 1.0298  | 0.494 | 1.0544  | 0.908 | 1.0881  | 1.480 |
| 83 | 1.0032  | 0.047 | 1.0132  | 0.188 | 1.0301  | 0.433 | 1.0550  | 0.797 | 1.0892  | 1.301 |
| 84 | 1.0033  | 0.040 | 1.0133  | 0.162 | 1.0304  | 0.372 | 1.0555  | 0.685 | 1.0901  | 1.119 |
| 85 | 1.0033  | 0.033 | 1.0134  | 0.136 | 1.0307  | 0.311 | 1.0560  | 0.573 | 1.0908  | 0.935 |
| 86 | 1.0033  | 0.026 | 1.0134  | 0.109 | 1.0309  | 0.249 | 1.0563  | 0.460 | 1.0914  | 0.749 |
| 87 | 1.0033  | 0.019 | 1.0135  | 0.082 | 1.0310  | 0.187 | 1.0566  | 0.346 | 1.0919  | 0.562 |
| 88 | 1.0033  | 0.012 | 1.0135  | 0.054 | 1.0311  | 0.125 | 1.0568  | 0.232 | 1.0923  | 0.374 |
| 89 | 1.0033  | 0.006 | 1.0135  | 0.027 | 1.0311  | 0.063 | 1.0570  | 0.117 | 1.0926  | 0.187 |
| 90 | 1.0033  | 0.000 | 1.0136  | 0.000 | 1.0311  | 0.000 | 1.0570  | 0.000 | 1.0926  | 0.000 |

Note. Negative quantities are in heavy type.

Example.  $\frac{\tanh (0.4 / 74^{\circ})}{0.4 / 74^{\circ}} = 1.0472 \sqrt{1^{\circ}.728} = 1.0472 \sqrt{1^{\circ}.43'.41''}.$

TABLE V. CORRECTING FACTOR.  $\frac{\tanh \theta}{\theta} = r/\gamma$ . CONTINUED

|    | 0.6     |       | 0.7     |       | 0.8     |        | 0.9     |        | 1.0     |        |
|----|---------|-------|---------|-------|---------|--------|---------|--------|---------|--------|
| °  | °       | °     | °       | °     | °       | °      | °       | °      | °       | °      |
| 45 | 0.99010 | 6.817 | 0.98189 | 9.214 | 0.96971 | 11.902 | 0.95284 | 14.839 | 0.93077 | 17.956 |
| 46 | 0.99417 | 6.854 | 0.98727 | 9.281 | 0.97646 | 12.020 | 0.96089 | 15.020 | 0.93999 | 18.216 |
| 47 | 0.99830 | 6.881 | 0.99279 | 9.337 | 0.98339 | 12.122 | 0.96921 | 15.185 | 0.94950 | 18.461 |
| 48 | 1.00248 | 6.901 | 0.99840 | 9.383 | 0.99050 | 12.210 | 0.97779 | 15.334 | 0.95938 | 18.692 |
| 49 | 1.00672 | 6.912 | 1.00410 | 9.419 | 0.99780 | 12.285 | 0.98665 | 15.469 | 0.96966 | 18.908 |
| 50 | 1.01100 | 6.915 | 1.00991 | 9.443 | 1.00526 | 12.347 | 0.99578 | 15.588 | 0.98032 | 19.108 |
| 51 | 1.01533 | 6.908 | 1.01581 | 9.455 | 1.01289 | 12.395 | 1.00518 | 15.690 | 0.99136 | 19.290 |
| 52 | 1.01970 | 6.893 | 1.02181 | 9.457 | 1.02069 | 12.427 | 1.01486 | 15.777 | 1.00282 | 19.455 |
| 53 | 1.02412 | 6.870 | 1.02789 | 9.445 | 1.02864 | 12.444 | 1.02479 | 15.845 | 1.01469 | 19.600 |
| 54 | 1.02855 | 6.838 | 1.03403 | 9.423 | 1.03675 | 12.446 | 1.03500 | 15.895 | 1.02697 | 19.726 |
| 55 | 1.03300 | 6.797 | 1.04024 | 9.388 | 1.04501 | 12.432 | 1.04549 | 15.925 | 1.03970 | 19.828 |
| 56 | 1.03747 | 6.746 | 1.04653 | 9.338 | 1.05343 | 12.401 | 1.05626 | 15.934 | 1.05287 | 19.909 |
| 57 | 1.04195 | 6.685 | 1.05286 | 9.275 | 1.06196 | 12.353 | 1.06729 | 15.922 | 1.06648 | 19.965 |
| 58 | 1.04642 | 6.616 | 1.05921 | 9.200 | 1.07061 | 12.288 | 1.07854 | 15.889 | 1.08054 | 19.996 |
| 59 | 1.05089 | 6.537 | 1.06561 | 9.112 | 1.07939 | 12.203 | 1.09007 | 15.833 | 1.09506 | 19.998 |
| 60 | 1.05537 | 6.448 | 1.07210 | 9.010 | 1.08829 | 12.100 | 1.10188 | 15.753 | 1.11009 | 19.973 |
| 61 | 1.05980 | 6.350 | 1.07856 | 8.893 | 1.09726 | 11.978 | 1.11390 | 15.647 | 1.12555 | 19.918 |
| 62 | 1.06420 | 6.242 | 1.08500 | 8.762 | 1.10630 | 11.835 | 1.12614 | 15.514 | 1.14144 | 19.832 |
| 63 | 1.06858 | 6.124 | 1.09143 | 8.617 | 1.11540 | 11.673 | 1.13856 | 15.355 | 1.15777 | 19.711 |
| 64 | 1.07293 | 5.998 | 1.09783 | 8.458 | 1.12456 | 11.491 | 1.15116 | 15.169 | 1.17454 | 19.555 |
| 65 | 1.07722 | 5.862 | 1.10420 | 8.285 | 1.13373 | 11.288 | 1.16392 | 14.956 | 1.19173 | 19.362 |
| 66 | 1.08143 | 5.716 | 1.11053 | 8.098 | 1.14291 | 11.064 | 1.17684 | 14.711 | 1.20935 | 19.131 |
| 67 | 1.08560 | 5.561 | 1.11683 | 7.895 | 1.15208 | 10.820 | 1.18989 | 14.438 | 1.22737 | 18.860 |
| 68 | 1.08967 | 5.397 | 1.12299 | 7.678 | 1.16118 | 10.552 | 1.20298 | 14.132 | 1.24569 | 18.545 |
| 69 | 1.09363 | 5.223 | 1.12904 | 7.447 | 1.17020 | 10.263 | 1.21608 | 13.796 | 1.26429 | 18.188 |
| 70 | 1.09750 | 5.041 | 1.13500 | 7.202 | 1.17915 | 9.953  | 1.22919 | 13.428 | 1.28316 | 17.785 |
| 71 | 1.10125 | 4.850 | 1.14083 | 6.943 | 1.18796 | 9.621  | 1.24226 | 13.026 | 1.30221 | 17.334 |
| 72 | 1.10490 | 4.650 | 1.14651 | 6.671 | 1.19659 | 9.268  | 1.25524 | 12.592 | 1.32140 | 16.836 |
| 73 | 1.10842 | 4.441 | 1.15201 | 6.386 | 1.20503 | 8.894  | 1.26807 | 12.126 | 1.34063 | 16.290 |
| 74 | 1.11180 | 4.225 | 1.15734 | 6.087 | 1.21325 | 8.499  | 1.28067 | 11.628 | 1.35986 | 15.693 |
| 75 | 1.11503 | 4.002 | 1.16244 | 5.775 | 1.22121 | 8.083  | 1.29300 | 11.097 | 1.37894 | 15.043 |
| 76 | 1.11810 | 3.772 | 1.16733 | 5.450 | 1.22889 | 7.648  | 1.30500 | 10.532 | 1.39775 | 14.342 |
| 77 | 1.12102 | 3.534 | 1.17197 | 5.114 | 1.23623 | 7.192  | 1.31659 | 9.936  | 1.41620 | 13.591 |
| 78 | 1.12375 | 3.289 | 1.17636 | 4.767 | 1.24319 | 6.718  | 1.32772 | 9.309  | 1.43412 | 12.788 |
| 79 | 1.12628 | 3.038 | 1.18046 | 4.409 | 1.24976 | 6.227  | 1.33830 | 8.653  | 1.45141 | 11.935 |
| 80 | 1.12863 | 2.781 | 1.18427 | 4.042 | 1.25591 | 5.718  | 1.34827 | 7.966  | 1.46790 | 11.032 |
| 81 | 1.13078 | 2.519 | 1.18777 | 3.666 | 1.26158 | 5.195  | 1.35754 | 7.254  | 1.48345 | 10.081 |
| 82 | 1.13273 | 2.251 | 1.19094 | 3.281 | 1.26674 | 4.656  | 1.36607 | 6.517  | 1.49790 | 9.087  |
| 83 | 1.13447 | 1.979 | 1.19377 | 2.888 | 1.27138 | 4.104  | 1.37378 | 5.757  | 1.51110 | 8.052  |
| 84 | 1.13598 | 1.704 | 1.19626 | 2.488 | 1.27545 | 3.540  | 1.38059 | 4.975  | 1.52289 | 6.977  |
| 85 | 1.13727 | 1.425 | 1.19839 | 2.083 | 1.27895 | 2.966  | 1.38646 | 4.175  | 1.53314 | 5.868  |
| 86 | 1.13832 | 1.144 | 1.20013 | 1.672 | 1.28185 | 2.383  | 1.39132 | 3.360  | 1.54170 | 4.731  |
| 87 | 1.13915 | 0.860 | 1.20150 | 1.257 | 1.28413 | 1.794  | 1.39518 | 2.532  | 1.54848 | 3.571  |
| 88 | 1.13975 | 0.574 | 1.20247 | 0.839 | 1.28575 | 1.199  | 1.39793 | 1.692  | 1.55339 | 2.391  |
| 89 | 1.14010 | 0.287 | 1.20306 | 0.420 | 1.28671 | 0.600  | 1.39959 | 0.847  | 1.55637 | 1.198  |
| 90 | 1.14022 | 0.000 | 1.20327 | 0.000 | 1.28700 | 0.000  | 1.40017 | 0.000  | 1.55740 | 0.000  |

Note. Negative quantities are in heavy type.

Example.  $\frac{\tanh (0.9 / 75^{\circ})}{0.9 / 75^{\circ}} = 1.293 \sqrt{11^{\circ}.097} = 1.293 \sqrt{11^{\circ}.05'.49''}$ .

TABLE V. CORRECTING FACTOR.  $\frac{\tanh \theta}{\theta} = r/\gamma$ . CONTINUED

|    | I.1     |        | I.2     |        | I.3     |        | I.4     |        | I.5     |        |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| °  | °       |        | °       |        | °       |        | °       |        | °       |        |
| 45 | 0.90354 | 21.167 | 0.87142 | 24.384 | 0.83515 | 27.536 | 0.79593 | 30.516 | 0.75487 | 33.288 |
| 46 | 0.91359 | 21.524 | 0.88183 | 24.856 | 0.84561 | 28.118 | 0.80600 | 31.225 | 0.76427 | 34.116 |
| 47 | 0.92400 | 21.869 | 0.89275 | 25.315 | 0.85661 | 28.699 | 0.81664 | 31.936 | 0.77420 | 34.952 |
| 48 | 0.93482 | 22.201 | 0.90425 | 25.764 | 0.86815 | 29.279 | 0.82786 | 32.648 | 0.78467 | 35.792 |
| 49 | 0.94618 | 22.520 | 0.91625 | 26.202 | 0.88038 | 29.852 | 0.83971 | 33.360 | 0.79573 | 36.638 |
| 50 | 0.95809 | 22.822 | 0.92892 | 26.629 | 0.89331 | 30.416 | 0.85228 | 34.070 | 0.80747 | 37.490 |
| 51 | 0.97055 | 23.109 | 0.94225 | 27.043 | 0.90685 | 30.973 | 0.86557 | 34.778 | 0.81993 | 38.348 |
| 52 | 0.98345 | 23.379 | 0.95617 | 27.442 | 0.92123 | 31.521 | 0.87964 | 35.485 | 0.83320 | 39.213 |
| 53 | 0.99700 | 23.630 | 0.97083 | 27.826 | 0.93638 | 32.059 | 0.89457 | 36.189 | 0.84727 | 40.086 |
| 54 | 1.0111  | 23.861 | 0.98625 | 28.193 | 0.95238 | 32.586 | 0.91043 | 36.890 | 0.86220 | 40.965 |
| 55 | 1.0258  | 24.070 | 1.0025  | 28.540 | 0.96938 | 33.100 | 0.92729 | 37.587 | 0.87813 | 41.849 |
| 56 | 1.0412  | 24.256 | 1.0196  | 28.867 | 0.98738 | 33.599 | 0.94529 | 38.279 | 0.89513 | 42.738 |
| 57 | 1.0572  | 24.417 | 1.0375  | 29.173 | 1.0065  | 34.082 | 0.96443 | 38.964 | 0.91333 | 43.634 |
| 58 | 1.0739  | 24.551 | 1.0564  | 29.454 | 1.0266  | 34.548 | 0.98479 | 39.642 | 0.93280 | 44.534 |
| 59 | 1.0914  | 24.657 | 1.0763  | 29.708 | 1.0482  | 34.993 | 1.0066  | 40.311 | 0.95367 | 45.441 |
| 60 | 1.1096  | 24.733 | 1.0973  | 29.932 | 1.0710  | 35.416 | 1.0301  | 40.968 | 0.97613 | 46.349 |
| 61 | 1.1286  | 24.774 | 1.1195  | 30.124 | 1.0953  | 35.813 | 1.0551  | 41.611 | 1.0003  | 47.262 |
| 62 | 1.1484  | 24.778 | 1.1428  | 30.280 | 1.1212  | 36.180 | 1.0820  | 42.239 | 1.0264  | 48.176 |
| 63 | 1.1689  | 24.745 | 1.1674  | 30.397 | 1.1488  | 36.515 | 1.1109  | 42.850 | 1.0546  | 49.092 |
| 64 | 1.1904  | 24.672 | 1.1934  | 30.473 | 1.1784  | 36.814 | 1.1421  | 43.438 | 1.0851  | 50.009 |
| 65 | 1.2126  | 24.554 | 1.2208  | 30.502 | 1.2099  | 37.072 | 1.1759  | 44.001 | 1.1184  | 50.924 |
| 66 | 1.2357  | 24.388 | 1.2466  | 30.480 | 1.2436  | 37.285 | 1.2124  | 44.535 | 1.1546  | 51.834 |
| 67 | 1.2596  | 24.173 | 1.2799  | 30.403 | 1.2798  | 37.446 | 1.2520  | 45.034 | 1.1943  | 52.739 |
| 68 | 1.2845  | 23.904 | 1.3119  | 30.263 | 1.3185  | 37.548 | 1.2951  | 45.493 | 1.2377  | 53.636 |
| 69 | 1.3101  | 23.579 | 1.3457  | 30.059 | 1.3600  | 37.585 | 1.3420  | 45.904 | 1.2857  | 54.520 |
| 70 | 1.3365  | 23.194 | 1.3811  | 29.781 | 1.4046  | 37.548 | 1.3933  | 46.260 | 1.3385  | 55.388 |
| 71 | 1.3637  | 22.744 | 1.4183  | 29.425 | 1.4525  | 37.430 | 1.4495  | 46.552 | 1.3973  | 56.235 |
| 72 | 1.3915  | 22.227 | 1.4573  | 28.983 | 1.5039  | 37.218 | 1.5113  | 46.768 | 1.4628  | 57.056 |
| 73 | 1.4200  | 21.639 | 1.4981  | 28.447 | 1.5592  | 36.899 | 1.5794  | 46.894 | 1.5362  | 57.841 |
| 74 | 1.4489  | 20.978 | 1.5407  | 27.810 | 1.6186  | 36.461 | 1.6546  | 46.916 | 1.6190  | 58.581 |
| 75 | 1.4783  | 20.238 | 1.5849  | 27.064 | 1.6822  | 35.890 | 1.7380  | 46.814 | 1.7129  | 59.266 |
| 76 | 1.5077  | 19.419 | 1.6307  | 26.202 | 1.7505  | 35.170 | 1.8308  | 46.567 | 1.8202  | 59.880 |
| 77 | 1.5373  | 18.518 | 1.6778  | 25.214 | 1.8235  | 34.280 | 1.9341  | 46.145 | 1.9438  | 60.403 |
| 78 | 1.5665  | 17.532 | 1.7261  | 24.094 | 1.9012  | 33.202 | 2.0498  | 45.514 | 2.0875  | 60.814 |
| 79 | 1.5955  | 16.462 | 1.7749  | 22.835 | 1.9835  | 31.912 | 2.1792  | 44.633 | 2.2562  | 61.076 |
| 80 | 1.6235  | 15.308 | 1.8239  | 21.429 | 2.0702  | 30.389 | 2.3240  | 43.454 | 2.4563  | 61.144 |
| 81 | 1.6505  | 14.069 | 1.8726  | 19.874 | 2.1604  | 28.609 | 2.4862  | 41.915 | 2.6973  | 60.956 |
| 82 | 1.6759  | 12.748 | 1.9200  | 18.168 | 2.2531  | 26.553 | 2.6669  | 39.944 | 2.9917  | 60.422 |
| 83 | 1.6996  | 11.350 | 1.9654  | 16.310 | 2.3496  | 24.200 | 2.8668  | 37.454 | 3.3577  | 59.412 |
| 84 | 1.7212  | 9.879  | 2.0078  | 14.306 | 2.4388  | 21.540 | 3.0848  | 34.354 | 3.8207  | 57.724 |
| 85 | 1.7402  | 8.343  | 2.0463  | 12.164 | 2.5267  | 18.568 | 3.3164  | 30.541 | 4.4198  | 55.045 |
| 86 | 1.7562  | 6.748  | 2.0796  | 9.898  | 2.6065  | 15.294 | 3.5521  | 25.931 | 5.2010  | 50.866 |
| 87 | 1.7691  | 5.105  | 2.1068  | 7.525  | 2.6744  | 11.747 | 3.7765  | 20.477 | 6.2275  | 44.356 |
| 88 | 1.7785  | 3.425  | 2.1269  | 5.067  | 2.7266  | 7.970  | 3.9665  | 14.214 | 7.5060  | 34.274 |
| 89 | 1.7843  | 1.719  | 2.1393  | 2.548  | 2.7596  | 4.029  | 4.0952  | 7.293  | 8.7880  | 19.305 |
| 90 | 1.7862  | 0.000  | 2.1434  | 0.000  | 2.7709  | 0.000  | 4.1413  | 0.000  | 9.4007  | 0.000  |

Note. Negative quantities are in heavy type.

Example.  $\frac{\tanh (1.3 / 45^{\circ})}{1.3 / 45^{\circ}} = 0.83515 \sqrt{27^{\circ}.536} = 0.83515 \sqrt{17^{\circ}.32'.10''}.$

TABLE V. CORRECTING FACTOR.  $\frac{\tanh \theta}{\theta} = r/\gamma$ . CONTINUED

|    | 1.6     |         | 1.7     |         | 1.8     |         | 1.9     |         | 2.0     |         |
|----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| °  | °       | °       | °       | °       | °       | °       | °       | °       | °       | °       |
| 45 | 0.71331 | 35.799  | 0.67224 | 38.016  | 0.63250 | 39.937  | 0.59484 | 41.562  | 0.55955 | 42.906  |
| 46 | 0.72175 | 36.733  | 0.67959 | 39.048  | 0.63883 | 41.050  | 0.60005 | 42.740  | 0.56375 | 44.133  |
| 47 | 0.73069 | 37.682  | 0.68741 | 40.098  | 0.64544 | 42.186  | 0.60553 | 43.942  | 0.56815 | 45.387  |
| 48 | 0.74013 | 38.642  | 0.69565 | 41.166  | 0.65239 | 43.341  | 0.61126 | 45.168  | 0.57275 | 46.667  |
| 49 | 0.75013 | 39.615  | 0.70435 | 42.251  | 0.65978 | 44.520  | 0.61726 | 46.422  | 0.57750 | 47.977  |
| 50 | 0.76069 | 40.600  | 0.71359 | 43.355  | 0.66756 | 45.725  | 0.62358 | 47.705  | 0.58250 | 49.319  |
| 51 | 0.77194 | 41.600  | 0.72329 | 44.480  | 0.67572 | 46.955  | 0.63026 | 49.019  | 0.58770 | 50.694  |
| 52 | 0.78381 | 42.616  | 0.73365 | 45.627  | 0.68433 | 48.213  | 0.63721 | 50.366  | 0.59315 | 52.103  |
| 53 | 0.79650 | 43.646  | 0.74465 | 46.797  | 0.69350 | 49.501  | 0.64458 | 51.747  | 0.59885 | 53.550  |
| 54 | 0.81000 | 44.691  | 0.75629 | 47.992  | 0.70317 | 50.820  | 0.65232 | 53.165  | 0.60480 | 55.039  |
| 55 | 0.82438 | 45.754  | 0.76865 | 49.212  | 0.71344 | 52.174  | 0.66047 | 54.621  | 0.61100 | 56.571  |
| 56 | 0.83969 | 46.834  | 0.78188 | 50.461  | 0.72433 | 53.564  | 0.66911 | 56.121  | 0.61750 | 58.149  |
| 57 | 0.85613 | 47.931  | 0.79600 | 51.739  | 0.73594 | 54.992  | 0.67821 | 57.666  | 0.62435 | 59.777  |
| 58 | 0.87369 | 49.047  | 0.81106 | 53.047  | 0.74828 | 56.460  | 0.68784 | 59.259  | 0.63150 | 61.458  |
| 59 | 0.89256 | 50.183  | 0.82724 | 54.389  | 0.76144 | 57.974  | 0.69805 | 60.905  | 0.63905 | 63.195  |
| 60 | 0.91288 | 51.339  | 0.84459 | 55.768  | 0.77550 | 59.535  | 0.70895 | 62.606  | 0.64695 | 64.995  |
| 61 | 0.93475 | 52.516  | 0.86324 | 57.181  | 0.79056 | 61.145  | 0.72047 | 64.366  | 0.65530 | 66.860  |
| 62 | 0.95834 | 53.714  | 0.88335 | 58.635  | 0.80672 | 62.810  | 0.73274 | 66.191  | 0.66405 | 68.793  |
| 63 | 0.98394 | 54.935  | 0.90506 | 60.131  | 0.82406 | 64.533  | 0.74599 | 68.085  | 0.67330 | 70.802  |
| 64 | 1.0117  | 56.181  | 0.92853 | 61.674  | 0.84272 | 66.318  | 0.75974 | 70.052  | 0.68310 | 72.891  |
| 65 | 1.0419  | 57.452  | 0.95400 | 63.266  | 0.86283 | 68.172  | 0.77463 | 72.099  | 0.69340 | 75.068  |
| 66 | 1.0749  | 58.748  | 0.98176 | 64.910  | 0.88461 | 70.098  | 0.79063 | 74.232  | 0.70430 | 77.338  |
| 67 | 1.1111  | 60.072  | 1.0121  | 66.613  | 0.90817 | 72.104  | 0.80774 | 76.459  | 0.71585 | 79.707  |
| 68 | 1.1508  | 61.425  | 1.0453  | 68.376  | 0.93372 | 74.196  | 0.82611 | 78.786  | 0.72810 | 82.184  |
| 69 | 1.1948  | 62.809  | 1.0817  | 70.209  | 0.96156 | 76.382  | 0.84579 | 81.221  | 0.74105 | 84.776  |
| 70 | 1.2433  | 64.224  | 1.1219  | 72.116  | 0.99189 | 78.671  | 0.86705 | 83.773  | 0.75475 | 87.490  |
| 71 | 1.2974  | 65.674  | 1.1663  | 74.106  | 1.0251  | 81.074  | 0.88989 | 86.454  | 0.76925 | 90.337  |
| 72 | 1.3580  | 67.160  | 1.2157  | 76.187  | 1.0614  | 83.602  | 0.91453 | 89.273  | 0.78465 | 93.326  |
| 73 | 1.4262  | 68.686  | 1.2708  | 78.371  | 1.1013  | 86.266  | 0.94111 | 92.244  | 0.80095 | 96.466  |
| 74 | 1.5033  | 70.256  | 1.3325  | 80.670  | 1.1452  | 89.084  | 0.96974 | 95.379  | 0.81815 | 99.769  |
| 75 | 1.5913  | 71.873  | 1.4020  | 83.101  | 1.1936  | 92.076  | 1.0005  | 98.692  | 0.83625 | 103.245 |
| 76 | 1.6924  | 73.543  | 1.4807  | 85.685  | 1.2470  | 95.261  | 1.0337  | 102.202 | 0.85525 | 106.905 |
| 77 | 1.8098  | 75.274  | 1.5704  | 88.447  | 1.3061  | 98.664  | 1.0693  | 105.925 | 0.87515 | 110.760 |
| 78 | 1.9473  | 77.078  | 1.6731  | 91.420  | 1.3714  | 102.315 | 1.1074  | 109.881 | 0.89585 | 114.820 |
| 79 | 2.1104  | 78.966  | 1.7915  | 94.644  | 1.4436  | 106.250 | 1.1478  | 114.088 | 0.91720 | 119.093 |
| 80 | 2.3068  | 80.960  | 1.9288  | 98.175  | 1.5231  | 110.510 | 1.1906  | 118.568 | 0.93905 | 123.590 |
| 81 | 2.5472  | 83.087  | 2.0894  | 102.083 | 1.6104  | 115.140 | 1.2352  | 123.344 | 0.96110 | 128.319 |
| 82 | 2.8474  | 85.390  | 2.2775  | 106.466 | 1.7056  | 120.191 | 1.2813  | 128.428 | 0.98305 | 133.281 |
| 83 | 3.2326  | 87.937  | 2.4992  | 111.446 | 1.8078  | 125.722 | 1.3279  | 133.842 | 1.0045  | 138.474 |
| 84 | 3.7435  | 90.839  | 2.7604  | 117.199 | 1.9155  | 131.793 | 1.3739  | 139.590 | 1.0249  | 143.894 |
| 85 | 4.4502  | 94.289  | 3.0648  | 123.945 | 2.0255  | 138.448 | 1.4179  | 145.673 | 1.0436  | 149.528 |
| 86 | 5.4825  | 98.668  | 3.4114  | 131.956 | 2.1329  | 145.720 | 1.4579  | 152.075 | 1.0601  | 155.359 |
| 87 | 7.1175  | 104.791 | 3.7856  | 141.527 | 2.2309  | 153.606 | 1.4921  | 158.768 | 1.0738  | 161.359 |
| 88 | 9.9938  | 114.760 | 4.1472  | 152.868 | 2.3107  | 162.043 | 1.5184  | 165.700 | 1.0840  | 167.494 |
| 89 | 15.528  | 135.011 | 4.4231  | 165.896 | 2.3628  | 170.905 | 1.5349  | 172.805 | 1.0904  | 173.723 |
| 90 | 21.395  | 180.000 | 4.5275  | 180.000 | 2.3813  | 180.000 | 1.5406  | 180.000 | 1.0925  | 180.000 |

Note. Negative quantities are in heavy type.

Example.  $\frac{\tanh (2.0 / 80^{\circ})}{2.0 / 80^{\circ}} = 0.93905 \sqrt{123^{\circ}.590} = 0.93905 \sqrt{123^{\circ}.35'.24''}$ .

TABLE V. CORRECTING FACTOR.  $\frac{\tanh \theta}{\theta} = r/\gamma$ . CONTINUED

|    | 2.1     |         | 2.2     |         | 2.3     |         | 2.4     |         | 2.5     |         |
|----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| °  | °       | °       | °       | °       | °       | °       | °       | °       | °       |         |
| 45 | 0.52690 | 43.993  | 0.49691 | 44.845  | 0.46952 | 45.492  | 0.44467 | 45.961  | 0.42212 | 46.283  |
| 46 | 0.53019 | 45.254  | 0.49941 | 46.127  | 0.47139 | 46.783  | 0.44592 | 47.252  | 0.42292 | 47.564  |
| 47 | 0.53362 | 46.542  | 0.50200 | 47.436  | 0.47322 | 48.101  | 0.44717 | 48.567  | 0.42368 | 48.868  |
| 48 | 0.53714 | 47.859  | 0.50465 | 48.774  | 0.47509 | 49.446  | 0.44842 | 49.908  | 0.42440 | 50.195  |
| 49 | 0.54081 | 49.206  | 0.50732 | 50.142  | 0.47696 | 50.820  | 0.44958 | 51.277  | 0.42504 | 51.548  |
| 50 | 0.54462 | 50.586  | 0.51009 | 51.542  | 0.47883 | 52.226  | 0.45075 | 52.676  | 0.42560 | 52.929  |
| 51 | 0.54852 | 52.001  | 0.51291 | 52.978  | 0.48070 | 53.666  | 0.45188 | 54.107  | 0.42612 | 54.340  |
| 52 | 0.55257 | 53.452  | 0.51578 | 54.452  | 0.48256 | 55.144  | 0.45292 | 55.572  | 0.42652 | 55.782  |
| 53 | 0.55681 | 54.943  | 0.51868 | 55.965  | 0.48443 | 56.659  | 0.45392 | 57.073  | 0.42684 | 57.256  |
| 54 | 0.56114 | 56.476  | 0.52168 | 57.520  | 0.48630 | 58.215  | 0.45488 | 58.614  | 0.42704 | 58.766  |
| 55 | 0.56562 | 58.055  | 0.52473 | 59.120  | 0.48813 | 59.815  | 0.45571 | 60.196  | 0.42712 | 60.314  |
| 56 | 0.57029 | 59.682  | 0.52782 | 60.769  | 0.48991 | 61.462  | 0.45646 | 61.822  | 0.42708 | 61.903  |
| 57 | 0.57515 | 61.361  | 0.53095 | 62.469  | 0.49170 | 63.160  | 0.45713 | 63.496  | 0.42688 | 63.534  |
| 58 | 0.58014 | 63.094  | 0.53414 | 64.226  | 0.49339 | 64.912  | 0.45767 | 65.220  | 0.42648 | 65.211  |
| 59 | 0.58533 | 64.888  | 0.53736 | 66.042  | 0.49504 | 66.722  | 0.45804 | 66.998  | 0.42588 | 66.938  |
| 60 | 0.59071 | 66.745  | 0.54064 | 67.921  | 0.49661 | 68.594  | 0.45829 | 68.835  | 0.42512 | 68.718  |
| 61 | 0.59629 | 68.671  | 0.54395 | 69.869  | 0.49813 | 70.531  | 0.45838 | 70.735  | 0.42412 | 70.556  |
| 62 | 0.60210 | 70.669  | 0.54732 | 71.889  | 0.49957 | 72.540  | 0.45829 | 72.701  | 0.42288 | 72.455  |
| 63 | 0.60814 | 72.744  | 0.55073 | 73.988  | 0.50087 | 74.625  | 0.45800 | 74.740  | 0.42140 | 74.419  |
| 64 | 0.61443 | 74.902  | 0.55418 | 76.171  | 0.50209 | 76.791  | 0.45750 | 76.855  | 0.41960 | 76.455  |
| 65 | 0.62090 | 77.151  | 0.55764 | 78.443  | 0.50317 | 79.044  | 0.45675 | 79.053  | 0.41752 | 78.567  |
| 66 | 0.62767 | 79.496  | 0.56109 | 80.810  | 0.50413 | 81.390  | 0.45579 | 81.341  | 0.41512 | 80.761  |
| 67 | 0.63471 | 81.942  | 0.56459 | 83.280  | 0.50491 | 83.836  | 0.45454 | 83.723  | 0.41240 | 83.044  |
| 68 | 0.64205 | 84.499  | 0.56814 | 85.858  | 0.50557 | 86.389  | 0.45300 | 86.208  | 0.40928 | 85.424  |
| 69 | 0.64962 | 87.172  | 0.57164 | 88.552  | 0.50604 | 89.056  | 0.45121 | 88.802  | 0.40580 | 87.906  |
| 70 | 0.65752 | 89.968  | 0.57518 | 91.370  | 0.50630 | 91.843  | 0.44913 | 91.514  | 0.40196 | 90.500  |
| 71 | 0.66571 | 92.898  | 0.57873 | 94.319  | 0.50639 | 94.759  | 0.44671 | 94.352  | 0.39771 | 93.215  |
| 72 | 0.67424 | 95.968  | 0.58223 | 97.405  | 0.50626 | 97.811  | 0.44396 | 97.324  | 0.39306 | 96.061  |
| 73 | 0.68305 | 99.185  | 0.58568 | 100.636 | 0.50596 | 101.006 | 0.44092 | 100.439 | 0.38802 | 99.048  |
| 74 | 0.69214 | 102.560 | 0.58914 | 104.020 | 0.50543 | 104.352 | 0.43754 | 103.706 | 0.38260 | 102.187 |
| 75 | 0.70152 | 106.099 | 0.59255 | 107.564 | 0.50470 | 107.858 | 0.43388 | 107.134 | 0.37681 | 105.489 |
| 76 | 0.71114 | 109.811 | 0.59586 | 111.274 | 0.50378 | 111.531 | 0.42992 | 110.730 | 0.37068 | 108.966 |
| 77 | 0.72096 | 113.702 | 0.59914 | 115.156 | 0.50265 | 115.374 | 0.42575 | 114.505 | 0.36427 | 112.633 |
| 78 | 0.73086 | 117.778 | 0.60232 | 119.212 | 0.50135 | 119.392 | 0.42138 | 118.467 | 0.35762 | 116.501 |
| 79 | 0.74090 | 122.043 | 0.60536 | 123.446 | 0.49991 | 123.590 | 0.41683 | 122.620 | 0.35080 | 120.582 |
| 80 | 0.75086 | 126.501 | 0.60832 | 127.859 | 0.49835 | 127.968 | 0.41221 | 126.968 | 0.34392 | 124.887 |
| 81 | 0.76067 | 131.151 | 0.61109 | 132.449 | 0.49674 | 132.527 | 0.40759 | 131.517 | 0.33706 | 129.427 |
| 82 | 0.77019 | 135.989 | 0.61364 | 137.212 | 0.49509 | 137.264 | 0.40305 | 136.267 | 0.33033 | 134.209 |
| 83 | 0.77924 | 141.011 | 0.61600 | 142.140 | 0.49343 | 142.171 | 0.39869 | 141.212 | 0.32391 | 139.233 |
| 84 | 0.78767 | 146.210 | 0.61814 | 147.223 | 0.49183 | 147.237 | 0.39463 | 146.344 | 0.31790 | 144.499 |
| 85 | 0.79524 | 151.570 | 0.62000 | 152.449 | 0.49039 | 152.450 | 0.39094 | 151.650 | 0.31248 | 149.994 |
| 86 | 0.80176 | 157.070 | 0.62155 | 157.800 | 0.48893 | 157.793 | 0.38775 | 157.113 | 0.30778 | 155.703 |
| 87 | 0.80710 | 162.690 | 0.62282 | 163.254 | 0.48804 | 163.245 | 0.38516 | 162.710 | 0.30396 | 161.598 |
| 88 | 0.81100 | 168.406 | 0.62374 | 168.790 | 0.48726 | 168.781 | 0.38325 | 168.413 | 0.30113 | 167.645 |
| 89 | 0.81338 | 174.187 | 0.62427 | 174.381 | 0.48678 | 174.376 | 0.38207 | 174.188 | 0.29940 | 173.796 |
| 90 | 0.81424 | 180.000 | 0.62445 | 180.000 | 0.48661 | 180.000 | 0.38167 | 180.000 | 0.29880 | 180.000 |

Note Negative quantities are in heavy type.

Example.  $\frac{\tanh (2.3 / 90^{\circ})}{2.3 / 90^{\circ}} = 0.48661 / \overline{180^{\circ}} = 0.48661 / 180^{\circ}.$

TABLE V. CORRECTING FACTOR.  $\frac{\tanh \theta}{\theta} = r/\gamma$ . CONTINUED

|    | 2.6             | 2.7             | 2.8             | 2.9              | 3.0              |
|----|-----------------|-----------------|-----------------|------------------|------------------|
| 45 | 0.40173 46.480  | 0.38326 46.576  | 0.36657 46.595  | 0.35145 46.554   | 0.33770 46.468   |
| 46 | 0.40215 47.744  | 0.38341 47.821  | 0.36650 47.816  | 0.35121 47.748   | 0.33733 47.637   |
| 47 | 0.40250 49.030  | 0.38348 49.083  | 0.36632 49.051  | 0.35086 48.955   | 0.33690 48.815   |
| 48 | 0.40281 50.337  | 0.38344 50.365  | 0.36607 50.303  | 0.35045 50.175   | 0.33637 50.002   |
| 49 | 0.40304 51.667  | 0.38333 51.666  | 0.36571 51.572  | 0.34997 51.409   | 0.33573 51.198   |
| 50 | 0.40319 53.022  | 0.38311 52.988  | 0.36525 52.857  | 0.34928 52.656   | 0.33497 52.405   |
| 51 | 0.40319 54.403  | 0.38278 54.332  | 0.36464 54.160  | 0.34848 53.916   | 0.33410 53.622   |
| 52 | 0.40308 55.812  | 0.38233 55.700  | 0.36389 55.483  | 0.34759 55.191   | 0.33307 54.849   |
| 53 | 0.40288 57.250  | 0.38174 57.094  | 0.36300 56.826  | 0.34652 56.481   | 0.33191 56.086   |
| 54 | 0.40254 58.717  | 0.38096 58.513  | 0.36196 58.190  | 0.34528 57.788   | 0.33058 57.334   |
| 55 | 0.40200 60.219  | 0.38000 59.959  | 0.36075 59.577  | 0.34386 59.112   | 0.32907 58.593   |
| 56 | 0.40131 61.756  | 0.37889 61.437  | 0.35932 60.987  | 0.34226 60.452   | 0.32737 59.862   |
| 57 | 0.40046 63.331  | 0.37756 62.945  | 0.35768 62.422  | 0.34043 61.810   | 0.32546 61.141   |
| 58 | 0.39942 64.947  | 0.37600 64.487  | 0.35580 63.884  | 0.33837 63.186   | 0.32332 62.432   |
| 59 | 0.39812 66.606  | 0.37422 66.066  | 0.35367 65.374  | 0.33602 64.582   | 0.32093 63.735   |
| 60 | 0.39658 68.313  | 0.37215 67.684  | 0.35126 66.895  | 0.33345 65.999   | 0.31827 65.049   |
| 61 | 0.39481 70.069  | 0.36980 69.343  | 0.34856 68.449  | 0.33056 67.439   | 0.31532 66.375   |
| 62 | 0.39273 71.879  | 0.36716 71.048  | 0.34554 70.036  | 0.32734 68.904   | 0.31206 67.713   |
| 63 | 0.39035 73.747  | 0.36416 72.801  | 0.34217 71.660  | 0.32378 70.392   | 0.30847 69.063   |
| 64 | 0.38765 75.677  | 0.36081 74.607  | 0.33843 73.325  | 0.31985 71.907   | 0.30451 70.426   |
| 65 | 0.38458 77.675  | 0.35709 76.470  | 0.33430 75.033  | 0.31552 73.452   | 0.30016 71.801   |
| 66 | 0.38115 79.747  | 0.35296 78.393  | 0.32974 76.789  | 0.31076 75.027   | 0.29539 73.192   |
| 67 | 0.37733 81.898  | 0.34841 80.383  | 0.32474 78.599  | 0.30556 76.639   | 0.29018 74.598   |
| 68 | 0.37310 84.136  | 0.34341 82.449  | 0.31927 80.464  | 0.29988 78.289   | 0.28450 76.021   |
| 69 | 0.36844 86.469  | 0.33794 84.597  | 0.31331 82.396  | 0.29369 79.982   | 0.27832 77.463   |
| 70 | 0.36333 88.902  | 0.33198 86.833  | 0.30683 84.399  | 0.28697 81.722   | 0.27161 78.926   |
| 71 | 0.35777 91.451  | 0.32553 89.168  | 0.29982 86.482  | 0.27970 83.518   | 0.26434 80.414   |
| 72 | 0.35176 94.122  | 0.31857 91.613  | 0.29226 88.655  | 0.27186 85.378   | 0.25649 81.932   |
| 73 | 0.34529 96.929  | 0.31109 94.184  | 0.28415 90.931  | 0.26343 87.310   | 0.24803 83.485   |
| 74 | 0.33837 99.884  | 0.30311 96.893  | 0.27548 93.325  | 0.25439 89.328   | 0.23894 85.081   |
| 75 | 0.33102 103.003 | 0.29464 99.757  | 0.26625 95.857  | 0.24475 91.448   | 0.22922 86.729   |
| 76 | 0.32328 106.302 | 0.28570 102.799 | 0.25648 98.547  | 0.23450 93.690   | 0.21883 88.443   |
| 77 | 0.31519 109.799 | 0.27634 106.038 | 0.24621 101.421 | 0.22366 96.078   | 0.20779 90.241   |
| 78 | 0.30680 113.513 | 0.26661 109.505 | 0.23546 104.512 | 0.21224 98.647   | 0.19610 92.146   |
| 79 | 0.29821 117.466 | 0.25659 113.231 | 0.22431 107.861 | 0.20030 101.437  | 0.18376 94.191   |
| 80 | 0.28950 121.678 | 0.24637 117.249 | 0.21284 111.519 | 0.18788 104.504  | 0.17082 96.419   |
| 81 | 0.28081 126.171 | 0.23609 121.599 | 0.20116 115.543 | 0.17507 107.921  | 0.15730 98.893   |
| 82 | 0.27227 130.966 | 0.22589 126.322 | 0.18943 120.004 | 0.16200 111.784  | 0.14329 101.700  |
| 83 | 0.26405 136.078 | 0.21597 131.464 | 0.17783 124.989 | 0.14880 116.221  | 0.12887 104.976  |
| 84 | 0.25633 141.517 | 0.20654 137.060 | 0.16660 130.591 | 0.13573 121.403  | 0.11419 108.916  |
| 85 | 0.24931 147.286 | 0.19786 143.143 | 0.15604 136.907 | 0.12307 127.549  | 0.099443 113.839 |
| 86 | 0.24319 153.370 | 0.19020 149.722 | 0.14651 144.025 | 0.11122 134.931  | 0.084970 120.254 |
| 87 | 0.23818 159.740 | 0.18384 156.778 | 0.13844 151.988 | 0.10074 143.833  | 0.071310 128.990 |
| 88 | 0.23446 166.349 | 0.17906 164.253 | 0.13224 160.762 | 0.092383 154.456 | 0.059383 141.294 |
| 89 | 0.23216 173.129 | 0.17609 172.042 | 0.12832 170.195 | 0.086890 166.709 | 0.050750 158.480 |
| 90 | 0.23138 180.000 | 0.17509 180.000 | 0.12698 180.000 | 0.084969 180.000 | 0.047517 180.000 |

Note. Negative quantities are in heavy type.

Example.  $\frac{\tanh (2.9 / 85^{\circ})}{2.9 / 85^{\circ}} = 0.12307 \sqrt{127^{\circ}.549} = 0.12307 \sqrt{127^{\circ}.32'.56''}.$



TABLE VI  
FUNCTIONS OF SEMI-IMAGINARIES.  $f(\rho / 45^\circ) = r / \gamma$

| $\rho$ | Sinh $\circ, ,$ |        | Cosh $\circ, ,$ |        | Tanh $\circ, ,$ |       |
|--------|-----------------|--------|-----------------|--------|-----------------|-------|
| 0      | 0.              | 45.00  | 1.              | 0.     | 0.              | 45.00 |
| 0.1    | 0.10000         | 45.06  | 1.00001         | 0.17   | 0.09999         | 44.49 |
| 0.2    | 0.20000         | 45.23  | 1.00013         | 1.09   | 0.19997         | 44.14 |
| 0.3    | 0.30001         | 45.52  | 1.0007          | 2.35   | 0.29981         | 43.17 |
| 0.4    | 0.40005         | 46.32  | 1.0021          | 4.35   | 0.39921         | 41.57 |
| 0.5    | 0.50016         | 47.23  | 1.0052          | 7.08   | 0.49757         | 40.15 |
| 0.6    | 0.60042         | 48.27  | 1.0107          | 10.15  | 0.59406         | 38.11 |
| 0.7    | 0.70094         | 49.40  | 1.0198          | 13.53  | 0.68732         | 35.47 |
| 0.8    | 0.80184         | 51.06  | 1.0336          | 18.00  | 0.77577         | 33.06 |
| 0.9    | 0.90327         | 52.44  | 1.0533          | 22.34  | 0.85756         | 30.10 |
| 1.0    | 1.0055          | 54.32  | 1.0803          | 27.29  | 0.93077         | 27.03 |
| 1.1    | 1.1089          | 56.31  | 1.1157          | 32.41  | 0.99389         | 23.50 |
| 1.2    | 1.2138          | 58.41  | 1.1608          | 38.05  | 1.0457          | 20.36 |
| 1.3    | 1.3205          | 61.02  | 1.2163          | 43.35  | 1.0857          | 17.27 |
| 1.4    | 1.4297          | 63.34  | 1.2830          | 49.05  | 1.1143          | 14.29 |
| 1.5    | 1.5418          | 66.15  | 1.3616          | 54.33  | 1.1323          | 11.42 |
| 1.6    | 1.6575          | 69.07  | 1.4524          | 59.55  | 1.1413          | 9.12  |
| 1.7    | 1.7776          | 72.08  | 1.5556          | 65.09  | 1.1428          | 6.59  |
| 1.8    | 1.9029          | 75.18  | 1.6714          | 70.14  | 1.1385          | 5.04  |
| 1.9    | 2.0343          | 78.36  | 1.7999          | 75.10  | 1.1302          | 3.26  |
| 2.0    | 2.1726          | 82.01  | 1.9413          | 79.56  | 1.1191          | 2.06  |
| 2.1    | 2.3190          | 85.34  | 2.0958          | 84.33  | 1.1065          | 1.01  |
| 2.2    | 2.4745          | 89.12  | 2.2636          | 89.03  | 1.0932          | 0.09  |
| 2.3    | 2.6404          | 92.57  | 2.4449          | 93.26  | 1.0799          | 0.29  |
| 2.4    | 2.8177          | 96.46  | 2.6403          | 97.44  | 1.0672          | 0.58  |
| 2.5    | 3.0079          | 100.39 | 2.8502          | 101.56 | 1.0553          | 1.17  |
| 2.6    | 3.2121          | 104.36 | 3.0753          | 106.05 | 1.0445          | 1.29  |
| 2.7    | 3.4318          | 108.36 | 3.3163          | 110.10 | 1.0348          | 1.34  |
| 2.8    | 3.6685          | 112.39 | 3.5741          | 114.15 | 1.0264          | 1.36  |
| 2.9    | 3.9236          | 116.43 | 3.8497          | 118.16 | 1.0192          | 1.33  |
| 3.0    | 4.1986          | 120.48 | 4.1443          | 122.16 | 1.0131          | 1.28  |
| 3.1    | 4.4948          | 124.56 | 4.4589          | 126.15 | 1.0080          | 1.19  |
| 3.2    | 4.8154          | 129.02 | 4.7955          | 130.15 | 1.0041          | 1.13  |
| 3.3    | 5.1586          | 133.09 | 5.1541          | 134.13 | 1.0008          | 1.04  |
| 3.4    | 5.5306          | 137.17 | 5.5393          | 138.13 | 0.9984          | 0.96  |
| 3.5    | 5.9305          | 141.24 | 5.9356          | 142.12 | 0.9967          | 0.48  |
| 3.6    | 6.3603          | 145.31 | 6.3900          | 146.11 | 0.9954          | 0.40  |
| 3.7    | 6.8244          | 149.38 | 6.8606          | 150.10 | 0.9947          | 0.32  |
| 3.8    | 7.3228          | 153.44 | 7.3646          | 154.09 | 0.9943          | 0.25  |
| 3.9    | 7.8590          | 157.50 | 7.9047          | 158.10 | 0.9942          | 0.20  |
| 4.0    | 8.4351          | 161.57 | 8.4831          | 162.11 | 0.9943          | 0.14  |
| 4.1    | 9.0535          | 166.02 | 9.1024          | 166.12 | 0.9946          | 0.10  |
| 4.2    | 9.7198          | 170.07 | 9.7704          | 170.13 | 0.9948          | 0.06  |
| 4.3    | 10.434          | 174.11 | 10.481          | 174.15 | 0.9955          | 0.04  |
| 4.4    | 11.201          | 178.16 | 11.246          | 178.16 | 0.9960          | 0.00  |

Note. Negative quantities are in heavy type.

Examples.  $\sinh (1.7 / 45^\circ) = 1.7776 / 72^\circ.08'.$   
 $\tanh (2.4 / 45^\circ) = 1.0672 / 0^\circ.58'.$

TABLE VI

FUNCTIONS OF SEMI-IMAGINARIES.  $f(\rho/45^\circ) = r/\gamma$ . CONTINUED

| $\rho$ | Cosech   |        | Sech    |        | Coth     |       |
|--------|----------|--------|---------|--------|----------|-------|
| 0.     | $\infty$ | 45.00  | 1.      | 0.     | $\infty$ | 45.00 |
| 0.1    | 10.0000  | 45.06  | 0.99999 | 0.17   | 10.0000  | 44.49 |
| 0.2    | 5.0000   | 45.23  | 0.99987 | 1.09   | 5.0008   | 44.14 |
| 0.3    | 3.3333   | 45.52  | 0.9993  | 2.35   | 3.3355   | 43.17 |
| 0.4    | 2.4997   | 46.32  | 0.9979  | 4.35   | 2.5050   | 41.57 |
| 0.5    | 1.9984   | 47.23  | 0.9948  | 7.08   | 2.0096   | 40.15 |
| 0.6    | 1.6654   | 48.27  | 0.9894  | 10.15  | 1.6830   | 38.11 |
| 0.7    | 1.4268   | 49.40  | 0.9806  | 13.53  | 1.4549   | 35.47 |
| 0.8    | 1.2471   | 51.06  | 0.9675  | 18.00  | 1.2890   | 33.06 |
| 0.9    | 1.1070   | 52.44  | 0.9494  | 22.34  | 1.1660   | 30.10 |
| 1.0    | 0.9945   | 54.32  | 0.9256  | 27.29  | 1.0746   | 27.03 |
| 1.1    | 0.9018   | 56.31  | 0.8963  | 32.41  | 1.0061   | 23.50 |
| 1.2    | 0.8238   | 58.41  | 0.8614  | 38.05  | 0.9564   | 20.36 |
| 1.3    | 0.7573   | 61.02  | 0.8222  | 43.35  | 0.9211   | 17.27 |
| 1.4    | 0.6995   | 63.34  | 0.7793  | 49.05  | 0.8996   | 14.29 |
| 1.5    | 0.6486   | 66.15  | 0.7344  | 54.33  | 0.8831   | 11.42 |
| 1.6    | 0.6033   | 69.07  | 0.6885  | 59.55  | 0.8763   | 9.12  |
| 1.7    | 0.5625   | 72.08  | 0.6429  | 65.09  | 0.8751   | 6.59  |
| 1.8    | 0.5256   | 75.18  | 0.5981  | 70.14  | 0.8788   | 5.04  |
| 1.9    | 0.4916   | 78.36  | 0.5556  | 75.10  | 0.8848   | 3.26  |
| 2.0    | 0.4603   | 82.01  | 0.5151  | 79.56  | 0.8936   | 2.06  |
| 2.1    | 0.4312   | 85.34  | 0.4772  | 84.33  | 0.9038   | 1.01  |
| 2.2    | 0.4041   | 89.12  | 0.4418  | 89.03  | 0.9147   | 0.09  |
| 2.3    | 0.3788   | 92.57  | 0.4090  | 93.26  | 0.9260   | 0.29  |
| 2.4    | 0.3549   | 96.46  | 0.3788  | 97.44  | 0.9370   | 0.58  |
| 2.5    | 0.3325   | 100.39 | 0.3509  | 101.56 | 0.9476   | 1.17  |
| 2.6    | 0.3114   | 104.36 | 0.3252  | 106.05 | 0.9574   | 1.29  |
| 2.7    | 0.2914   | 108.36 | 0.3016  | 110.10 | 0.9663   | 1.34  |
| 2.8    | 0.2726   | 112.39 | 0.2798  | 114.15 | 0.9743   | 1.36  |
| 2.9    | 0.2549   | 116.43 | 0.2598  | 118.16 | 0.9812   | 1.33  |
| 3.0    | 0.2382   | 120.48 | 0.2413  | 122.16 | 0.9871   | 1.28  |
| 3.1    | 0.2225   | 124.56 | 0.2243  | 126.15 | 0.9920   | 1.19  |
| 3.2    | 0.2077   | 129.02 | 0.2085  | 130.15 | 0.9959   | 1.13  |
| 3.3    | 0.1939   | 133.09 | 0.1940  | 134.13 | 0.9992   | 1.04  |
| 3.4    | 0.1808   | 137.17 | 0.1805  | 138.13 | 1.0016   | 0.96  |
| 3.5    | 0.1686   | 141.24 | 0.1681  | 142.12 | 1.0033   | 0.48  |
| 3.6    | 0.1572   | 145.31 | 0.1565  | 146.11 | 1.0047   | 0.40  |
| 3.7    | 0.1465   | 149.38 | 0.1458  | 150.10 | 1.0053   | 0.32  |
| 3.8    | 0.1366   | 153.44 | 0.1358  | 154.09 | 1.0057   | 0.25  |
| 3.9    | 0.1272   | 157.50 | 0.1265  | 158.10 | 1.0058   | 0.20  |
| 4.0    | 0.1186   | 161.57 | 0.1179  | 162.11 | 1.0057   | 0.14  |
| 4.1    | 0.1105   | 166.02 | 0.1099  | 166.12 | 1.0054   | 0.10  |
| 4.2    | 0.1029   | 170.07 | 0.1024  | 170.13 | 1.0052   | 0.06  |
| 4.3    | 0.09584  | 174.11 | 0.09541 | 174.15 | 1.0045   | 0.04  |
| 4.4    | 0.08927  | 178.16 | 0.08892 | 178.16 | 1.0040   | 0.00  |

Note. Negative quantities are in heavy type.

Examples. cosech  $(2.0/45^\circ) = 0.4603 \sqrt{82^\circ.01'}$ .  
 coth  $(2.5/45^\circ) = 0.9476 \sqrt{1^\circ.17'}$ .

TABLE VI  
FUNCTIONS OF SEMI-IMAGINARIES.  $f(\rho / 45^\circ) = r / \gamma$ . CONTINUED

| $\rho$ | Sinh $^\circ$ , |        | Cosh $^\circ$ , |        | Tanh $^\circ$ , |      |
|--------|-----------------|--------|-----------------|--------|-----------------|------|
| 4.5    | 12.026          | 182.19 | 12.067          | 182.19 | 0.9966          | 0.00 |
| 4.6    | 12.909          | 186.23 | 12.948          | 186.21 | 0.9970          | 0.02 |
| 4.7    | 13.858          | 190.27 | 13.894          | 190.23 | 0.9974          | 0.04 |
| 4.8    | 14.876          | 194.30 | 14.909          | 194.26 | 0.9978          | 0.04 |
| 4.9    | 15.968          | 198.33 | 15.999          | 198.29 | 0.9980          | 0.04 |
| 5.0    | 17.140          | 202.36 | 17.169          | 202.32 | 0.9983          | 0.04 |
| 5.1    | 18.397          | 206.39 | 18.425          | 206.35 | 0.9985          | 0.04 |
| 5.2    | 19.747          | 210.42 | 19.772          | 210.38 | 0.9987          | 0.04 |
| 5.3    | 21.195          | 214.45 | 21.219          | 214.41 | 0.9989          | 0.04 |
| 5.4    | 22.750          | 218.48 | 22.772          | 218.44 | 0.9990          | 0.04 |
| 5.5    | 24.418          | 222.50 | 24.439          | 222.47 | 0.9992          | 0.03 |
| 5.6    | 26.219          | 226.53 | 26.238          | 226.51 | 0.9993          | 0.02 |
| 5.7    | 28.141          | 230.56 | 28.159          | 230.54 | 0.9994          | 0.02 |
| 5.8    | 30.192          | 234.59 | 30.209          | 234.57 | 0.9995          | 0.02 |
| 5.9    | 32.405          | 239.02 | 32.421          | 239.00 | 0.9996          | 0.02 |
| 6.0    | 34.784          | 243.05 | 34.798          | 243.04 | 0.9996          | 0.01 |

| $\rho$ | Cosech $^\circ$ , |        | Sech $^\circ$ , |        | Coth $^\circ$ , |      |
|--------|-------------------|--------|-----------------|--------|-----------------|------|
| 4.5    | 0.08316           | 182.19 | 0.08288         | 182.19 | 1.0034          | 0.00 |
| 4.6    | 0.07746           | 186.23 | 0.07723         | 186.21 | 1.0030          | 0.02 |
| 4.7    | 0.07216           | 190.27 | 0.07197         | 190.23 | 1.0026          | 0.04 |
| 4.8    | 0.06722           | 194.30 | 0.06707         | 194.26 | 1.0022          | 0.04 |
| 4.9    | 0.06263           | 198.33 | 0.06250         | 198.29 | 1.0020          | 0.04 |
| 5.0    | 0.05834           | 202.36 | 0.05824         | 202.32 | 1.0017          | 0.04 |
| 5.1    | 0.05436           | 206.39 | 0.05428         | 206.35 | 1.0015          | 0.04 |
| 5.2    | 0.05064           | 210.42 | 0.05058         | 210.38 | 1.0013          | 0.04 |
| 5.3    | 0.04718           | 214.45 | 0.04713         | 214.41 | 1.0011          | 0.04 |
| 5.4    | 0.04396           | 218.48 | 0.04391         | 218.44 | 1.0010          | 0.04 |
| 5.5    | 0.04095           | 222.50 | 0.04092         | 222.47 | 1.0008          | 0.03 |
| 5.6    | 0.03814           | 226.53 | 0.03811         | 226.51 | 1.0007          | 0.02 |
| 5.7    | 0.03554           | 230.56 | 0.03551         | 230.54 | 1.0006          | 0.02 |
| 5.8    | 0.03312           | 234.59 | 0.03310         | 234.57 | 1.0005          | 0.02 |
| 5.9    | 0.03086           | 239.02 | 0.03085         | 239.00 | 1.0004          | 0.02 |
| 6.0    | 0.02875           | 243.05 | 0.02874         | 243.04 | 1.0004          | 0.01 |

Note. Negative quantities are in heavy type.

Examples.  $\tanh (6.0 / 45^\circ) = 0.9996 / 0.01$ .  
 $\operatorname{sech} (5.0 / 45^\circ) = 0.05824 / 202.32$ .

TABLE VI

FUNCTIONS OF SEMI-IMAGINARIES.  $f(\rho/45^\circ) = r/\gamma$ . CONTINUED

| $\rho$ | Sinh and cosh |        | Tanh and coth |      | Sech and cosech        |        |
|--------|---------------|--------|---------------|------|------------------------|--------|
|        | ° /           |        | °             |      | ° /                    |        |
| 6.05   | 36.047        | 245.06 | 1.000         | 0.00 | $2.774 \times 10^{-2}$ | 245.06 |
| 6.10   | 37.349        | 247.08 | 1.000         | 0.00 | 2.678 "                | 247.08 |
| 6.15   | 38.693        | 249.09 | 1.000         | 0.00 | 2.583 "                | 249.09 |
| 6.20   | 40.084        | 251.11 | 1.000         | 0.00 | 2.495 "                | 251.11 |
| 6.25   | 41.524        | 253.12 | 1.000         | 0.00 | 2.408 "                | 253.12 |
| 6.30   | 43.020        | 255.14 | 1.000         | 0.00 | 2.325 "                | 255.14 |
| 6.35   | 44.563        | 257.15 | 1.000         | 0.00 | 2.244 "                | 257.15 |
| 6.40   | 46.171        | 259.17 | 1.000         | 0.00 | 2.166 "                | 259.17 |
| 6.45   | 47.832        | 261.18 | 1.000         | 0.00 | 2.091 "                | 261.18 |
| 6.50   | 49.553        | 263.20 | 1.000         | 0.00 | 2.018 "                | 263.20 |
| 6.55   | 51.336        | 265.22 | 1.000         | 0.00 | 1.948 "                | 265.22 |
| 6.60   | 53.183        | 267.24 | 1.000         | 0.00 | 1.880 "                | 267.24 |
| 6.65   | 55.110        | 269.25 | 1.000         | 0.00 | 1.815 "                | 269.25 |
| 6.70   | 57.058        | 271.27 | 1.000         | 0.00 | 1.752 "                | 271.27 |
| 6.75   | 59.136        | 273.28 | 1.000         | 0.00 | 1.691 "                | 273.28 |
| 6.80   | 61.259        | 275.30 | 1.000         | 0.00 | 1.632 "                | 275.30 |
| 6.85   | 63.463        | 277.31 | 1.000         | 0.00 | 1.576 "                | 277.31 |
| 6.90   | 65.746        | 279.33 | 1.000         | 0.00 | 1.521 "                | 279.33 |
| 6.95   | 68.119        | 281.34 | 1.000         | 0.00 | 1.468 "                | 281.34 |
| 7.00   | 70.570        | 283.36 | 1.000         | 0.00 | 1.417 "                | 283.36 |
| 7.05   | 73.109        | 285.37 | 1.000         | 0.00 | 1.368 "                | 285.37 |
| 7.10   | 75.739        | 287.39 | 1.000         | 0.00 | 1.312 "                | 287.39 |
| 7.15   | 78.473        | 289.40 | 1.000         | 0.00 | 1.274 "                | 289.40 |
| 7.20   | 81.296        | 291.42 | 1.000         | 0.00 | 1.230 "                | 291.42 |
| 7.25   | 84.215        | 293.43 | 1.000         | 0.00 | 1.187 "                | 293.43 |
| 7.30   | 87.250        | 295.45 | 1.000         | 0.00 | 1.146 "                | 295.45 |
| 7.35   | 90.386        | 297.46 | 1.000         | 0.00 | 1.016 "                | 297.46 |
| 7.40   | 93.083        | 299.48 | 1.000         | 0.00 | 1.074 "                | 299.48 |
| 7.45   | 97.009        | 301.49 | 1.000         | 0.00 | 1.031 "                | 301.49 |
| 7.50   | 100.50        | 303.51 | 1.000         | 0.00 | $9.950 \times 10^{-3}$ | 303.51 |
| 7.55   | 104.12        | 305.52 | 1.000         | 0.00 | 9.605 "                | 305.52 |
| 7.60   | 107.86        | 307.54 | 1.000         | 0.00 | 9.271 "                | 307.54 |
| 7.65   | 111.74        | 309.56 | 1.000         | 0.00 | 8.949 "                | 309.56 |
| 7.70   | 115.67        | 311.57 | 1.000         | 0.00 | 8.638 "                | 311.57 |
| 7.75   | 119.94        | 313.59 | 1.000         | 0.00 | 8.337 "                | 313.59 |
| 7.80   | 124.26        | 316.00 | 1.000         | 0.00 | 8.048 "                | 316.00 |
| 7.85   | 128.71        | 318.02 | 1.000         | 0.00 | 7.769 "                | 318.02 |
| 7.90   | 133.35        | 320.03 | 1.000         | 0.00 | 7.499 "                | 320.03 |
| 7.95   | 138.16        | 322.05 | 1.000         | 0.00 | 7.238 "                | 322.05 |
| 8.00   | 143.12        | 324.06 | 1.000         | 0.00 | 6.987 "                | 324.06 |
| 8.05   | 148.28        | 326.07 | 1.000         | 0.00 | 6.744 "                | 326.07 |
| 8.10   | 153.61        | 328.09 | 1.000         | 0.00 | 6.510 "                | 328.09 |
| 8.15   | 159.14        | 330.11 | 1.000         | 0.00 | 6.284 "                | 330.11 |
| 8.20   | 164.87        | 332.12 | 1.000         | 0.00 | 6.066 "                | 332.12 |
| 8.25   | 170.80        | 334.14 | 1.000         | 0.00 | 5.855 "                | 334.14 |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(7.55/45^\circ) = \cosh(7.55/45^\circ) = 104.12/305^\circ.52'$ .  
 $\operatorname{sech}(7.50/45^\circ) = \operatorname{cosech}(7.50/45^\circ) = 9.950 \times 10^{-3}/303^\circ.51'$ .

TABLE VI  
FUNCTIONS OF SEMI-IMAGINARIES.  $f(\rho/45^\circ) = r/\gamma$ . CONTINUED

| $\rho$ | Sinh and cosh |        | Tanh and coth |      | Sech and cosech        |        |
|--------|---------------|--------|---------------|------|------------------------|--------|
|        | °             | '      | °             |      | °                      | '      |
| 8.30   | 176.95        | 336.15 | 1.000         | 0.00 | $5.651 \times 10^{-3}$ | 336.15 |
| 8.35   | 183.31        | 338.17 | 1.000         | 0.00 | 5.455                  | 338.17 |
| 8.40   | 189.91        | 340.18 | 1.000         | 0.00 | 5.266                  | 340.18 |
| 8.45   | 196.75        | 342.20 | 1.000         | 0.00 | 5.083                  | 342.20 |
| 8.50   | 203.83        | 344.22 | 1.000         | 0.00 | 4.906                  | 344.22 |
| 8.55   | 211.16        | 346.24 | 1.000         | 0.00 | 4.736                  | 346.24 |
| 8.60   | 218.76        | 348.25 | 1.000         | 0.00 | 4.571                  | 348.25 |
| 8.65   | 226.63        | 350.27 | 1.000         | 0.00 | 4.413                  | 350.27 |
| 8.70   | 234.79        | 352.28 | 1.000         | 0.00 | 4.259                  | 352.28 |
| 8.75   | 243.23        | 354.30 | 1.000         | 0.00 | 4.111                  | 354.30 |
| 8.80   | 251.99        | 356.31 | 1.000         | 0.00 | 3.968                  | 356.31 |
| 8.85   | 261.06        | 358.33 | 1.000         | 0.00 | 3.830                  | 358.33 |
| 8.90   | 270.46        | 360.34 | 1.000         | 0.00 | 3.698                  | 360.34 |
| 8.95   | 280.19        | 362.36 | 1.000         | 0.00 | 3.569                  | 362.36 |
| 9.00   | 290.28        | 364.38 | 1.000         | 0.00 | 3.445                  | 364.38 |
| 9.05   | 300.73        | 366.39 | 1.000         | 0.00 | 3.3253                 | 366.39 |
| 9.10   | 311.54        | 368.41 | 1.000         | 0.00 | 3.2099                 | 368.41 |
| 9.15   | 322.75        | 370.42 | 1.000         | 0.00 | 3.0983                 | 370.42 |
| 9.20   | 334.37        | 372.44 | 1.000         | 0.00 | 2.9908                 | 372.44 |
| 9.25   | 346.39        | 374.46 | 1.000         | 0.00 | 2.8869                 | 374.46 |
| 9.30   | 358.85        | 376.47 | 1.000         | 0.00 | 2.7867                 | 376.47 |
| 9.35   | 371.81        | 378.48 | 1.000         | 0.00 | 2.6895                 | 378.48 |
| 9.40   | 385.15        | 380.50 | 1.000         | 0.00 | 2.5964                 | 380.50 |
| 9.45   | 399.04        | 382.51 | 1.000         | 0.00 | 2.5060                 | 382.51 |
| 9.50   | 413.38        | 384.53 | 1.000         | 0.00 | 2.4191                 | 384.53 |
| 9.55   | 428.26        | 386.55 | 1.000         | 0.00 | 2.3350                 | 386.55 |
| 9.60   | 443.67        | 388.56 | 1.000         | 0.00 | 2.2540                 | 388.56 |
| 9.65   | 446.93        | 390.57 | 1.000         | 0.00 | 2.2263                 | 390.57 |
| 9.70   | 476.18        | 392.59 | 1.000         | 0.00 | 2.1001                 | 392.59 |
| 9.75   | 493.31        | 395.01 | 1.000         | 0.00 | 2.0271                 | 395.01 |
| 9.80   | 511.07        | 397.02 | 1.000         | 0.00 | 1.9567                 | 397.02 |
| 9.85   | 529.46        | 399.03 | 1.000         | 0.00 | 1.8887                 | 399.03 |
| 9.90   | 548.52        | 401.05 | 1.000         | 0.00 | 1.8231                 | 401.05 |
| 9.95   | 568.25        | 403.07 | 1.000         | 0.00 | 1.7598                 | 403.07 |
| 10.00  | 588.69        | 405.08 | 1.000         | 0.00 | 1.6987                 | 405.08 |
| 10.05  | 609.89        | 407.09 | 1.000         | 0.00 | 1.6397                 | 407.09 |
| 10.10  | 631.84        | 409.11 | 1.000         | 0.00 | 1.5827                 | 409.11 |
| 10.15  | 654.58        | 411.13 | 1.000         | 0.00 | 1.5277                 | 411.13 |
| 10.20  | 678.14        | 413.14 | 1.000         | 0.00 | 1.4746                 | 413.14 |
| 10.25  | 702.53        | 415.15 | 1.000         | 0.00 | 1.4234                 | 415.15 |
| 10.30  | 727.81        | 417.17 | 1.000         | 0.00 | 1.3740                 | 417.17 |
| 10.35  | 754.01        | 419.19 | 1.000         | 0.00 | 1.3262                 | 419.19 |
| 10.40  | 781.14        | 421.21 | 1.000         | 0.00 | 1.2802                 | 421.21 |
| 10.45  | 809.26        | 423.23 | 1.000         | 0.00 | 1.2357                 | 423.23 |
| 10.50  | 838.38        | 425.24 | 1.000         | 0.00 | 1.1928                 | 425.24 |

Note. Negative quantities are in heavy type.

Examples.

$$\sinh(10.0/45^\circ) = \cosh(10.0/45^\circ) = 588.69/405^\circ.08'.$$

$$\operatorname{sech}(10.0/45^\circ) = \operatorname{cosech}(10.0/45^\circ) = 1.6987 \times 10^{-3}/405^\circ.08' = 1.6987 \times 10^{-3}/45^\circ.08'.$$

TABLE VI

FUNCTIONS OF SEMI-IMAGINARIES.  $f(\rho/45^\circ) = r/\gamma$ . CONTINUED.

| $\rho$ | Sinh and cosh |           | Tanh and coth |         | Sech and cosech         |           |
|--------|---------------|-----------|---------------|---------|-------------------------|-----------|
|        |               | $\circ$ / |               | $\circ$ |                         | $\circ$ / |
| 10.55  | 868.56        | 427.26    | 1.000         | 0.00    | $1.1513 \times 10^{-3}$ | 427.26    |
| 10.60  | 899.81        | 429.27    | 1.000         | 0.00    | 1.1113                  | 429.27    |
| 10.65  | 932.18        | 431.29    | 1.000         | 0.00    | 1.0728                  | 431.29    |
| 10.70  | 965.74        | 433.30    | 1.000         | 0.00    | 1.0555                  | 433.30    |
| 10.75  | 1,000.5       | 435.32    | 1.000         | 0.00    | $9.9952 \times 10^{-4}$ | 435.32    |
| 10.80  | 1,036.5       | 437.33    | 1.000         | 0.00    | 9.6478                  | 437.33    |
| 10.85  | 1,073.8       | 439.35    | 1.000         | 0.00    | 9.3128                  | 439.35    |
| 10.90  | 1,112.4       | 441.36    | 1.000         | 0.00    | 8.9892                  | 441.36    |
| 10.95  | 1,152.5       | 443.38    | 1.000         | 0.00    | 8.6770                  | 443.38    |
| 11.00  | 1,194.0       | 445.39    | 1.000         | 0.00    | 8.3750                  | 445.39    |
| 11.05  | 1,237.0       | 447.41    | 1.000         | 0.00    | 8.0845                  | 447.41    |
| 11.10  | 1,281.5       | 449.42    | 1.000         | 0.00    | 7.8037                  | 449.42    |
| 11.15  | 1,327.5       | 451.44    | 1.000         | 0.00    | 7.5327                  | 451.44    |
| 11.20  | 1,375.3       | 453.46    | 1.000         | 0.00    | 7.2711                  | 453.46    |
| 11.25  | 1,424.8       | 455.47    | 1.000         | 0.00    | 7.0184                  | 455.47    |
| 11.30  | 1,476.1       | 457.48    | 1.000         | 0.00    | 6.7747                  | 457.48    |
| 11.35  | 1,529.2       | 459.50    | 1.000         | 0.00    | 6.5393                  | 459.50    |
| 11.40  | 1,584.3       | 561.52    | 1.000         | 0.00    | 6.3120                  | 461.52    |
| 11.45  | 1,641.4       | 463.53    | 1.000         | 0.00    | 6.0929                  | 463.53    |
| 11.50  | 1,700.3       | 465.54    | 1.000         | 0.00    | 5.8811                  | 465.54    |
| 11.55  | 1,761.5       | 467.56    | 1.000         | 0.00    | 5.6769                  | 467.56    |
| 11.60  | 1,824.9       | 469.57    | 1.000         | 0.00    | 5.4797                  | 469.57    |
| 11.65  | 1,890.6       | 471.59    | 1.000         | 0.00    | 5.2893                  | 471.59    |
| 11.70  | 1,958.6       | 474.01    | 1.000         | 0.00    | 5.1056                  | 474.01    |
| 11.75  | 2,029.1       | 476.03    | 1.000         | 0.00    | 4.9282                  | 476.03    |
| 11.80  | 2,102.1       | 478.04    | 1.000         | 0.00    | 4.7571                  | 478.04    |
| 11.85  | 2,177.8       | 480.05    | 1.000         | 0.00    | 4.5910                  | 480.05    |
| 11.90  | 2,256.1       | 482.07    | 1.000         | 0.00    | 4.4323                  | 482.07    |
| 11.95  | 2,337.3       | 484.09    | 1.000         | 0.00    | 4.2784                  | 484.09    |
| 12.00  | 2,421.5       | 486.10    | 1.000         | 0.00    | 4.1297                  | 486.10    |
| 12.05  | 2,508.6       | 488.12    | 1.000         | 0.00    | 3.9862                  | 488.12    |
| 12.10  | 2,598.9       | 490.14    | 1.000         | 0.00    | 3.8478                  | 490.14    |
| 12.15  | 2,692.6       | 492.15    | 1.000         | 0.00    | 3.7141                  | 492.15    |
| 12.20  | 2,789.0       | 494.17    | 1.000         | 0.00    | 3.5856                  | 494.17    |
| 12.25  | 2,889.7       | 496.18    | 1.000         | 0.00    | 3.4605                  | 496.18    |
| 12.30  | 2,993.7       | 498.20    | 1.000         | 0.00    | 3.3403                  | 498.20    |
| 12.35  | 3,101.4       | 500.21    | 1.000         | 0.00    | 3.2243                  | 500.21    |
| 12.40  | 3,213.1       | 502.23    | 1.000         | 0.00    | 3.0143                  | 502.23    |
| 12.45  | 3,328.3       | 504.24    | 1.000         | 0.00    | 3.0042                  | 504.24    |
| 12.50  | 3,448.5       | 506.26    | 1.000         | 0.00    | 2.8998                  | 506.26    |
| 12.55  | 3,572.6       | 508.27    | 1.000         | 0.00    | 2.7991                  | 508.27    |
| 12.60  | 3,701.1       | 510.29    | 1.000         | 0.00    | 2.7019                  | 510.29    |
| 12.65  | 3,834.3       | 512.31    | 1.000         | 0.00    | 2.6080                  | 512.31    |
| 12.70  | 3,972.6       | 514.32    | 1.000         | 0.00    | 2.5172                  | 514.32    |
| 12.75  | 4,115.3       | 516.33    | 1.000         | 0.00    | 2.4300                  | 516.33    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(12.0/45^\circ) = \cosh(12.0/45^\circ) = 2421.5/486^\circ.10' = 2421.5/126^\circ.10'$   
 $\operatorname{sech}(12.75/45^\circ) = \operatorname{cosech}(12.75/45^\circ) = 2.43 \times 10^{-3}/516^\circ.33'$

TABLE VI

FUNCTIONS OF SEMI-IMAGINARIES.  $f(\rho/45^\circ) = r/\gamma$ . CONTINUED

| $\rho$ | Sinh and cosh |        | Tanh and coth |      | Sech and cosech         |        |
|--------|---------------|--------|---------------|------|-------------------------|--------|
|        | °             | '      | °             | '    | °                       | '      |
| 12.80  | 4,263.4       | 518.35 | 1.000         | 0.00 | 2.3455 $\times 10^{-4}$ | 518.35 |
| 12.85  | 4,416.8       | 520.37 | 1.000         | 0.00 | 2.2641 "                | 520.37 |
| 12.90  | 4,575.7       | 522.38 | 1.000         | 0.00 | 2.1854 "                | 522.38 |
| 12.95  | 4,740.5       | 524.39 | 1.000         | 0.00 | 2.1065 "                | 524.39 |
| 13.00  | 4,911.0       | 526.41 | 1.000         | 0.00 | 2.0362 "                | 526.41 |
| 13.05  | 5,087.8       | 528.43 | 1.000         | 0.00 | 1.9655 "                | 528.43 |
| 13.10  | 5,270.9       | 530.44 | 1.000         | 0.00 | 1.8972 "                | 530.44 |
| 13.15  | 5,460.6       | 532.45 | 1.000         | 0.00 | 1.8313 "                | 532.45 |
| 13.20  | 5,657.0       | 534.47 | 1.000         | 0.00 | 1.7677 "                | 534.47 |
| 13.25  | 5,858.5       | 536.49 | 1.000         | 0.00 | 1.7061 "                | 536.49 |
| 13.30  | 6,071.6       | 538.50 | 1.000         | 0.00 | 1.6470 "                | 538.50 |
| 13.35  | 6,290.1       | 540.51 | 1.000         | 0.00 | 1.5898 "                | 540.51 |
| 13.40  | 6,516.5       | 542.53 | 1.000         | 0.00 | 1.5346 "                | 542.53 |
| 13.45  | 6,751.0       | 544.55 | 1.000         | 0.00 | 1.4813 "                | 544.55 |
| 13.50  | 6,993.9       | 546.57 | 1.000         | 0.00 | 1.4298 "                | 546.57 |
| 13.55  | 7,245.5       | 548.58 | 1.000         | 0.00 | 1.3801 "                | 548.58 |
| 13.60  | 7,506.4       | 551.00 | 1.000         | 0.00 | 1.3322 "                | 551.00 |
| 13.65  | 7,776.4       | 553.01 | 1.000         | 0.00 | 1.2859 "                | 553.01 |
| 13.70  | 8,056.4       | 555.03 | 1.000         | 0.00 | 1.2412 "                | 555.03 |
| 13.75  | 8,346.2       | 557.05 | 1.000         | 0.00 | 1.1982 "                | 557.05 |
| 13.80  | 8,646.7       | 559.06 | 1.000         | 0.00 | 1.1565 "                | 559.06 |
| 13.85  | 8,957.8       | 561.07 | 1.000         | 0.00 | 1.1164 "                | 561.07 |
| 13.90  | 9,280.3       | 563.09 | 1.000         | 0.00 | 1.0776 "                | 563.09 |
| 13.95  | 9,614.1       | 565.11 | 1.000         | 0.00 | 1.0405 "                | 565.11 |
| 14.00  | 9,960.2       | 567.12 | 1.000         | 0.00 | 1.0040 "                | 567.12 |
| 14.05  | 10,318        | 569.14 | 1.000         | 0.00 | 9.6914 $\times 10^{-5}$ | 569.14 |
| 14.10  | 10,690        | 571.15 | 1.000         | 0.00 | 9.3547 "                | 571.15 |
| 14.15  | 11,075        | 573.16 | 1.000         | 0.00 | 9.0296 "                | 573.16 |
| 14.20  | 11,473        | 575.18 | 1.000         | 0.00 | 8.7160 "                | 575.18 |
| 14.25  | 11,886        | 577.20 | 1.000         | 0.00 | 8.4132 "                | 577.20 |
| 14.30  | 12,314        | 579.21 | 1.000         | 0.00 | 8.1210 "                | 579.21 |
| 14.35  | 12,757        | 581.22 | 1.000         | 0.00 | 7.8388 "                | 581.22 |
| 14.40  | 13,216        | 583.24 | 1.000         | 0.00 | 7.5666 "                | 583.24 |
| 14.45  | 13,692        | 585.26 | 1.000         | 0.00 | 7.3037 "                | 585.26 |
| 14.50  | 14,184        | 587.27 | 1.000         | 0.00 | 7.0500 "                | 587.27 |
| 14.55  | 14,695        | 589.29 | 1.000         | 0.00 | 6.8050 "                | 589.29 |
| 14.60  | 15,224        | 591.30 | 1.000         | 0.00 | 6.5687 "                | 591.30 |
| 14.65  | 15,772        | 593.32 | 1.000         | 0.00 | 6.3405 "                | 593.32 |
| 14.70  | 16,339        | 595.34 | 1.000         | 0.00 | 6.1203 "                | 595.34 |
| 14.75  | 16,927        | 597.35 | 1.000         | 0.00 | 5.9077 "                | 597.35 |
| 14.80  | 17,536        | 599.37 | 1.000         | 0.00 | 5.7024 "                | 599.37 |
| 14.85  | 18,167        | 601.39 | 1.000         | 0.00 | 5.5044 "                | 601.39 |
| 14.90  | 18,822        | 603.40 | 1.000         | 0.00 | 5.3130 "                | 603.40 |
| 14.95  | 19,498        | 605.41 | 1.000         | 0.00 | 5.1286 "                | 605.41 |
| 15.00  | 20,200        | 607.43 | 1.000         | 0.00 | 4.9504 "                | 607.43 |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(14.0/45^\circ) = \cosh(14.0/45^\circ) = 9960.2/567^\circ.12'$   
 $\operatorname{sech}(14.0/45^\circ) = \operatorname{cosech}(14.0/45^\circ) = 1.0040 \times 10^{-4}/567^\circ.12'$

TABLE VI  
FUNCTIONS OF SEMI-IMAGINARIES.  $f(\rho / 45^\circ) = r / \gamma$ . CONTINUED

| $\rho$ | Sinh and cosh<br>o / |        | Tanh and coth<br>o |      | Sech and cosech<br>o /  |          |
|--------|----------------------|--------|--------------------|------|-------------------------|----------|
| 15.05  | 20,927               | 609.44 | 1.000              | 0.00 | $4.7785 \times 10^{-5}$ | 609.44   |
| 15.10  | 21,680               | 611.46 | 1.000              | 0.00 | 4.6120                  | " 611.46 |
| 15.15  | 22,460               | 613.48 | 1.000              | 0.00 | 4.4523                  | " 613.48 |
| 15.20  | 23,269               | 615.49 | 1.000              | 0.00 | 4.2980                  | " 615.49 |
| 15.25  | 24,106               | 617.50 | 1.000              | 0.00 | 4.1482                  | " 617.50 |
| 15.30  | 24,973               | 619.52 | 1.000              | 0.00 | 4.0040                  | " 619.52 |
| 15.35  | 25,873               | 621.54 | 1.000              | 0.00 | 3.8651                  | " 621.54 |
| 15.40  | 26,802               | 623.55 | 1.000              | 0.00 | 3.7310                  | " 623.55 |
| 15.45  | 27,768               | 625.57 | 1.000              | 0.00 | 3.6012                  | " 625.57 |
| 15.50  | 28,765               | 627.59 | 1.000              | 0.00 | 3.4760                  | " 627.59 |
| 15.55  | 29,803               | 630.00 | 1.000              | 0.00 | 3.3554                  | " 630.00 |
| 15.60  | 30,872               | 632.02 | 1.000              | 0.00 | 3.2390                  | " 632.02 |
| 15.65  | 31,987               | 634.04 | 1.000              | 0.00 | 3.1263                  | " 634.04 |
| 15.70  | 33,140               | 636.05 | 1.000              | 0.00 | 3.0170                  | " 636.05 |
| 15.75  | 34,331               | 638.06 | 1.000              | 0.00 | 2.9129                  | " 638.06 |
| 15.80  | 35,569               | 640.08 | 1.000              | 0.00 | 2.8110                  | " 640.08 |
| 15.85  | 36,846               | 642.10 | 1.000              | 0.00 | 2.7140                  | " 642.10 |
| 15.90  | 38,174               | 644.11 | 1.000              | 0.00 | 2.6200                  | " 644.11 |
| 15.95  | 39,546               | 646.12 | 1.000              | 0.00 | 2.5287                  | " 646.12 |
| 16.00  | 40,970               | 648.14 | 1.000              | 0.00 | 2.4410                  | " 648.14 |
| 16.05  | 42,443               | 650.16 | 1.000              | 0.00 | 2.3561                  | " 650.16 |
| 16.10  | 43,971               | 652.17 | 1.000              | 0.00 | 2.2740                  | " 652.17 |
| 16.15  | 45,553               | 654.18 | 1.000              | 0.00 | 2.1952                  | " 654.18 |
| 16.20  | 47,192               | 656.20 | 1.000              | 0.00 | 2.1190                  | " 656.20 |
| 16.25  | 48,890               | 658.22 | 1.000              | 0.00 | 2.0454                  | " 658.22 |
| 16.30  | 50,649               | 660.23 | 1.000              | 0.00 | 1.9740                  | " 660.23 |
| 16.35  | 52,473               | 662.24 | 1.000              | 0.00 | 1.9055                  | " 662.24 |
| 16.40  | 54,359               | 664.26 | 1.000              | 0.00 | 1.8400                  | " 664.26 |
| 16.45  | 56,316               | 666.28 | 1.000              | 0.00 | 1.7757                  | " 666.28 |
| 16.50  | 58,475               | 668.29 | 1.000              | 0.00 | 1.7100                  | " 668.29 |
| 16.55  | 60,444               | 670.31 | 1.000              | 0.00 | 1.6544                  | " 670.31 |
| 16.60  | 62,619               | 672.32 | 1.000              | 0.00 | 1.5969                  | " 672.32 |
| 16.65  | 64,872               | 674.34 | 1.000              | 0.00 | 1.5415                  | " 674.34 |
| 16.70  | 57,208               | 676.35 | 1.000              | 0.00 | 1.4879                  | " 676.35 |
| 16.75  | 69,626               | 678.36 | 1.000              | 0.00 | 1.4362                  | " 678.36 |
| 16.80  | 72,132               | 680.38 | 1.000              | 0.00 | 1.3863                  | " 680.38 |
| 16.85  | 74,727               | 682.40 | 1.000              | 0.00 | 1.3382                  | " 682.40 |
| 16.90  | 77,418               | 684.41 | 1.000              | 0.00 | 1.2917                  | " 684.41 |
| 16.95  | 80,203               | 686.43 | 1.000              | 0.00 | 1.2468                  | " 686.43 |
| 17.00  | 83,088               | 688.45 | 1.000              | 0.00 | 1.2035                  | " 688.45 |
| 17.05  | 86,080               | 690.47 | 1.000              | 0.00 | 1.1617                  | " 690.47 |
| 17.10  | 89,176               | 692.48 | 1.000              | 0.00 | 1.1214                  | " 692.48 |
| 17.15  | 92,387               | 694.49 | 1.000              | 0.00 | 1.0824                  | " 694.49 |
| 17.20  | 95,711               | 696.51 | 1.000              | 0.00 | 1.0448                  | " 696.51 |
| 17.25  | 99,149               | 698.53 | 1.000              | 0.00 | 1.0086                  | " 698.53 |

Note. Negative quantities are in heavy type.

Examples.  $\sinh (17.0 / 45^\circ) = \cosh (17.0 / 45^\circ) = 83,088 / 688^\circ.45' = 83,088 / 328^\circ.45'.$   
 $\operatorname{sech} (17.0 / 45^\circ) = \operatorname{cosech} (17.0 / 45^\circ) = 1.2035 \times 10^{-5} \sqrt{688^\circ.45'}$



TABLE VI  
FUNCTIONS OF SEMI-IMAGINARIES.  $f(\rho / 45^\circ) = r / \gamma$ . CONTINUED

| $\rho$ | Sinh and cosh<br>° , |        | Tanh and coth<br>° |      | Sech and cosech<br>° ,  |        |
|--------|----------------------|--------|--------------------|------|-------------------------|--------|
| 17.30  | 102,720              | 700.54 | 1.000              | 0.00 | $9.7349 \times 10^{-6}$ | 700.54 |
| 17.35  | 106,420              | 702.55 | 1.000              | 0.00 | 9.3968 "                | 702.55 |
| 17.40  | 110,250              | 704.57 | 1.000              | 0.00 | 9.0703 "                | 704.57 |
| 17.45  | 114,220              | 706.59 | 1.000              | 0.00 | 8.7551 "                | 706.59 |
| 17.50  | 118,330              | 709.00 | 1.000              | 0.00 | 8.4510 "                | 709.00 |
| 17.55  | 122,590              | 711.01 | 1.000              | 0.00 | 8.1576 "                | 711.01 |
| 17.60  | 127,000              | 713.03 | 1.000              | 0.00 | 7.8741 "                | 713.03 |
| 17.65  | 131,570              | 715.05 | 1.000              | 0.00 | 7.6006 "                | 715.05 |
| 17.70  | 136,300              | 717.06 | 1.000              | 0.00 | 7.3365 "                | 717.06 |
| 17.75  | 141,210              | 719.07 | 1.000              | 0.00 | 7.0817 "                | 719.07 |
| 17.80  | 146,290              | 721.09 | 1.000              | 0.00 | 6.8356 "                | 721.09 |
| 17.85  | 151,550              | 723.11 | 1.000              | 0.00 | 6.5983 "                | 723.11 |
| 17.90  | 157,000              | 725.12 | 1.000              | 0.00 | 6.3710 "                | 725.12 |
| 17.95  | 162,660              | 727.13 | 1.000              | 0.00 | 6.1478 "                | 727.13 |
| 18.00  | 168,520              | 729.15 | 1.000              | 0.00 | 5.9383 "                | 729.15 |
| 18.05  | 174,580              | 731.17 | 1.000              | 0.00 | 5.7281 "                | 731.17 |
| 18.10  | 180,860              | 733.18 | 1.000              | 0.00 | 5.5292 "                | 733.18 |
| 18.15  | 183,530              | 735.20 | 1.000              | 0.00 | 5.4488 "                | 735.20 |
| 18.20  | 194,110              | 737.21 | 1.000              | 0.00 | 5.1517 "                | 737.21 |
| 18.25  | 201,100              | 739.23 | 1.000              | 0.00 | 4.9727 "                | 739.23 |
| 18.30  | 208,330              | 741.24 | 1.000              | 0.00 | 4.8000 "                | 741.24 |
| 18.35  | 215,830              | 743.26 | 1.000              | 0.00 | 4.6332 "                | 743.26 |
| 18.40  | 223,600              | 745.27 | 1.000              | 0.00 | 4.4723 "                | 745.27 |
| 18.45  | 231,650              | 747.29 | 1.000              | 0.00 | 4.3168 "                | 747.29 |
| 18.50  | 239,980              | 749.31 | 1.000              | 0.00 | 4.1671 "                | 749.31 |
| 18.55  | 248,620              | 751.32 | 1.000              | 0.00 | 4.0222 "                | 751.32 |
| 18.60  | 257,570              | 753.34 | 1.000              | 0.00 | 3.8825 "                | 753.34 |
| 18.65  | 266,840              | 755.35 | 1.000              | 0.00 | 3.7476 "                | 755.35 |
| 18.70  | 276,440              | 757.37 | 1.000              | 0.00 | 3.6174 "                | 757.37 |
| 18.75  | 286,390              | 759.38 | 1.000              | 0.00 | 3.4918 "                | 759.38 |
| 18.80  | 296,690              | 761.40 | 1.000              | 0.00 | 3.3628 "                | 761.40 |
| 18.85  | 307,380              | 763.41 | 1.000              | 0.00 | 3.2533 "                | 763.41 |
| 18.90  | 318,570              | 765.43 | 1.000              | 0.00 | 3.1404 "                | 765.43 |
| 18.95  | 329,890              | 767.44 | 1.000              | 0.00 | 3.0313 "                | 767.44 |
| 19.00  | 341,770              | 769.46 | 1.000              | 0.00 | 2.9260 "                | 769.46 |
| 19.05  | 354,060              | 771.47 | 1.000              | 0.00 | 2.8244 "                | 771.47 |
| 19.10  | 366,810              | 773.49 | 1.000              | 0.00 | 2.7262 "                | 773.49 |
| 19.15  | 380,010              | 775.50 | 1.000              | 0.00 | 2.6315 "                | 775.50 |
| 19.20  | 393,690              | 777.52 | 1.000              | 0.00 | 2.5401 "                | 777.52 |
| 19.25  | 407,850              | 779.53 | 1.000              | 0.00 | 2.4519 "                | 779.53 |
| 19.30  | 422,530              | 781.55 | 1.000              | 0.00 | 2.3667 "                | 781.55 |
| 19.35  | 437,730              | 783.57 | 1.000              | 0.00 | 2.2845 "                | 783.57 |
| 19.40  | 453,490              | 785.59 | 1.000              | 0.00 | 2.2051 "                | 785.59 |
| 19.45  | 469,810              | 788.00 | 1.000              | 0.00 | 2.1285 "                | 788.00 |
| 19.50  | 486,720              | 790.02 | 1.000              | 0.00 | 2.0546 "                | 790.02 |

Note. Negative quantities are in heavy type.

Examples.  $\sinh (19.05 / 45^\circ) = \cosh (19.05 / 45^\circ) = 354,060 / 771^\circ.47' = 354,060 / 51^\circ.47'.$   
 $\operatorname{sech} (19.30 / 45^\circ) = \operatorname{cosech} (19.3 / 45^\circ) = 2.3667 \times 10^{-6} / 781^\circ.55'.$

TABLE VI

FUNCTIONS OF SEMI-IMAGINARIES.  $f(\rho / \underline{45^\circ}) = r / \underline{\gamma}$ . CONTINUED

| $\rho$ | Sinh and cosh |        | Tanh and coth |      | Sech and cosech         |        |
|--------|---------------|--------|---------------|------|-------------------------|--------|
|        | °             | '      | °             | '    | °                       | '      |
| 19.55  | 504,230       | 792.03 | 1.000         | 0.00 | 1.9832 $\times 10^{-6}$ | 792.03 |
| 19.60  | 522,380       | 794.05 | 1.000         | 0.00 | 1.9153 "                | 794.05 |
| 19.65  | 541,220       | 796.06 | 1.000         | 0.00 | 1.8478 "                | 796.06 |
| 19.70  | 560,650       | 798.08 | 1.000         | 0.00 | 1.7837 "                | 798.08 |
| 19.75  | 599,830       | 800.09 | 1.000         | 0.00 | 1.6671 "                | 800.09 |
| 19.80  | 601,730       | 802.11 | 1.000         | 0.00 | 1.6619 "                | 802.11 |
| 19.85  | 623,390       | 804.12 | 1.000         | 0.00 | 1.6041 "                | 804.12 |
| 19.90  | 645,820       | 806.14 | 1.000         | 0.00 | 1.5484 "                | 806.14 |
| 19.95  | 669,070       | 808.15 | 1.000         | 0.00 | 1.4946 "                | 808.15 |
| 20.00  | 693,150       | 810.17 | 1.000         | 0.00 | 1.4426 "                | 810.17 |
| 20.05  | 718,090       | 812.18 | 1.000         | 0.00 | 1.3926 "                | 812.18 |
| 20.10  | 743,930       | 814.20 | 1.000         | 0.00 | 1.3442 "                | 814.20 |
| 20.15  | 770,710       | 816.21 | 1.000         | 0.00 | 1.2975 "                | 816.21 |
| 20.20  | 798,440       | 818.23 | 1.000         | 0.00 | 1.2525 "                | 818.23 |
| 20.25  | 827,160       | 820.24 | 1.000         | 0.00 | 1.2090 "                | 820.24 |
| 20.30  | 856,940       | 822.26 | 1.000         | 0.00 | 1.1669 "                | 822.26 |
| 20.35  | 887,770       | 824.27 | 1.000         | 0.00 | 1.1264 "                | 824.27 |
| 20.40  | 919,730       | 826.29 | 1.000         | 0.00 | 1.0873 "                | 826.29 |
| 20.45  | 952,820       | 828.30 | 1.000         | 0.00 | 1.0496 "                | 828.30 |
| 20.50  | 987,120       | 830.32 | 1.000         | 0.00 | 1.0130 "                | 830.32 |

Note. Negative quantities are in heavy type.

Example.  $\sinh(20.0 / \underline{45^\circ}) = \cosh(20.0 / \underline{45^\circ}) = 693,150 / \underline{810^\circ.17'} = 693,150 / \underline{90^\circ.17'}$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$

| $q$  | $x = 0$ |         | $x = 0.05$ |         | $x = 0.1$ |         | $x = 0.15$ |         | $x = 0.2$ |         |
|------|---------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0.0  | 0.00    | 0.00    | 0.05002    | 0.00    | 0.10017   | 0.00    | 0.15056    | 0.00    | 0.20134   | 0.00    |
| 0.05 | 0.00    | 0.07846 | 0.04987    | 0.07856 | 0.09986   | 0.07885 | 0.15010    | 0.07934 | 0.20072   | 0.08003 |
| 0.1  | 0.00    | 0.15643 | 0.04945    | 0.15663 | 0.09893   | 0.15722 | 0.14871    | 0.15820 | 0.19886   | 0.15957 |
| 0.15 | 0.00    | 0.23345 | 0.04864    | 0.23374 | 0.09740   | 0.23461 | 0.14640    | 0.23608 | 0.19577   | 0.23813 |
| 0.2  | 0.00    | 0.30902 | 0.04757    | 0.30940 | 0.09526   | 0.31056 | 0.14319    | 0.31250 | 0.19148   | 0.31522 |
| 0.25 | 0.00    | 0.38268 | 0.04621    | 0.38316 | 0.09254   | 0.38460 | 0.13910    | 0.38700 | 0.18601   | 0.39036 |
| 0.3  | 0.00    | 0.45399 | 0.04457    | 0.45454 | 0.08925   | 0.45626 | 0.13415    | 0.45911 | 0.17939   | 0.46310 |
| 0.35 | 0.00    | 0.52250 | 0.04265    | 0.52313 | 0.08541   | 0.52511 | 0.12838    | 0.52839 | 0.17167   | 0.53298 |
| 0.4  | 0.00    | 0.58778 | 0.04047    | 0.58850 | 0.08104   | 0.59073 | 0.12181    | 0.59441 | 0.16288   | 0.59958 |
| 0.45 | 0.00    | 0.64944 | 0.03804    | 0.65023 | 0.07617   | 0.65270 | 0.11449    | 0.65677 | 0.15310   | 0.66248 |
| 0.5  | 0.00    | 0.70711 | 0.03537    | 0.70796 | 0.07083   | 0.71065 | 0.10646    | 0.71508 | 0.14237   | 0.72130 |
| 0.55 | 0.00    | 0.76041 | 0.03249    | 0.76133 | 0.06505   | 0.76421 | 0.09778    | 0.76898 | 0.13076   | 0.77567 |
| 0.6  | 0.00    | 0.80902 | 0.02940    | 0.81000 | 0.05888   | 0.81307 | 0.08850    | 0.81814 | 0.11834   | 0.82525 |
| 0.65 | 0.00    | 0.85264 | 0.02614    | 0.85367 | 0.05234   | 0.85691 | 0.07867    | 0.86225 | 0.10520   | 0.86975 |
| 0.7  | 0.00    | 0.89101 | 0.02271    | 0.89208 | 0.04547   | 0.89547 | 0.06835    | 0.90105 | 0.09141   | 0.90889 |
| 0.75 | 0.00    | 0.92388 | 0.01914    | 0.92503 | 0.03833   | 0.92850 | 0.05762    | 0.93429 | 0.07705   | 0.94242 |
| 0.8  | 0.00    | 0.95106 | 0.01546    | 0.95225 | 0.03095   | 0.95582 | 0.04653    | 0.96178 | 0.06222   | 0.97014 |
| 0.85 | 0.00    | 0.97237 | 0.01168    | 0.97359 | 0.02338   | 0.97724 | 0.03515    | 0.98333 | 0.04700   | 0.99188 |
| 0.9  | 0.00    | 0.98769 | 0.00783    | 0.98892 | 0.01567   | 0.99263 | 0.02355    | 0.99882 | 0.03150   | 1.00751 |
| 0.95 | 0.00    | 0.99692 | 0.00392    | 0.99816 | 0.00786   | 1.00191 | 0.01181    | 1.00815 | 0.01580   | 1.01692 |
| 1.0  | 0.00    | 1.00000 | 0.00       | 1.00125 | 0.00      | 1.00500 | 0.00       | 1.01127 | 0.00      | 1.02007 |
| 1.05 | 0.00    | 0.99692 | 0.00392    | 0.99816 | 0.00786   | 1.00191 | 0.01181    | 1.00815 | 0.01580   | 1.01692 |
| 1.1  | 0.00    | 0.98769 | 0.00783    | 0.98892 | 0.01567   | 0.99263 | 0.02355    | 0.99882 | 0.03150   | 1.00751 |
| 1.15 | 0.00    | 0.97237 | 0.01168    | 0.97359 | 0.02338   | 0.97724 | 0.03515    | 0.98333 | 0.04700   | 0.99188 |
| 1.2  | 0.00    | 0.95106 | 0.01546    | 0.95225 | 0.03095   | 0.95582 | 0.04653    | 0.96178 | 0.06222   | 0.97014 |
| 1.25 | 0.00    | 0.92388 | 0.01914    | 0.92503 | 0.03833   | 0.92850 | 0.05762    | 0.93429 | 0.07705   | 0.94242 |
| 1.3  | 0.00    | 0.89101 | 0.02271    | 0.89208 | 0.04547   | 0.89547 | 0.06835    | 0.90105 | 0.09141   | 0.90889 |
| 1.35 | 0.00    | 0.85264 | 0.02614    | 0.85367 | 0.05234   | 0.85691 | 0.07867    | 0.86225 | 0.10520   | 0.86975 |
| 1.4  | 0.00    | 0.80902 | 0.02940    | 0.81000 | 0.05888   | 0.81307 | 0.08850    | 0.81814 | 0.11834   | 0.82525 |
| 1.45 | 0.00    | 0.76041 | 0.03249    | 0.76133 | 0.06505   | 0.76421 | 0.09778    | 0.76898 | 0.13076   | 0.77567 |
| 1.5  | 0.00    | 0.70711 | 0.03537    | 0.70796 | 0.07083   | 0.71065 | 0.10646    | 0.71508 | 0.14237   | 0.72130 |
| 1.55 | 0.00    | 0.64944 | 0.03804    | 0.65023 | 0.07617   | 0.65270 | 0.11449    | 0.65677 | 0.15310   | 0.66248 |
| 1.6  | 0.00    | 0.58778 | 0.04047    | 0.58850 | 0.08104   | 0.59073 | 0.12181    | 0.59441 | 0.16288   | 0.59958 |
| 1.65 | 0.00    | 0.52250 | 0.04265    | 0.52313 | 0.08541   | 0.52511 | 0.12838    | 0.52839 | 0.17167   | 0.53298 |
| 1.7  | 0.00    | 0.45399 | 0.04457    | 0.45454 | 0.08925   | 0.45626 | 0.13415    | 0.45911 | 0.17939   | 0.46310 |
| 1.75 | 0.00    | 0.38268 | 0.04621    | 0.38316 | 0.09254   | 0.38460 | 0.13910    | 0.38700 | 0.18601   | 0.39036 |
| 1.80 | 0.00    | 0.30902 | 0.04757    | 0.30940 | 0.09526   | 0.31056 | 0.14319    | 0.31250 | 0.19148   | 0.31522 |
| 1.85 | 0.00    | 0.23345 | 0.04864    | 0.23374 | 0.09740   | 0.23461 | 0.14640    | 0.23608 | 0.19577   | 0.23813 |
| 1.9  | 0.00    | 0.15643 | 0.04945    | 0.15663 | 0.09893   | 0.15722 | 0.14871    | 0.15820 | 0.19886   | 0.15957 |
| 1.95 | 0.00    | 0.07846 | 0.04987    | 0.07856 | 0.09986   | 0.07885 | 0.15010    | 0.07934 | 0.20072   | 0.08003 |
| 2.0  | 0.00    | 0.00    | 0.05002    | 0.00    | 0.10017   | 0.00    | 0.15056    | 0.00    | 0.20134   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(0.1 + i \underline{0.5}) = 0.07083 + i \underline{0.71065}$ .  
 $\sinh(0.1 + i \underline{1.2}) = -0.03095 + i \underline{0.95582}$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 0.25$ |         | $x = 0.3$ |         | $x = 0.35$ |         | $x = 0.4$ |         | $x = 0.45$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 0.25261    | 0.00    | 0.30452   | 0.00    | 0.35719    | 0.00    | 0.41075   | 0.00    | 0.46534    | 0.00    |
| 0.05 | 0.25183    | 0.08092 | 0.30358   | 0.08202 | 0.35609    | 0.08331 | 0.40949   | 0.08482 | 0.46391    | 0.08654 |
| 0.1  | 0.24950    | 0.16135 | 0.30077   | 0.16353 | 0.35279    | 0.16611 | 0.40570   | 0.16912 | 0.45961    | 0.17254 |
| 0.15 | 0.24563    | 0.24078 | 0.29611   | 0.24403 | 0.34732    | 0.24789 | 0.39940   | 0.25237 | 0.45249    | 0.25748 |
| 0.2  | 0.24025    | 0.31872 | 0.28962   | 0.32303 | 0.33971    | 0.32814 | 0.39065   | 0.33407 | 0.44257    | 0.34084 |
| 0.25 | 0.23338    | 0.39471 | 0.28134   | 0.40003 | 0.33000    | 0.40636 | 0.37949   | 0.41371 | 0.42992    | 0.42209 |
| 0.3  | 0.22508    | 0.46825 | 0.27133   | 0.47457 | 0.31826    | 0.48208 | 0.36598   | 0.49080 | 0.41462    | 0.50074 |
| 0.35 | 0.21539    | 0.53891 | 0.25965   | 0.54619 | 0.30455    | 0.55483 | 0.35022   | 0.56486 | 0.39677    | 0.57630 |
| 0.4  | 0.20437    | 0.60625 | 0.24636   | 0.61444 | 0.28897    | 0.62416 | 0.33231   | 0.63544 | 0.37647    | 0.64831 |
| 0.45 | 0.19208    | 0.66985 | 0.23156   | 0.67889 | 0.27161    | 0.68964 | 0.31234   | 0.70210 | 0.35385    | 0.71632 |
| 0.5  | 0.17862    | 0.72932 | 0.21533   | 0.73917 | 0.25257    | 0.75086 | 0.29045   | 0.76443 | 0.32905    | 0.77992 |
| 0.55 | 0.16406    | 0.78429 | 0.19777   | 0.79488 | 0.23198    | 0.80746 | 0.26676   | 0.82206 | 0.30222    | 0.83871 |
| 0.6  | 0.14848    | 0.83443 | 0.17899   | 0.84570 | 0.20995    | 0.85908 | 0.24143   | 0.87461 | 0.27352    | 0.89232 |
| 0.65 | 0.13199    | 0.87942 | 0.15911   | 0.89130 | 0.18663    | 0.90540 | 0.21462   | 0.92177 | 0.24314    | 0.94044 |
| 0.7  | 0.11468    | 0.91900 | 0.13825   | 0.93140 | 0.16216    | 0.94614 | 0.18648   | 0.96324 | 0.21126    | 0.98275 |
| 0.75 | 0.09667    | 0.95290 | 0.11654   | 0.96577 | 0.13669    | 0.98105 | 0.15719   | 0.99878 | 0.17808    | 1.01901 |
| 0.8  | 0.07801    | 0.98093 | 0.09410   | 0.99418 | 0.11038    | 1.00991 | 0.12693   | 1.02816 | 0.14380    | 1.04899 |
| 0.85 | 0.05897    | 1.00292 | 0.07109   | 1.01646 | 0.08338    | 1.03254 | 0.09589   | 1.05120 | 0.10863    | 1.07250 |
| 0.9  | 0.03952    | 1.01871 | 0.04764   | 1.03247 | 0.05588    | 1.04880 | 0.06426   | 1.06776 | 0.07280    | 1.08939 |
| 0.95 | 0.01982    | 1.02823 | 0.02389   | 1.04212 | 0.02803    | 1.05860 | 0.03223   | 1.07774 | 0.03651    | 1.09957 |
| 1.0  | 0.00       | 1.03141 | 0.00      | 1.04534 | 0.00       | 1.06188 | 0.00      | 1.08107 | 0.00       | 1.10297 |
| 1.05 | 0.01982    | 1.02823 | 0.02389   | 1.04212 | 0.02803    | 1.05860 | 0.03223   | 1.07774 | 0.03651    | 1.09957 |
| 1.1  | 0.03952    | 1.01871 | 0.04764   | 1.03247 | 0.05588    | 1.04880 | 0.06426   | 1.06776 | 0.07280    | 1.08939 |
| 1.15 | 0.05897    | 1.00292 | 0.07109   | 1.01646 | 0.08338    | 1.03254 | 0.09589   | 1.05120 | 0.10863    | 1.07250 |
| 1.2  | 0.07801    | 0.98093 | 0.09410   | 0.99418 | 0.11038    | 1.00991 | 0.12693   | 1.02816 | 0.14380    | 1.04899 |
| 1.25 | 0.09667    | 0.95290 | 0.11654   | 0.96577 | 0.13669    | 0.98105 | 0.15719   | 0.99878 | 0.17808    | 1.01901 |
| 1.3  | 0.11468    | 0.91900 | 0.13825   | 0.93140 | 0.16216    | 0.94614 | 0.18648   | 0.96324 | 0.21126    | 0.98275 |
| 1.35 | 0.13199    | 0.87942 | 0.15911   | 0.89130 | 0.18663    | 0.90540 | 0.21462   | 0.92177 | 0.24314    | 0.94044 |
| 1.4  | 0.14848    | 0.83443 | 0.17899   | 0.84570 | 0.20995    | 0.85908 | 0.24143   | 0.87461 | 0.27352    | 0.89232 |
| 1.45 | 0.16406    | 0.78429 | 0.19777   | 0.79488 | 0.23198    | 0.80746 | 0.26676   | 0.82206 | 0.30222    | 0.83871 |
| 1.5  | 0.17862    | 0.72932 | 0.21533   | 0.73917 | 0.25257    | 0.75086 | 0.29045   | 0.76443 | 0.32905    | 0.77992 |
| 1.55 | 0.19208    | 0.66985 | 0.23156   | 0.67889 | 0.27161    | 0.68964 | 0.31234   | 0.70210 | 0.35385    | 0.71632 |
| 1.6  | 0.20437    | 0.60625 | 0.24636   | 0.61444 | 0.28897    | 0.62416 | 0.33231   | 0.63544 | 0.37647    | 0.64831 |
| 1.65 | 0.21539    | 0.53891 | 0.25965   | 0.54619 | 0.30455    | 0.55483 | 0.35022   | 0.56486 | 0.39677    | 0.57630 |
| 1.7  | 0.22508    | 0.46825 | 0.27133   | 0.47457 | 0.31826    | 0.48208 | 0.36598   | 0.49080 | 0.41462    | 0.50074 |
| 1.75 | 0.23338    | 0.39471 | 0.28134   | 0.40003 | 0.33000    | 0.40636 | 0.37949   | 0.41371 | 0.42992    | 0.42209 |
| 1.8  | 0.24025    | 0.31872 | 0.28962   | 0.32303 | 0.33971    | 0.32814 | 0.39065   | 0.33407 | 0.44257    | 0.34084 |
| 1.85 | 0.24563    | 0.24078 | 0.29611   | 0.24403 | 0.34732    | 0.24789 | 0.39940   | 0.25237 | 0.45249    | 0.25748 |
| 1.9  | 0.24950    | 0.16135 | 0.30077   | 0.16353 | 0.35279    | 0.16611 | 0.40570   | 0.16912 | 0.45961    | 0.17254 |
| 1.95 | 0.25183    | 0.08092 | 0.30358   | 0.08202 | 0.35609    | 0.08331 | 0.40949   | 0.08482 | 0.46391    | 0.08654 |
| 2.0  | 0.25261    | 0.00    | 0.30452   | 0.00    | 0.35719    | 0.00    | 0.41075   | 0.00    | 0.46534    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(0.4 + i0) = 0.41075 + i0$ .  
 $\sinh(0.4 + i1) = 0. + i1.08107$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 0.5$ |         | $x = 0.55$ |         | $x = 0.6$ |         | $x = 0.65$ |         | $x = 0.7$ |         |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 0.52110   | 0.00    | 0.57815    | 0.00    | 0.63665   | 0.00    | 0.69675    | 0.00    | 0.75858   | 0.00    |
| 0.05 | 0.51940   | 0.08847 | 0.57637    | 0.09063 | 0.63469   | 0.09301 | 0.69460    | 0.09563 | 0.75625   | 0.09848 |
| 0.1  | 0.51468   | 0.17640 | 0.57103    | 0.18070 | 0.62882   | 0.18545 | 0.68817    | 0.19066 | 0.74925   | 0.19635 |
| 0.15 | 0.50670   | 0.26324 | 0.56218    | 0.26965 | 0.61906   | 0.27674 | 0.67750    | 0.28452 | 0.73763   | 0.29301 |
| 0.2  | 0.49559   | 0.34846 | 0.54986    | 0.35695 | 0.60549   | 0.36633 | 0.66265    | 0.37663 | 0.72146   | 0.38787 |
| 0.25 | 0.48143   | 0.43152 | 0.53414    | 0.44204 | 0.58819   | 0.45366 | 0.64371    | 0.46641 | 0.70084   | 0.48033 |
| 0.3  | 0.46430   | 0.51193 | 0.51514    | 0.52441 | 0.56726   | 0.53819 | 0.62081    | 0.55332 | 0.67590   | 0.56984 |
| 0.35 | 0.44431   | 0.58918 | 0.49296    | 0.60354 | 0.54284   | 0.61940 | 0.59408    | 0.63682 | 0.64680   | 0.65582 |
| 0.40 | 0.42158   | 0.66280 | 0.46773    | 0.67895 | 0.51506   | 0.69680 | 0.56368    | 0.71639 | 0.61371   | 0.73777 |
| 0.45 | 0.39624   | 0.73233 | 0.43963    | 0.75018 | 0.48412   | 0.76990 | 0.52981    | 0.79154 | 0.57683   | 0.81517 |
| 0.5  | 0.36847   | 0.79735 | 0.40882    | 0.81678 | 0.45018   | 0.83825 | 0.49268    | 0.86182 | 0.53640   | 0.88754 |
| 0.55 | 0.33842   | 0.85745 | 0.37548    | 0.87835 | 0.41347   | 0.90144 | 0.45250    | 0.92678 | 0.49266   | 0.95444 |
| 0.6  | 0.30629   | 0.91227 | 0.33983    | 0.93450 | 0.37422   | 0.95906 | 0.40954    | 0.98602 | 0.44589   | 1.01545 |
| 0.65 | 0.27227   | 0.96146 | 0.30208    | 0.98489 | 0.33265   | 1.01078 | 0.36405    | 1.03919 | 0.39036   | 1.07021 |
| 0.7  | 0.23657   | 1.00472 | 0.26248    | 1.02920 | 0.28904   | 1.05626 | 0.31632    | 1.08595 | 0.34439   | 1.11836 |
| 0.75 | 0.19942   | 1.04179 | 0.22125    | 1.06717 | 0.24364   | 1.09523 | 0.26663    | 1.12602 | 0.29030   | 1.15962 |
| 0.8  | 0.16103   | 1.07244 | 0.17866    | 1.09857 | 0.19674   | 1.12744 | 0.21531    | 1.15912 | 0.23442   | 1.19374 |
| 0.85 | 0.12165   | 1.09647 | 0.13497    | 1.12319 | 0.14862   | 1.15271 | 0.16265    | 1.18512 | 0.17709   | 1.22049 |
| 0.9  | 0.08152   | 1.11374 | 0.09044    | 1.14088 | 0.09959   | 1.17087 | 0.10900    | 1.20379 | 0.11867   | 1.23972 |
| 0.95 | 0.04088   | 1.12415 | 0.04536    | 1.15154 | 0.04995   | 1.18181 | 0.05467    | 1.21504 | 0.05952   | 1.25130 |
| 1.0  | 0.00      | 1.12763 | 0.00       | 1.15510 | 0.00      | 1.18547 | 0.00       | 1.21879 | 0.00      | 1.25517 |
| 1.05 | 0.04088   | 1.12415 | 0.04536    | 1.15154 | 0.04995   | 1.18181 | 0.05467    | 1.21504 | 0.05952   | 1.25130 |
| 1.1  | 0.08152   | 1.11374 | 0.09044    | 1.14088 | 0.09959   | 1.17087 | 0.10900    | 1.20379 | 0.11867   | 1.23972 |
| 1.15 | 0.12165   | 1.09647 | 0.13497    | 1.12319 | 0.14862   | 1.15271 | 0.16265    | 1.18512 | 0.17709   | 1.22049 |
| 1.2  | 0.16103   | 1.07244 | 0.17866    | 1.09857 | 0.19674   | 1.12744 | 0.21531    | 1.15912 | 0.23442   | 1.19374 |
| 1.25 | 0.19942   | 1.04179 | 0.22125    | 1.06717 | 0.24364   | 1.09523 | 0.26663    | 1.12602 | 0.29030   | 1.15962 |
| 1.3  | 0.23657   | 1.00472 | 0.26248    | 1.02920 | 0.28904   | 1.05626 | 0.31632    | 1.08595 | 0.34439   | 1.11836 |
| 1.35 | 0.27227   | 0.96146 | 0.30208    | 0.98489 | 0.33265   | 1.01078 | 0.36405    | 1.03919 | 0.39036   | 1.07021 |
| 1.4  | 0.30629   | 0.91227 | 0.33983    | 0.93450 | 0.37422   | 0.95906 | 0.40954    | 0.98602 | 0.44589   | 1.01545 |
| 1.45 | 0.33842   | 0.85745 | 0.37548    | 0.87835 | 0.41347   | 0.90144 | 0.45250    | 0.92678 | 0.49266   | 0.95444 |
| 1.5  | 0.36847   | 0.79735 | 0.40882    | 0.81678 | 0.45018   | 0.83825 | 0.49268    | 0.86182 | 0.53640   | 0.88754 |
| 1.55 | 0.39624   | 0.73233 | 0.43963    | 0.75018 | 0.48412   | 0.76990 | 0.52981    | 0.79154 | 0.57683   | 0.81517 |
| 1.6  | 0.42158   | 0.66280 | 0.46773    | 0.67895 | 0.51506   | 0.69680 | 0.56368    | 0.71639 | 0.61371   | 0.73777 |
| 1.65 | 0.44431   | 0.58918 | 0.49296    | 0.60354 | 0.54284   | 0.61940 | 0.59408    | 0.63682 | 0.64680   | 0.65582 |
| 1.7  | 0.46430   | 0.51193 | 0.51514    | 0.52441 | 0.56726   | 0.53819 | 0.62081    | 0.55332 | 0.67590   | 0.56984 |
| 1.75 | 0.48143   | 0.43152 | 0.53414    | 0.44204 | 0.58819   | 0.45366 | 0.64371    | 0.46641 | 0.70084   | 0.48033 |
| 1.8  | 0.49559   | 0.34846 | 0.54986    | 0.35695 | 0.60549   | 0.36633 | 0.66265    | 0.37663 | 0.72146   | 0.38787 |
| 1.85 | 0.50670   | 0.26324 | 0.56218    | 0.26965 | 0.61906   | 0.27674 | 0.67750    | 0.28452 | 0.73763   | 0.29301 |
| 1.9  | 0.51468   | 0.17640 | 0.57103    | 0.18070 | 0.62882   | 0.18545 | 0.68817    | 0.19066 | 0.74925   | 0.19635 |
| 1.95 | 0.51949   | 0.08847 | 0.57637    | 0.09063 | 0.63469   | 0.09301 | 0.69460    | 0.09563 | 0.75625   | 0.09848 |
| 2.0  | 0.52110   | 0.00    | 0.57815    | 0.00    | 0.63665   | 0.00    | 0.69675    | 0.00    | 0.75858   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(0.65 + i \underline{0.75}) = 0.26663 + i \underline{1.12602}$ .  
 $\sinh(0.55 + i \underline{1.40}) = -0.33983 + i \underline{0.93450}$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 0.75$ |         | $x = 0.8$ |         | $x = 0.85$ |         | $x = 0.9$ |         | $x = 0.95$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 0.82232    | 0.00    | 0.88811   | 0.00    | 0.95612    | 0.00    | 1.02652   | 0.00    | 1.09948    | 0.00    |
| 0.05 | 0.81978    | 0.10158 | 0.88537   | 0.10493 | 0.95317    | 0.10855 | 1.02335   | 0.11244 | 1.09610    | 0.11661 |
| 0.1  | 0.81219    | 0.20253 | 0.87717   | 0.20922 | 0.94435    | 0.21643 | 1.01388   | 0.22418 | 1.08595    | 0.23250 |
| 0.15 | 0.79960    | 0.30224 | 0.86357   | 0.31222 | 0.92970    | 0.32298 | 0.99816   | 0.33455 | 1.06911    | 0.34695 |
| 0.2  | 0.78207    | 0.40079 | 0.84464   | 0.41329 | 0.90932    | 0.42753 | 0.97628   | 0.44285 | 1.04567    | 0.45927 |
| 0.25 | 0.75972    | 0.49545 | 0.82050   | 0.51182 | 0.88334    | 0.52945 | 0.94838   | 0.54842 | 1.01579    | 0.56875 |
| 0.3  | 0.73269    | 0.58777 | 0.79131   | 0.60718 | 0.85191    | 0.62811 | 0.91463   | 0.65061 | 0.97965    | 0.67473 |
| 0.35 | 0.70114    | 0.67647 | 0.75724   | 0.69881 | 0.81522    | 0.72289 | 0.87525   | 0.74879 | 0.93747    | 0.77655 |
| 0.4  | 0.66527    | 0.76100 | 0.71849   | 0.78613 | 0.77351    | 0.81322 | 0.83047   | 0.84235 | 0.88950    | 0.87358 |
| 0.45 | 0.62529    | 0.84083 | 0.67532   | 0.86859 | 0.72704    | 0.89853 | 0.78057   | 0.93071 | 0.83605    | 0.96523 |
| 0.5  | 0.58146    | 0.91548 | 0.62799   | 0.94571 | 0.67608    | 0.97830 | 0.72586   | 1.01334 | 0.77745    | 1.05092 |
| 0.55 | 0.53495    | 0.98449 | 0.57678   | 1.01700 | 0.62095    | 1.05205 | 0.66667   | 1.08973 | 0.71406    | 1.13013 |
| 0.6  | 0.48335    | 1.04742 | 0.52202   | 1.08201 | 0.56109    | 1.11930 | 0.60337   | 1.15939 | 0.64621    | 1.20238 |
| 0.65 | 0.42966    | 1.10390 | 0.46403   | 1.14035 | 0.49957    | 1.17965 | 0.53635   | 1.22191 | 0.57448    | 1.26722 |
| 0.7  | 0.37332    | 1.15356 | 0.40319   | 1.19166 | 0.43407    | 1.23274 | 0.46603   | 1.27689 | 0.49916    | 1.32425 |
| 0.75 | 0.31469    | 1.19613 | 0.33986   | 1.23563 | 0.36589    | 1.27822 | 0.39283   | 1.32400 | 0.42076    | 1.37309 |
| 0.8  | 0.25411    | 1.23132 | 0.27444   | 1.27198 | 0.29546    | 1.31582 | 0.31721   | 1.36294 | 0.33976    | 1.41348 |
| 0.85 | 0.19197    | 1.25891 | 0.20732   | 1.30048 | 0.22320    | 1.34530 | 0.23964   | 1.39349 | 0.25667    | 1.44516 |
| 0.9  | 0.12864    | 1.27874 | 0.13893   | 1.32097 | 0.14957    | 1.36650 | 0.16058   | 1.41544 | 0.17200    | 1.46793 |
| 0.95 | 0.06452    | 1.29069 | 0.06968   | 1.33331 | 0.07502    | 1.37927 | 0.08054   | 1.42867 | 0.08627    | 1.48164 |
| 1.0  | 0.00       | 1.29468 | 0.00      | 1.33743 | 0.00       | 1.38353 | 0.00      | 1.43309 | 0.00       | 1.48623 |
| 1.05 | 0.06452    | 1.29069 | 0.06968   | 1.33331 | 0.07502    | 1.37927 | 0.08054   | 1.42867 | 0.08627    | 1.48164 |
| 1.1  | 0.12864    | 1.27874 | 0.13893   | 1.32097 | 0.14957    | 1.36650 | 0.16058   | 1.41544 | 0.17200    | 1.46793 |
| 1.15 | 0.19197    | 1.25891 | 0.20732   | 1.30048 | 0.22320    | 1.34530 | 0.23964   | 1.39349 | 0.25667    | 1.44516 |
| 1.2  | 0.25411    | 1.23132 | 0.27444   | 1.27198 | 0.29546    | 1.31582 | 0.31721   | 1.36294 | 0.33976    | 1.41348 |
| 1.25 | 0.31469    | 1.19613 | 0.33986   | 1.23563 | 0.36589    | 1.27822 | 0.39283   | 1.32400 | 0.42076    | 1.37309 |
| 1.3  | 0.37332    | 1.15356 | 0.40319   | 1.19166 | 0.43407    | 1.23274 | 0.46603   | 1.27689 | 0.49916    | 1.32425 |
| 1.35 | 0.42966    | 1.10390 | 0.46403   | 1.14035 | 0.49957    | 1.17965 | 0.53635   | 1.22191 | 0.57448    | 1.26722 |
| 1.4  | 0.48335    | 1.04742 | 0.52202   | 1.08201 | 0.56109    | 1.11930 | 0.60337   | 1.15939 | 0.64621    | 1.20238 |
| 1.45 | 0.53495    | 0.98449 | 0.57678   | 1.01700 | 0.62095    | 1.05205 | 0.66667   | 1.08973 | 0.71406    | 1.13013 |
| 1.5  | 0.58146    | 0.91548 | 0.62799   | 0.94571 | 0.67608    | 0.97830 | 0.72586   | 1.01334 | 0.77745    | 1.05092 |
| 1.55 | 0.62529    | 0.84083 | 0.67532   | 0.86859 | 0.72704    | 0.89853 | 0.78057   | 0.93071 | 0.83605    | 0.96523 |
| 1.6  | 0.66527    | 0.76100 | 0.71849   | 0.78613 | 0.77351    | 0.81322 | 0.83047   | 0.84235 | 0.88950    | 0.87358 |
| 1.65 | 0.70114    | 0.67647 | 0.75724   | 0.69881 | 0.81522    | 0.72289 | 0.87525   | 0.74879 | 0.93747    | 0.77655 |
| 1.7  | 0.73269    | 0.58777 | 0.79131   | 0.60718 | 0.85191    | 0.62811 | 0.91463   | 0.65061 | 0.97965    | 0.67473 |
| 1.75 | 0.75972    | 0.49545 | 0.82050   | 0.51182 | 0.88334    | 0.52945 | 0.94838   | 0.54842 | 1.01579    | 0.56875 |
| 1.8  | 0.78207    | 0.40079 | 0.84464   | 0.41329 | 0.90932    | 0.42753 | 0.97628   | 0.44285 | 1.04567    | 0.45927 |
| 1.85 | 0.79960    | 0.30224 | 0.86357   | 0.31222 | 0.92970    | 0.32298 | 0.99816   | 0.33455 | 1.06911    | 0.34695 |
| 1.9  | 0.81219    | 0.20253 | 0.87717   | 0.20922 | 0.94435    | 0.21643 | 1.01388   | 0.22418 | 1.08595    | 0.23250 |
| 1.95 | 0.81978    | 0.10158 | 0.88537   | 0.10493 | 0.95317    | 0.10855 | 1.02335   | 0.11244 | 1.09610    | 0.11661 |
| 2.0  | 0.82232    | 0.00    | 0.88811   | 0.00    | 0.95612    | 0.00    | 1.02652   | 0.00    | 1.09948    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(0.8 + i0.7) = 0.40319 + i1.19166$ .  
 $\sinh(0.8 + i1.7) = -0.79131 + i0.60718$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.0$ |         | $x = 1.05$ |         | $x = 1.1$ |         | $x = 1.15$ |         | $x = 1.2$ |          |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|----------|
| 0    | 1.17520   | 0.00    | 1.25386    | 0.00    | 1.33565   | 0.00    | 1.42078    | 0.00    | 1.50946   | 0.00     |
| 0.05 | 1.17158   | 0.12107 | 1.24999    | 0.12583 | 1.33153   | 0.13091 | 1.41640    | 0.13632 | 1.50481   | 0.14206  |
| 0.1  | 1.16073   | 0.24139 | 1.23842    | 0.25089 | 1.31920   | 0.26101 | 1.40329    | 0.27179 | 1.49088   | 0.28325  |
| 0.15 | 1.14273   | 0.36023 | 1.21921    | 0.37440 | 1.29875   | 0.38951 | 1.38152    | 0.40559 | 1.46776   | 0.42269  |
| 0.2  | 1.11768   | 0.47684 | 1.19249    | 0.49560 | 1.27028   | 0.51560 | 1.35124    | 0.53689 | 1.43558   | 0.55952  |
| 0.25 | 1.08574   | 0.59051 | 1.15841    | 0.61375 | 1.23398   | 0.63852 | 1.31263    | 0.66488 | 1.39456   | 0.69291  |
| 0.3  | 1.04711   | 0.70055 | 1.11719    | 0.72811 | 1.19007   | 0.75749 | 1.26592    | 0.78877 | 1.34494   | 0.82202  |
| 0.35 | 1.00202   | 0.80626 | 1.06909    | 0.83798 | 1.13883   | 0.87180 | 1.21141    | 0.90780 | 1.28703   | 0.94607  |
| 0.4  | 0.95076   | 0.90700 | 1.01439    | 0.94269 | 1.08056   | 0.98073 | 1.14943    | 1.02123 | 1.22118   | 1.06428  |
| 0.45 | 0.89363   | 1.00215 | 0.95344    | 1.04158 | 1.01564   | 1.08362 | 1.08037    | 1.12836 | 1.14781   | 1.171593 |
| 0.5  | 0.83099   | 1.09112 | 0.88661    | 1.13405 | 0.94445   | 1.17982 | 1.00464    | 1.22854 | 1.06735   | 1.28033  |
| 0.55 | 0.76323   | 1.17337 | 0.81432    | 1.21953 | 0.86743   | 1.26875 | 0.92272    | 1.32114 | 0.98032   | 1.37684  |
| 0.6  | 0.69077   | 1.24838 | 0.73700    | 1.29750 | 0.78508   | 1.34986 | 0.83511    | 1.40560 | 0.88724   | 1.46485  |
| 0.65 | 0.61404   | 1.31569 | 0.65514    | 1.36746 | 0.69787   | 1.42265 | 0.74235    | 1.48139 | 0.78869   | 1.54384  |
| 0.7  | 0.53353   | 1.37490 | 0.56924    | 1.42899 | 0.60637   | 1.48666 | 0.64502    | 1.54805 | 0.68528   | 1.61331  |
| 0.75 | 0.44973   | 1.42562 | 0.47983    | 1.48171 | 0.51113   | 1.54151 | 0.54371    | 1.60517 | 0.57765   | 1.67283  |
| 0.8  | 0.36316   | 1.46756 | 0.38746    | 1.52530 | 0.41274   | 1.58685 | 0.43904    | 1.65238 | 0.46645   | 1.72204  |
| 0.85 | 0.27435   | 1.50045 | 0.29271    | 1.55948 | 0.31180   | 1.62242 | 0.33167    | 1.68941 | 0.35238   | 1.76063  |
| 0.9  | 0.18384   | 1.52408 | 0.19615    | 1.58405 | 0.20894   | 1.64798 | 0.22226    | 1.71602 | 0.23613   | 1.78836  |
| 0.95 | 0.09221   | 1.53832 | 0.09838    | 1.59885 | 0.10479   | 1.66337 | 0.11147    | 1.73206 | 0.11843   | 1.80507  |
| 1.0  | 0.00      | 1.54308 | 0.00       | 1.60379 | 0.00      | 1.66852 | 0.00       | 1.73741 | 0.00      | 1.81066  |
| 1.05 | 0.09221   | 1.53832 | 0.09838    | 1.59885 | 0.10479   | 1.66337 | 0.11147    | 1.73206 | 0.11843   | 1.80507  |
| 1.1  | 0.18384   | 1.52408 | 0.19615    | 1.58405 | 0.20894   | 1.64798 | 0.22226    | 1.71602 | 0.23613   | 1.78836  |
| 1.15 | 0.27435   | 1.50045 | 0.29271    | 1.55948 | 0.31180   | 1.62242 | 0.33167    | 1.68941 | 0.35238   | 1.76063  |
| 1.2  | 0.36316   | 1.46756 | 0.38746    | 1.52530 | 0.41274   | 1.58685 | 0.43904    | 1.65238 | 0.46645   | 1.72204  |
| 1.25 | 0.44973   | 1.42562 | 0.47983    | 1.48171 | 0.51113   | 1.54151 | 0.54371    | 1.60517 | 0.57765   | 1.67283  |
| 1.3  | 0.53353   | 1.37490 | 0.56924    | 1.42899 | 0.60637   | 1.48666 | 0.64502    | 1.54805 | 0.68528   | 1.61331  |
| 1.35 | 0.61404   | 1.31569 | 0.65514    | 1.36746 | 0.69787   | 1.42265 | 0.74235    | 1.48139 | 0.78869   | 1.54384  |
| 1.4  | 0.69077   | 1.24838 | 0.73700    | 1.29750 | 0.78508   | 1.34986 | 0.83511    | 1.40560 | 0.88724   | 1.46485  |
| 1.45 | 0.76323   | 1.17337 | 0.81432    | 1.21953 | 0.86743   | 1.26875 | 0.92272    | 1.32114 | 0.98032   | 1.37684  |
| 1.5  | 0.83099   | 1.09112 | 0.88661    | 1.13405 | 0.94445   | 1.17982 | 1.00464    | 1.22854 | 1.06735   | 1.28033  |
| 1.55 | 0.89363   | 1.00215 | 0.95344    | 1.04158 | 1.01564   | 1.08362 | 1.08037    | 1.12836 | 1.14781   | 1.171593 |
| 1.6  | 0.95076   | 0.90700 | 1.01439    | 0.94269 | 1.08056   | 0.98073 | 1.14943    | 1.02123 | 1.22118   | 1.06428  |
| 1.65 | 1.00202   | 0.80626 | 1.06909    | 0.83798 | 1.13883   | 0.87180 | 1.21141    | 0.90780 | 1.28703   | 0.94607  |
| 1.7  | 1.04711   | 0.70055 | 1.11719    | 0.72811 | 1.19007   | 0.75749 | 1.26592    | 0.78877 | 1.34494   | 0.82202  |
| 1.75 | 1.08574   | 0.59051 | 1.15841    | 0.61375 | 1.23398   | 0.63852 | 1.31263    | 0.66488 | 1.39456   | 0.69291  |
| 1.8  | 1.11768   | 0.47684 | 1.19249    | 0.49560 | 1.27028   | 0.51560 | 1.35124    | 0.53689 | 1.43558   | 0.55952  |
| 1.85 | 1.14273   | 0.36023 | 1.21921    | 0.37440 | 1.29875   | 0.38951 | 1.38152    | 0.40559 | 1.46776   | 0.42269  |
| 1.9  | 1.16073   | 0.24139 | 1.23842    | 0.25089 | 1.31920   | 0.26101 | 1.40329    | 0.27179 | 1.49088   | 0.28325  |
| 1.95 | 1.17158   | 0.12107 | 1.24999    | 0.12583 | 1.33153   | 0.13091 | 1.41640    | 0.13632 | 1.50481   | 0.14206  |
| 2.0  | 1.17520   | 0.00    | 1.25386    | 0.00    | 1.33565   | 0.00    | 1.42078    | 0.00    | 1.50946   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(1.0 + i 1.0) = 0 + i 1.54308$ .  
 $\sinh(1.0 + i 1.5) = -0.83099 + i 1.09112$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.25$ |         | $x = 1.3$ |         | $x = 1.35$ |         | $x = 1.4$ |         | $x = 1.45$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 1.60192    | 0.00    | 1.69838   | 0.00    | 1.79909    | 0.00    | 1.90430   | 0.00    | 2.01427    | 0.00    |
| 0.05 | 1.59698    | 0.14816 | 1.69315   | 0.15464 | 1.79354    | 0.16150 | 1.89843   | 0.16876 | 2.00806    | 0.17644 |
| 0.1  | 1.58220    | 0.29541 | 1.67747   | 0.30832 | 1.77694    | 0.32199 | 1.88086   | 0.33647 | 1.98947    | 0.35180 |
| 0.15 | 1.55766    | 0.44084 | 1.65146   | 0.46010 | 1.74938    | 0.48051 | 1.85169   | 0.50212 | 1.95862    | 0.52498 |
| 0.2  | 1.52352    | 0.58355 | 1.61526   | 0.60905 | 1.71104    | 0.63606 | 1.81110   | 0.66466 | 1.91569    | 0.69493 |
| 0.25 | 1.47998    | 0.72267 | 1.56910   | 0.75424 | 1.66215    | 0.78769 | 1.75934   | 0.82311 | 1.86094    | 0.86060 |
| 0.3  | 1.42732    | 0.85733 | 1.51327   | 0.89478 | 1.60300    | 0.93446 | 1.69675   | 0.97649 | 1.79473    | 1.02095 |
| 0.35 | 1.36586    | 0.98670 | 1.44811   | 1.02080 | 1.53398    | 1.07548 | 1.62369   | 1.12384 | 1.71745    | 1.17502 |
| 0.4  | 1.29598    | 1.10999 | 1.37402   | 1.15848 | 1.45550    | 1.20986 | 1.54061   | 1.26427 | 1.62958    | 1.32184 |
| 0.45 | 1.21811    | 1.22643 | 1.29146   | 1.28001 | 1.36804    | 1.33678 | 1.44804   | 1.39690 | 1.53166    | 1.46051 |
| 0.5  | 1.13273    | 1.33532 | 1.20094   | 1.39365 | 1.27215    | 1.45546 | 1.34655   | 1.52092 | 1.42431    | 1.59017 |
| 0.55 | 1.04036    | 1.43597 | 1.10301   | 1.49870 | 1.16842    | 1.56517 | 1.23674   | 1.63556 | 1.30817    | 1.71007 |
| 0.6  | 0.94158    | 1.52777 | 0.99829   | 1.59451 | 1.05748    | 1.66523 | 1.11932   | 1.74012 | 1.18396    | 1.81935 |
| 0.65 | 0.83700    | 1.61014 | 0.88740   | 1.68048 | 0.94002    | 1.75502 | 0.99500   | 1.83394 | 1.05245    | 1.91745 |
| 0.7  | 0.72726    | 1.68260 | 0.77105   | 1.75610 | 0.81677    | 1.83399 | 0.86454   | 1.91646 | 0.91446    | 2.00373 |
| 0.75 | 0.61303    | 1.74467 | 0.64994   | 1.82089 | 0.68848    | 1.90165 | 0.72875   | 1.98717 | 0.77083    | 2.07766 |
| 0.8  | 0.49502    | 1.79600 | 0.52483   | 1.87445 | 0.55595    | 1.95759 | 0.58846   | 2.04562 | 0.62244    | 2.13878 |
| 0.85 | 0.37396    | 1.83624 | 0.39648   | 1.91646 | 0.41999    | 2.00146 | 0.44455   | 2.09147 | 0.47022    | 2.18671 |
| 0.9  | 0.25060    | 1.86517 | 0.26569   | 1.94665 | 0.28144    | 2.03299 | 0.29790   | 2.12442 | 0.31510    | 2.22115 |
| 0.95 | 0.12569    | 1.88260 | 0.13325   | 1.96484 | 0.14116    | 2.05199 | 0.14427   | 2.14427 | 0.15804    | 2.24191 |
| 1.0  | 0.00       | 1.88842 | 0.00      | 1.97091 | 0.00       | 2.05833 | 0.00      | 2.15090 | 0.00       | 2.24884 |
| 1.05 | 0.12569    | 1.88260 | 0.13325   | 1.96484 | 0.14116    | 2.05199 | 0.14427   | 2.14427 | 0.15804    | 2.24191 |
| 1.1  | 0.25060    | 1.86517 | 0.26569   | 1.94665 | 0.28144    | 2.03299 | 0.29790   | 2.12442 | 0.31510    | 2.22115 |
| 1.15 | 0.37396    | 1.83624 | 0.39648   | 1.91646 | 0.41999    | 2.00146 | 0.44455   | 2.09147 | 0.47022    | 2.18671 |
| 1.2  | 0.49502    | 1.79600 | 0.52483   | 1.87445 | 0.55595    | 1.95759 | 0.58846   | 2.04562 | 0.62244    | 2.13878 |
| 1.25 | 0.61303    | 1.74467 | 0.64994   | 1.82089 | 0.68848    | 1.90165 | 0.72875   | 1.98717 | 0.77083    | 2.07766 |
| 1.3  | 0.72726    | 1.68260 | 0.77105   | 1.75610 | 0.81677    | 1.83399 | 0.86454   | 1.91646 | 0.91446    | 2.00373 |
| 1.35 | 0.83700    | 1.61014 | 0.88740   | 1.68048 | 0.94002    | 1.75502 | 0.99500   | 1.83394 | 1.05245    | 1.91745 |
| 1.4  | 0.94158    | 1.52777 | 0.99829   | 1.59451 | 1.05748    | 1.66523 | 1.11932   | 1.74012 | 1.18396    | 1.81935 |
| 1.45 | 1.04036    | 1.43597 | 1.10301   | 1.49870 | 1.16842    | 1.56517 | 1.23674   | 1.63556 | 1.30817    | 1.71007 |
| 1.5  | 1.13273    | 1.33532 | 1.20094   | 1.39365 | 1.27215    | 1.45546 | 1.34655   | 1.52092 | 1.42431    | 1.59017 |
| 1.55 | 1.21811    | 1.22643 | 1.29146   | 1.28001 | 1.36804    | 1.33678 | 1.44804   | 1.39690 | 1.53166    | 1.46051 |
| 1.6  | 1.29598    | 1.10999 | 1.37402   | 1.15848 | 1.45550    | 1.20986 | 1.54061   | 1.26427 | 1.62958    | 1.32184 |
| 1.65 | 1.36586    | 0.98670 | 1.44811   | 1.02080 | 1.53398    | 1.07548 | 1.62369   | 1.12384 | 1.71745    | 1.17502 |
| 1.7  | 1.42732    | 0.85733 | 1.51327   | 0.89478 | 1.60300    | 0.93446 | 1.69675   | 0.97649 | 1.79473    | 1.02095 |
| 1.75 | 1.47998    | 0.72267 | 1.56910   | 0.75424 | 1.66215    | 0.78769 | 1.75934   | 0.82311 | 1.86094    | 0.86060 |
| 1.8  | 1.52352    | 0.58355 | 1.61526   | 0.60905 | 1.71104    | 0.63606 | 1.81110   | 0.66466 | 1.91569    | 0.69493 |
| 1.85 | 1.55766    | 0.44084 | 1.65146   | 0.46010 | 1.74938    | 0.48051 | 1.85169   | 0.50212 | 1.95862    | 0.52498 |
| 1.9  | 1.58220    | 0.29541 | 1.67747   | 0.30832 | 1.77694    | 0.32199 | 1.88086   | 0.33647 | 1.98947    | 0.35180 |
| 1.95 | 1.59698    | 0.14816 | 1.69315   | 0.15464 | 1.79354    | 0.16150 | 1.89843   | 0.16876 | 2.00806    | 0.17644 |
| 2.0  | 1.60192    | 0.00    | 1.69838   | 0.00    | 1.79909    | 0.00    | 1.90430   | 0.00    | 2.01427    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(1.35 + i0) = 1.70909 + i0$ .  
 $\sinh(1.4 + i1.15) = -0.44455 + i2.09147$ .



TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.5$ |         | $x = 1.55$ |         | $x = 1.6$ |         | $x = 1.65$ |         | $x = 1.7$ |         |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 2.12928   | 0.00    | 2.24961    | 0.00    | 2.37557   | 0.00    | 2.50746    | 0.00    | 2.64563   | 0.00    |
| 0.05 | 2.12272   | 0.18457 | 2.24268    | 0.19316 | 2.36824   | 0.20223 | 2.49973    | 0.21180 | 2.63747   | 0.22191 |
| 0.1  | 2.10307   | 0.36800 | 2.22191    | 0.38512 | 2.34632   | 0.40320 | 2.47659    | 0.42230 | 2.61306   | 0.44245 |
| 0.15 | 2.07045   | 0.54916 | 2.18745    | 0.57471 | 2.30993   | 0.60170 | 2.43818    | 0.63019 | 2.57253   | 0.66026 |
| 0.2  | 2.02507   | 0.72693 | 2.13951    | 0.76076 | 2.25930   | 0.79648 | 2.38474    | 0.83420 | 2.51614   | 0.87400 |
| 0.25 | 1.96720   | 0.90023 | 2.07837    | 0.94211 | 2.19473   | 0.98636 | 2.31660    | 1.03306 | 2.44424   | 1.08235 |
| 0.3  | 1.89720   | 1.06797 | 2.00442    | 1.11766 | 2.11604   | 1.17015 | 2.23417    | 1.22556 | 2.35727   | 1.28403 |
| 0.35 | 1.81551   | 1.22913 | 1.91811    | 1.28632 | 2.02550   | 1.34672 | 2.13797    | 1.41050 | 2.25577   | 1.47779 |
| 0.4  | 1.72263   | 1.38271 | 1.81997    | 1.44704 | 1.92187   | 1.51500 | 2.02858    | 1.58674 | 2.14036   | 1.66244 |
| 0.45 | 1.61912   | 1.52777 | 1.71062    | 1.59885 | 1.80640   | 1.67393 | 1.90669    | 1.75320 | 2.01175   | 1.83684 |
| 0.5  | 1.50563   | 1.66341 | 1.59071    | 1.74080 | 1.67978   | 1.82254 | 1.77305    | 1.90885 | 1.87074   | 1.99992 |
| 0.55 | 1.38286   | 1.78879 | 1.46101    | 1.87201 | 1.54281   | 1.95992 | 1.62847    | 2.05274 | 1.71820   | 2.15067 |
| 0.6  | 1.25156   | 1.90314 | 1.32229    | 1.99169 | 1.39632   | 2.08522 | 1.47385    | 2.18396 | 1.55506   | 2.28816 |
| 0.65 | 1.11255   | 2.00576 | 1.17542    | 2.09908 | 1.24123   | 2.19765 | 1.31015    | 2.30173 | 1.38234   | 2.41154 |
| 0.7  | 0.96667   | 2.09601 | 1.02130    | 2.19353 | 1.07848   | 2.29654 | 1.13837    | 2.40529 | 1.20109   | 2.52005 |
| 0.75 | 0.81484   | 2.17334 | 0.86089    | 2.27446 | 0.90909   | 2.38127 | 0.95957    | 2.49404 | 1.01244   | 2.61302 |
| 0.8  | 0.65798   | 2.23727 | 0.69517    | 2.34137 | 0.73409   | 2.45131 | 0.77485    | 2.56740 | 0.81754   | 2.68989 |
| 0.85 | 0.49707   | 2.28742 | 0.52516    | 2.39384 | 0.55456   | 2.50625 | 0.58536    | 2.62494 | 0.61761   | 2.75017 |
| 0.9  | 0.33309   | 2.32345 | 0.35192    | 2.43155 | 0.37163   | 2.54573 | 0.39225    | 2.66629 | 0.41387   | 2.79350 |
| 0.95 | 0.16706   | 2.34516 | 0.17650    | 2.45427 | 0.18639   | 2.56952 | 0.19673    | 2.69121 | 0.20757   | 2.81960 |
| 1.0  | 0.00      | 2.35241 | 0.00       | 2.46186 | 0.00      | 2.57746 | 0.00       | 2.69951 | 0.00      | 2.82832 |
| 1.05 | 0.16706   | 2.34516 | 0.17650    | 2.45427 | 0.18639   | 2.56952 | 0.19673    | 2.69121 | 0.20757   | 2.81960 |
| 1.1  | 0.33309   | 2.32345 | 0.35192    | 2.43155 | 0.37163   | 2.54573 | 0.39225    | 2.66629 | 0.41387   | 2.79350 |
| 1.15 | 0.49707   | 2.28742 | 0.52516    | 2.39384 | 0.55456   | 2.50625 | 0.58536    | 2.62494 | 0.61761   | 2.75017 |
| 1.2  | 0.65798   | 2.23727 | 0.69517    | 2.34137 | 0.73409   | 2.45131 | 0.77485    | 2.56740 | 0.81754   | 2.68989 |
| 1.25 | 0.81484   | 2.17334 | 0.86089    | 2.27446 | 0.90909   | 2.38127 | 0.95957    | 2.49404 | 1.01244   | 2.61302 |
| 1.3  | 0.96667   | 2.09601 | 1.02130    | 2.19353 | 1.07848   | 2.29654 | 1.13837    | 2.40529 | 1.20109   | 2.52005 |
| 1.35 | 1.11255   | 2.00576 | 1.17542    | 2.09908 | 1.24123   | 2.19765 | 1.31015    | 2.30173 | 1.38234   | 2.41154 |
| 1.4  | 1.25156   | 1.90314 | 1.32229    | 1.99169 | 1.39632   | 2.08522 | 1.47385    | 2.18396 | 1.55506   | 2.28816 |
| 1.45 | 1.38286   | 1.78879 | 1.46101    | 1.87201 | 1.54281   | 1.95992 | 1.62847    | 2.05274 | 1.71820   | 2.15067 |
| 1.5  | 1.50563   | 1.66341 | 1.59071    | 1.74080 | 1.67978   | 1.82254 | 1.77305    | 1.90885 | 1.87074   | 1.99992 |
| 1.55 | 1.61912   | 1.52777 | 1.71062    | 1.59885 | 1.80640   | 1.67393 | 1.90669    | 1.75320 | 2.01175   | 1.83684 |
| 1.6  | 1.72263   | 1.38271 | 1.81997    | 1.44704 | 1.92187   | 1.51500 | 2.02858    | 1.58674 | 2.14036   | 1.66244 |
| 1.65 | 1.81551   | 1.22913 | 1.91811    | 1.28632 | 2.02550   | 1.34672 | 2.13797    | 1.41050 | 2.25577   | 1.47779 |
| 1.7  | 1.89720   | 1.06797 | 2.00442    | 1.11766 | 2.11664   | 1.17015 | 2.23417    | 1.22556 | 2.35727   | 1.28403 |
| 1.75 | 1.96720   | 0.90023 | 2.07837    | 0.94211 | 2.19473   | 0.98636 | 2.31660    | 1.03306 | 2.44424   | 1.08235 |
| 1.8  | 2.02507   | 0.72693 | 2.13951    | 0.76076 | 2.25930   | 0.79648 | 2.38474    | 0.83420 | 2.51614   | 0.87400 |
| 1.85 | 2.07045   | 0.54916 | 2.18745    | 0.57471 | 2.30993   | 0.60170 | 2.43818    | 0.63019 | 2.57253   | 0.66026 |
| 1.9  | 2.10307   | 0.36800 | 2.22191    | 0.38512 | 2.34632   | 0.40320 | 2.47659    | 0.42230 | 2.61306   | 0.44245 |
| 1.95 | 2.12272   | 0.18457 | 2.24268    | 0.19316 | 2.36824   | 0.20223 | 2.49973    | 0.21180 | 2.63747   | 0.22191 |
| 2.0  | 2.12928   | 0.00    | 2.24961    | 0.00    | 2.37557   | 0.00    | 2.50746    | 0.00    | 2.64563   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(1.7 + i\textcolor{red}{0.7}) = 1.20109 + i\textcolor{red}{2.52005}$ .  
 $\sinh(1.7 + i\textcolor{red}{1.7}) = -2.35727 + i\textcolor{red}{1.28403}$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.75$ |         | $x = 1.8$ |         | $x = 1.85$ |         | $x = 1.9$ |         | $x = 1.95$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 2.79041    | 0.00    | 2.94217   | 0.00    | 3.10129    | 0.00    | 3.26816   | 0.00    | 3.44321    | 0.00    |
| 0.05 | 2.78181    | 0.23257 | 2.93310   | 0.24381 | 3.09173    | 0.25566 | 3.25809   | 0.26815 | 3.43259    | 0.28131 |
| 0.1  | 2.75006    | 0.46370 | 2.90595   | 0.48612 | 3.06311    | 0.50975 | 3.22793   | 0.53465 | 3.40081    | 0.56089 |
| 0.15 | 2.71331    | 0.69198 | 2.86088   | 0.72542 | 3.01560    | 0.76069 | 3.17787   | 0.79785 | 3.34807    | 0.83701 |
| 0.2  | 2.65384    | 0.91599 | 2.79817   | 0.96026 | 2.94950    | 1.00694 | 3.10821   | 1.05614 | 3.27468    | 1.10797 |
| 0.25 | 2.57800    | 1.13435 | 2.71821   | 1.18918 | 2.86522    | 1.24698 | 3.01939   | 1.30791 | 3.18111    | 1.37210 |
| 0.3  | 2.48627    | 1.34571 | 2.62149   | 1.41076 | 2.76327    | 1.47934 | 2.91196   | 1.55162 | 3.06792    | 1.62777 |
| 0.35 | 2.37922    | 1.54878 | 2.50862   | 1.62365 | 2.64429    | 1.70258 | 2.78657   | 1.78576 | 2.93582    | 1.87341 |
| 0.4  | 2.25749    | 1.74231 | 2.38027   | 1.82653 | 2.50900    | 1.91531 | 2.64400   | 2.00889 | 2.78561    | 2.10749 |
| 0.45 | 2.12185    | 1.92509 | 2.23725   | 2.01814 | 2.35824    | 2.11624 | 2.48513   | 2.21964 | 2.61823    | 2.32858 |
| 0.5  | 1.97312    | 2.09600 | 2.08043   | 2.19731 | 2.19294    | 2.30413 | 2.31094   | 2.41670 | 2.43471    | 2.53532 |
| 0.55 | 1.81223    | 2.25399 | 1.91079   | 2.30294 | 2.01413    | 2.47780 | 2.12250   | 2.59887 | 2.23618    | 2.72642 |
| 0.6  | 1.64016    | 2.39808 | 1.72937   | 2.51400 | 1.82289    | 2.63620 | 1.92098   | 2.76501 | 2.02387    | 2.90071 |
| 0.65 | 1.45799    | 2.52739 | 1.53728   | 2.64956 | 1.62042    | 2.77835 | 1.70761   | 2.91409 | 1.79997    | 3.05713 |
| 0.7  | 1.26682    | 2.64111 | 1.33572   | 2.76878 | 1.40796    | 2.90337 | 1.48372   | 3.04522 | 1.56318    | 3.19469 |
| 0.75 | 1.06784    | 2.73855 | 1.12592   | 2.87093 | 1.18681    | 3.01049 | 1.25067   | 3.15757 | 1.31766    | 3.31255 |
| 0.8  | 0.86229    | 2.81911 | 0.90918   | 2.95538 | 0.95835    | 3.09904 | 1.00992   | 3.25045 | 1.06401    | 3.41000 |
| 0.85 | 0.65141    | 2.88229 | 0.68684   | 3.02161 | 0.72398    | 3.16850 | 0.76294   | 3.32330 | 0.80380    | 3.48641 |
| 0.9  | 0.43652    | 2.92769 | 0.46026   | 3.06921 | 0.48627    | 3.21841 | 0.51125   | 3.37565 | 0.53864    | 3.54134 |
| 0.95 | 0.21893    | 2.95505 | 0.23084   | 3.09789 | 0.24332    | 3.24848 | 0.25642   | 3.40719 | 0.27015    | 3.57443 |
| 1.0  | 0.00       | 2.96419 | 0.00      | 3.10747 | 0.00       | 3.25853 | 0.00      | 3.41773 | 0.00       | 3.58548 |
| 1.05 | 0.21893    | 2.95505 | 0.23084   | 3.09789 | 0.24332    | 3.24848 | 0.25642   | 3.40719 | 0.27015    | 3.57443 |
| 1.1  | 0.43652    | 2.92769 | 0.46026   | 3.06921 | 0.48627    | 3.21841 | 0.51125   | 3.37565 | 0.53864    | 3.54134 |
| 1.15 | 0.65141    | 2.88229 | 0.68684   | 3.02161 | 0.72398    | 3.16850 | 0.76294   | 3.32330 | 0.80380    | 3.48641 |
| 1.2  | 0.86229    | 2.81911 | 0.90918   | 2.95538 | 0.95835    | 3.09904 | 1.00992   | 3.25045 | 1.06401    | 3.41000 |
| 1.25 | 1.06784    | 2.73855 | 1.12592   | 2.87093 | 1.18681    | 3.01049 | 1.25067   | 3.15757 | 1.31766    | 3.31255 |
| 1.3  | 1.26682    | 2.64111 | 1.33572   | 2.76878 | 1.40796    | 2.90337 | 1.48372   | 3.04522 | 1.56318    | 3.19469 |
| 1.35 | 1.45799    | 2.52739 | 1.53728   | 2.64956 | 1.62042    | 2.77835 | 1.70761   | 2.91409 | 1.79997    | 3.05713 |
| 1.4  | 1.64016    | 2.39808 | 1.72937   | 2.51400 | 1.82289    | 2.63620 | 1.92098   | 2.76501 | 2.02387    | 2.90071 |
| 1.45 | 1.81223    | 2.25399 | 1.91079   | 2.36294 | 2.01413    | 2.47780 | 2.12250   | 2.59887 | 2.23618    | 2.72642 |
| 1.5  | 1.97312    | 2.09600 | 2.08043   | 2.19731 | 2.19294    | 2.30413 | 2.31094   | 2.41670 | 2.43471    | 2.53532 |
| 1.55 | 2.12185    | 1.92509 | 2.23725   | 2.01814 | 2.35824    | 2.11624 | 2.48513   | 2.21964 | 2.61823    | 2.32858 |
| 1.6  | 2.25749    | 1.74231 | 2.38027   | 1.82653 | 2.50900    | 1.91531 | 2.64400   | 2.00889 | 2.78561    | 2.10749 |
| 1.65 | 2.37922    | 1.54878 | 2.50862   | 1.62365 | 2.64429    | 1.70258 | 2.78657   | 1.78576 | 2.93582    | 1.87341 |
| 1.7  | 2.48627    | 1.34571 | 2.62149   | 1.41076 | 2.76327    | 1.47934 | 2.91196   | 1.55162 | 3.06792    | 1.62777 |
| 1.75 | 2.57800    | 1.13435 | 2.71821   | 1.18918 | 2.86522    | 1.24698 | 3.01939   | 1.30791 | 3.18111    | 1.37210 |
| 1.8  | 2.65384    | 0.91599 | 2.79817   | 0.96026 | 2.94950    | 1.00694 | 3.10821   | 1.05614 | 3.27468    | 1.10797 |
| 1.85 | 2.71331    | 0.69198 | 2.86088   | 0.72542 | 3.01560    | 0.76069 | 3.17787   | 0.79785 | 3.34807    | 0.83701 |
| 1.9  | 2.75006    | 0.46370 | 2.90595   | 0.48612 | 3.06311    | 0.50975 | 3.22793   | 0.53465 | 3.40081    | 0.56089 |
| 1.95 | 2.78181    | 0.23257 | 2.93310   | 0.24381 | 3.09173    | 0.25566 | 3.25809   | 0.26815 | 3.43259    | 0.28131 |
| 2.0  | 2.79041    | 0.00    | 2.94217   | 0.00    | 3.10129    | 0.00    | 3.26816   | 0.00    | 3.44321    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(1.85 + i \underline{0.75}) = 1.18681 + i \underline{3.01049}$ .  
 $\sinh(1.85 + i \underline{1.35}) = -1.62042 + i \underline{2.77835}$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.0$ |         | $x = 2.05$ |         | $x = 2.1$ |         | $x = 2.15$ |         | $x = 2.2$ |         |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 3.62686   | 0.00    | 3.81958    | 0.00    | 4.02186   | 0.00    | 4.23419    | 0.00    | 4.45711   | 0.00    |
| 0.05 | 3.61568   | 0.29518 | 3.80781    | 0.30978 | 4.00946   | 0.32516 | 4.22113    | 0.34135 | 4.44337   | 0.35839 |
| 0.1  | 3.58221   | 0.58854 | 3.77256    | 0.61765 | 3.97235   | 0.64831 | 4.18206    | 0.68060 | 4.40223   | 0.71458 |
| 0.15 | 3.52666   | 0.87827 | 3.71404    | 0.92172 | 3.91074   | 0.96747 | 4.11720    | 1.01564 | 4.33396   | 1.06636 |
| 0.2  | 3.44935   | 1.16258 | 3.63264    | 1.22010 | 3.82501   | 1.28066 | 4.02695    | 1.34443 | 4.23896   | 1.41156 |
| 0.25 | 3.35078   | 1.43973 | 3.52883    | 1.51096 | 3.71571   | 1.58596 | 3.91188    | 1.66493 | 4.11783   | 1.74806 |
| 0.3  | 3.23156   | 1.70800 | 3.40327    | 1.79250 | 3.58351   | 1.88148 | 3.77269    | 1.97516 | 3.97131   | 2.07379 |
| 0.35 | 3.09241   | 1.96574 | 3.23673    | 2.06299 | 3.42920   | 2.16540 | 3.61024    | 2.27322 | 3.80031   | 2.38672 |
| 0.4  | 2.93420   | 2.21136 | 3.09011    | 2.32076 | 3.25376   | 2.43597 | 3.42553    | 2.55726 | 3.60588   | 2.68495 |
| 0.45 | 2.75789   | 2.44335 | 2.90443    | 2.56423 | 3.05825   | 2.69152 | 3.21970    | 2.82553 | 3.38921   | 2.96661 |
| 0.5  | 2.56458   | 2.66027 | 2.70085    | 2.79188 | 2.84389   | 2.93048 | 2.99402    | 3.07639 | 3.15165   | 3.23000 |
| 0.55 | 2.35546   | 2.86080 | 2.48062    | 3.00232 | 2.61199   | 3.15137 | 2.74988    | 3.30828 | 2.89466   | 3.47347 |
| 0.6  | 2.13182   | 3.04368 | 2.24509    | 3.19426 | 2.36399   | 3.35283 | 2.48879    | 3.51977 | 2.61982   | 3.69552 |
| 0.65 | 1.89503   | 3.20780 | 1.99573    | 3.36649 | 2.10142   | 3.53361 | 2.21236    | 3.70956 | 2.32883   | 3.89478 |
| 0.7  | 1.64656   | 3.35214 | 1.73405    | 3.51798 | 1.82589   | 3.69261 | 1.92228    | 3.87647 | 2.02349   | 4.07003 |
| 0.75 | 1.38794   | 3.47581 | 1.46169    | 3.64777 | 1.53910   | 3.82885 | 1.62035    | 4.01950 | 1.70566   | 4.22019 |
| 0.8  | 1.12076   | 3.57806 | 1.18032    | 3.75507 | 1.24282   | 3.94148 | 1.30844    | 4.13773 | 1.37732   | 4.34433 |
| 0.85 | 0.84667   | 3.65825 | 0.89166    | 3.83923 | 0.93888   | 4.02981 | 0.98845    | 4.23046 | 1.04049   | 4.44170 |
| 0.9  | 0.56737   | 3.71587 | 0.59751    | 3.89971 | 0.62916   | 4.09329 | 0.66237    | 4.29711 | 0.69724   | 4.51167 |
| 0.95 | 0.28456   | 3.75059 | 0.29968    | 3.93615 | 0.31555   | 4.13154 | 0.33221    | 4.33726 | 0.34970   | 4.55382 |
| 1.0  | 0.00      | 3.76220 | 0.00       | 3.94832 | 0.00      | 4.14431 | 0.00       | 4.35067 | 0.00      | 4.56791 |
| 1.05 | 0.28456   | 3.75059 | 0.29968    | 3.93615 | 0.31555   | 4.13154 | 0.33221    | 4.33726 | 0.34970   | 4.55382 |
| 1.1  | 0.56737   | 3.71587 | 0.59751    | 3.89971 | 0.62916   | 4.09329 | 0.66237    | 4.29711 | 0.69724   | 4.51167 |
| 1.15 | 0.84667   | 3.65825 | 0.89166    | 3.83923 | 0.93888   | 4.02981 | 0.98845    | 4.23046 | 1.04049   | 4.44170 |
| 1.2  | 1.12076   | 3.57806 | 1.18032    | 3.75507 | 1.24282   | 3.94148 | 1.30844    | 4.13773 | 1.37732   | 4.34433 |
| 1.25 | 1.38794   | 3.47581 | 1.46169    | 3.64777 | 1.53910   | 3.82885 | 1.62035    | 4.01950 | 1.70566   | 4.22019 |
| 1.3  | 1.64656   | 3.35214 | 1.73405    | 3.51798 | 1.82589   | 3.69261 | 1.92228    | 3.87647 | 2.02349   | 4.07003 |
| 1.35 | 1.89503   | 3.20780 | 1.99573    | 3.36649 | 2.10142   | 3.53361 | 2.21236    | 3.70956 | 2.32883   | 3.89478 |
| 1.4  | 2.13182   | 3.04368 | 2.24509    | 3.19426 | 2.36399   | 3.35283 | 2.48879    | 3.51977 | 2.61982   | 3.69552 |
| 1.45 | 2.35546   | 2.86080 | 2.48062    | 3.00232 | 2.61199   | 3.15137 | 2.74988    | 3.30828 | 2.89466   | 3.47347 |
| 1.5  | 2.56458   | 2.66027 | 2.70085    | 2.79188 | 2.84389   | 2.93048 | 2.99402    | 3.07639 | 3.15165   | 3.23000 |
| 1.55 | 2.75789   | 2.44335 | 2.90443    | 2.56423 | 3.05825   | 2.69152 | 3.21970    | 2.82553 | 3.38921   | 2.96661 |
| 1.6  | 2.93420   | 2.21136 | 3.09011    | 2.32076 | 3.25376   | 2.43597 | 3.42553    | 2.55726 | 3.60588   | 2.68495 |
| 1.65 | 3.09241   | 1.96574 | 3.23673    | 2.06299 | 3.42920   | 2.16540 | 3.61024    | 2.27322 | 3.80031   | 2.38672 |
| 1.7  | 3.23156   | 1.70800 | 3.40327    | 1.79250 | 3.58351   | 1.88148 | 3.77269    | 1.97516 | 3.97131   | 2.07379 |
| 1.75 | 3.35078   | 1.43973 | 3.52883    | 1.51096 | 3.71571   | 1.58596 | 3.91188    | 1.66493 | 4.11783   | 1.74806 |
| 1.8  | 3.44935   | 1.16258 | 3.63264    | 1.22010 | 3.82501   | 1.28066 | 4.02695    | 1.34443 | 4.23896   | 1.41156 |
| 1.85 | 3.52666   | 0.87827 | 3.71404    | 0.92172 | 3.91074   | 0.96747 | 4.11720    | 1.01564 | 4.33396   | 1.06636 |
| 1.9  | 3.58221   | 0.58854 | 3.77256    | 0.61765 | 3.97235   | 0.64831 | 4.18206    | 0.68060 | 4.40223   | 0.71458 |
| 1.95 | 3.61568   | 0.29518 | 3.80781    | 0.30978 | 4.00946   | 0.32516 | 4.22113    | 0.34135 | 4.44337   | 0.35839 |
| 2.0  | 3.62686   | 0.00    | 3.81958    | 0.00    | 4.02186   | 0.00    | 4.23419    | 0.00    | 4.45711   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(2.2 + i \underline{1.0}) = 0 + i 4.56791$ .  
 $\sinh(2.2 + i \underline{1.5}) = -3.15165 + i 3.23000$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.25$ |         | $x = 2.3$ |         | $x = 2.35$ |         | $x = 2.4$ |         | $x = 2.45$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 4.69117    | 0.00    | 4.93696   | 0.00    | 5.19510    | 0.00    | 5.46623   | 0.00    | 5.75103    | 0.00    |
| 0.05 | 4.67671    | 0.37633 | 4.92174   | 0.39522 | 5.17909    | 0.41509 | 5.44938   | 0.43599 | 5.73330    | 0.45799 |
| 0.1  | 4.63341    | 0.75035 | 4.87618   | 0.78799 | 5.13114    | 0.82761 | 5.39893   | 0.86930 | 5.68022    | 0.91316 |
| 0.15 | 4.56155    | 1.11974 | 4.80056   | 1.17592 | 5.05156    | 1.23504 | 5.31521   | 1.29724 | 5.59213    | 1.36269 |
| 0.2  | 4.46157    | 1.48222 | 4.69533   | 1.55659 | 4.94083    | 1.63485 | 5.19869   | 1.71719 | 5.46955    | 1.80383 |
| 0.25 | 4.33407    | 1.83557 | 4.56116   | 1.92766 | 4.79965    | 2.02457 | 5.05014   | 2.12655 | 5.31325    | 2.23385 |
| 0.3  | 4.17986    | 2.17760 | 4.39887   | 2.28685 | 4.62887    | 2.40182 | 4.87045   | 2.52280 | 5.12420    | 2.65009 |
| 0.35 | 3.99987    | 2.50620 | 4.20946   | 2.63104 | 4.42955    | 2.76426 | 4.66073   | 2.90349 | 4.90356    | 3.04999 |
| 0.4  | 3.79523    | 2.81935 | 3.99409   | 2.96081 | 4.20292    | 3.10966 | 4.42228   | 3.26629 | 4.65268    | 3.43109 |
| 0.45 | 3.56719    | 3.11512 | 3.75410   | 3.27141 | 3.95038    | 3.43588 | 4.15656   | 3.60894 | 4.37311    | 3.79104 |
| 0.5  | 3.31716    | 3.39168 | 3.49096   | 3.56185 | 3.67351    | 3.74093 | 3.86521   | 3.92935 | 4.06659    | 4.12761 |
| 0.55 | 3.04677    | 3.64734 | 3.20630   | 3.83034 | 3.37395    | 4.02290 | 3.55003   | 4.22554 | 3.73499    | 4.43873 |
| 0.6  | 2.75740    | 3.88050 | 2.90188   | 4.07520 | 3.05360    | 4.28008 | 3.21297   | 4.49567 | 3.38036    | 4.72249 |
| 0.65 | 2.45113    | 4.08975 | 2.57956   | 4.29494 | 2.71443    | 4.51087 | 2.85610   | 4.73808 | 3.00490    | 4.97713 |
| 0.7  | 2.12975    | 4.27377 | 2.24134   | 4.48820 | 2.35853    | 4.71384 | 2.48162   | 4.95127 | 2.61091    | 5.20109 |
| 0.75 | 1.79523    | 4.43145 | 1.88930   | 4.65378 | 1.98808    | 4.88776 | 2.09184   | 5.13394 | 2.20082    | 5.39298 |
| 0.8  | 1.44965    | 4.56181 | 1.52560   | 4.79058 | 1.60537    | 5.03153 | 1.68916   | 5.28496 | 1.77716    | 5.55162 |
| 0.85 | 1.09513    | 4.66404 | 1.15251   | 4.89805 | 1.21277    | 5.14429 | 1.27607   | 5.40341 | 1.34255    | 5.67603 |
| 0.9  | 0.73386    | 4.73751 | 0.77231   | 4.97521 | 0.81269    | 5.22533 | 0.85511   | 5.48853 | 0.89966    | 5.76545 |
| 0.95 | 0.36807    | 4.78178 | 0.38735   | 5.02169 | 0.40760    | 5.27416 | 0.42888   | 5.53981 | 0.45122    | 5.81933 |
| 1.0  | 0.00       | 4.79657 | 0.00      | 5.03722 | 0.00       | 5.29047 | 0.00      | 5.55695 | 0.00       | 5.83732 |
| 1.05 | 0.36807    | 4.78178 | 0.38735   | 5.02169 | 0.40760    | 5.27416 | 0.42888   | 5.53981 | 0.45122    | 5.81933 |
| 1.1  | 0.73386    | 4.73751 | 0.77231   | 4.97521 | 0.81269    | 5.22533 | 0.85511   | 5.48853 | 0.89966    | 5.76545 |
| 1.15 | 1.09513    | 4.66404 | 1.15251   | 4.89805 | 1.21277    | 5.14429 | 1.27607   | 5.40341 | 1.34255    | 5.67603 |
| 1.2  | 1.44965    | 4.56181 | 1.52560   | 4.79058 | 1.60537    | 5.03153 | 1.68916   | 5.28496 | 1.77716    | 5.55162 |
| 1.25 | 1.79523    | 4.43145 | 1.88930   | 4.65378 | 1.98808    | 4.88776 | 2.09184   | 5.13394 | 2.20082    | 5.39298 |
| 1.3  | 2.12975    | 4.27377 | 2.24134   | 4.48820 | 2.35853    | 4.71384 | 2.48162   | 4.95127 | 2.61091    | 5.20109 |
| 1.35 | 2.45113    | 4.08975 | 2.57956   | 4.29494 | 2.71443    | 4.51087 | 2.85610   | 4.73808 | 3.00490    | 4.97713 |
| 1.4  | 2.75740    | 3.88050 | 2.90188   | 4.07520 | 3.05360    | 4.28008 | 3.21297   | 4.49567 | 3.38036    | 4.72249 |
| 1.45 | 3.04677    | 3.64734 | 3.20630   | 3.83034 | 3.37395    | 4.02290 | 3.55003   | 4.22554 | 3.73499    | 4.43873 |
| 1.5  | 3.31716    | 3.39168 | 3.49096   | 3.56185 | 3.67351    | 3.74093 | 3.86521   | 3.92935 | 4.06659    | 4.12761 |
| 1.55 | 3.56719    | 3.11512 | 3.75410   | 3.27141 | 3.95038    | 3.43588 | 4.15656   | 3.60894 | 4.37311    | 3.79104 |
| 1.6  | 3.79523    | 2.81935 | 3.99409   | 2.96081 | 4.20292    | 3.10966 | 4.42228   | 3.26629 | 4.65268    | 3.43109 |
| 1.65 | 3.99987    | 2.50620 | 4.20946   | 2.63104 | 4.42955    | 2.76426 | 4.66073   | 2.90349 | 4.90356    | 3.04999 |
| 1.7  | 4.17986    | 2.17760 | 4.39887   | 2.28685 | 4.62887    | 2.40182 | 4.87045   | 2.52280 | 5.12420    | 2.65009 |
| 1.75 | 4.33407    | 1.83557 | 4.56116   | 1.92766 | 4.79965    | 2.02457 | 5.05014   | 2.12655 | 5.31325    | 2.23385 |
| 1.8  | 4.46157    | 1.48222 | 4.69533   | 1.55659 | 4.94083    | 1.63485 | 5.19869   | 1.71719 | 5.46955    | 1.80383 |
| 1.85 | 4.56155    | 1.11974 | 4.80056   | 1.17592 | 5.05156    | 1.23504 | 5.31521   | 1.29724 | 5.59213    | 1.36269 |
| 1.9  | 4.63341    | 0.75035 | 4.87618   | 0.78799 | 5.13114    | 0.82761 | 5.39893   | 0.86930 | 5.68022    | 0.91316 |
| 1.95 | 4.67671    | 0.37633 | 4.92174   | 0.39522 | 5.17909    | 0.41509 | 5.44938   | 0.43599 | 5.73330    | 0.45799 |
| 2.0  | 4.69117    | 0.00    | 4.93696   | 0.00    | 5.19510    | 0.00    | 5.46623   | 0.00    | 5.75103    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(2.4 + i\mathbf{0.05}) = 5.44938 + i\mathbf{0.43599}$ .  
 $\sinh(2.4 + i\mathbf{1.95}) = -5.44938 + i\mathbf{0.43599}$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.5$ |         | $x = 2.55$ |         | $x = 2.6$ |         | $x = 2.65$ |         | $x = 2.7$ |         |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 6.05020   | 0.00    | 6.36451    | 0.00    | 6.69473   | 0.00    | 7.04169    | 0.00    | 7.40626   | 0.00    |
| 0.05 | 6.03155   | 0.48113 | 6.34489    | 0.50548 | 6.67409   | 0.53109 | 7.01998    | 0.55803 | 7.38343   | 0.58636 |
| 0.1  | 5.97572   | 0.95930 | 6.28615    | 1.00784 | 6.61231   | 1.05891 | 6.95500    | 1.11262 | 7.31508   | 1.16911 |
| 0.15 | 5.88304   | 1.43155 | 6.18866    | 1.50399 | 6.50976   | 1.58019 | 6.84713    | 1.66034 | 7.20163   | 1.74465 |
| 0.2  | 5.75408   | 1.89498 | 6.05301    | 1.99087 | 6.36706   | 2.09174 | 6.69705    | 2.19784 | 7.04377   | 2.30943 |
| 0.25 | 5.58966   | 2.34673 | 5.88004    | 2.46547 | 6.18512   | 2.59039 | 6.50567    | 2.72178 | 6.84249   | 2.85997 |
| 0.3  | 5.39077   | 2.78401 | 5.67082    | 2.92488 | 5.96505   | 3.07306 | 6.27419    | 3.22894 | 6.59903   | 3.39288 |
| 0.35 | 5.15865   | 3.20411 | 5.42664    | 3.36624 | 5.70819   | 3.53680 | 6.00403    | 3.71619 | 6.31488   | 3.90488 |
| 0.4  | 4.89472   | 3.60448 | 5.14900    | 3.78686 | 5.41615   | 3.97872 | 5.60685    | 4.18053 | 5.99179   | 4.39279 |
| 0.45 | 4.60062   | 3.98260 | 4.83961    | 4.18413 | 5.09071   | 4.39612 | 5.35454    | 4.61910 | 5.63177   | 4.85363 |
| 0.5  | 4.27814   | 4.33619 | 4.50039    | 4.55560 | 4.73389   | 4.78641 | 4.97923    | 5.02919 | 5.23702   | 5.28454 |
| 0.55 | 3.92029   | 4.60304 | 4.13342    | 4.89898 | 4.34788   | 5.14719 | 4.57321    | 5.40827 | 4.80998   | 5.68287 |
| 0.6  | 3.55622   | 4.96113 | 3.74097    | 5.21217 | 3.93506   | 5.47624 | 4.13900    | 5.75401 | 4.35329   | 6.04606 |
| 0.65 | 3.16122   | 5.22864 | 3.32545    | 5.49321 | 3.49799   | 5.77153 | 3.67927    | 6.06427 | 3.86976   | 6.37218 |
| 0.7  | 2.74674   | 5.46392 | 2.88943    | 5.74039 | 3.03934   | 6.03123 | 3.19686    | 6.33714 | 3.36237   | 6.65891 |
| 0.75 | 2.31531   | 5.66550 | 2.43559    | 5.95218 | 2.56196   | 6.25374 | 2.69474    | 6.57095 | 2.83425   | 6.90458 |
| 0.8  | 1.86961   | 5.83215 | 1.96674    | 6.12727 | 2.06878   | 6.43770 | 2.17600    | 6.76424 | 2.28866   | 7.10769 |
| 0.85 | 1.41239   | 5.96287 | 1.48577    | 6.26458 | 1.56285   | 6.58199 | 1.64385    | 6.91583 | 1.72866   | 7.26697 |
| 0.9  | 0.94646   | 6.05680 | 0.99563    | 6.36327 | 1.04729   | 6.68567 | 1.10156    | 7.02478 | 1.15859   | 7.38146 |
| 0.95 | 0.47469   | 6.11339 | 0.49935    | 6.42273 | 0.52526   | 6.74814 | 0.55249    | 7.09042 | 0.58109   | 7.45043 |
| 1.0  | 0.00      | 6.13229 | 0.00       | 6.44259 | 0.00      | 6.76901 | 0.00       | 7.11234 | 0.00      | 7.47347 |
| 1.05 | 0.47469   | 6.11339 | 0.49935    | 6.42273 | 0.52526   | 6.74814 | 0.55249    | 7.09042 | 0.58109   | 7.45043 |
| 1.1  | 0.94646   | 6.05680 | 0.99563    | 6.36327 | 1.04729   | 6.68567 | 1.10156    | 7.02478 | 1.15859   | 7.38146 |
| 1.15 | 1.41239   | 5.96287 | 1.48577    | 6.26458 | 1.56285   | 6.58199 | 1.64385    | 6.91583 | 1.72866   | 7.26697 |
| 1.2  | 1.86961   | 5.83215 | 1.96674    | 6.12727 | 2.06878   | 6.43770 | 2.17600    | 6.76424 | 2.28866   | 7.10769 |
| 1.25 | 2.31531   | 5.66550 | 2.43559    | 5.95218 | 2.56196   | 6.25374 | 2.69474    | 6.57095 | 2.83425   | 6.90458 |
| 1.3  | 2.74674   | 5.46392 | 2.88943    | 5.74039 | 3.03934   | 6.03123 | 3.19686    | 6.33714 | 3.36237   | 6.65891 |
| 1.35 | 3.16122   | 5.22864 | 3.32545    | 5.49321 | 3.49799   | 5.77153 | 3.67927    | 6.06427 | 3.86976   | 6.37218 |
| 1.4  | 3.55622   | 4.96113 | 3.74097    | 5.21217 | 3.93506   | 5.47624 | 4.13900    | 5.75401 | 4.35329   | 6.04606 |
| 1.45 | 3.92929   | 4.66304 | 4.13342    | 4.89898 | 4.34788   | 5.14719 | 4.57321    | 5.40827 | 4.80998   | 5.68287 |
| 1.5  | 4.27814   | 4.33619 | 4.50039    | 4.55560 | 4.73389   | 4.78641 | 4.97923    | 5.02919 | 5.23702   | 5.28454 |
| 1.55 | 4.60062   | 3.98260 | 4.83961    | 4.18413 | 5.09071   | 4.39612 | 5.35454    | 4.61910 | 5.63177   | 4.85363 |
| 1.6  | 4.89472   | 3.60448 | 5.14900    | 3.78686 | 5.41615   | 3.97872 | 5.69685    | 4.18053 | 5.99179   | 4.39279 |
| 1.65 | 5.15865   | 3.20411 | 5.42664    | 3.36624 | 5.70819   | 3.53680 | 6.00403    | 3.71619 | 6.31488   | 3.90488 |
| 1.7  | 5.39077   | 2.78401 | 5.67082    | 2.92488 | 5.96505   | 3.07306 | 6.27419    | 3.22894 | 6.59903   | 3.39288 |
| 1.75 | 5.58966   | 2.34673 | 5.88004    | 2.46547 | 6.18512   | 2.59039 | 6.50567    | 2.72178 | 6.84249   | 2.85997 |
| 1.8  | 5.75408   | 1.89498 | 6.05301    | 1.99087 | 6.36706   | 2.09174 | 6.69705    | 2.19784 | 7.04377   | 2.30943 |
| 1.85 | 5.88304   | 1.43155 | 6.18866    | 1.50399 | 6.50976   | 1.58019 | 6.84713    | 1.66034 | 7.20163   | 1.74465 |
| 1.9  | 5.97572   | 0.95930 | 6.28615    | 1.00784 | 6.61231   | 1.05891 | 6.95500    | 1.11262 | 7.31508   | 1.16911 |
| 1.95 | 6.03155   | 0.48113 | 6.34489    | 0.50548 | 6.67409   | 0.53109 | 7.01998    | 0.55803 | 7.38343   | 0.58636 |
| 2.0  | 6.05020   | 0.00    | 6.36451    | 0.00    | 6.69473   | 0.00    | 7.04169    | 0.00    | 7.40626   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(2.7 + i 0.7) = 3.36237 + i 6.65891$ .  
 $\sinh(2.5 + i 1.25) = -2.31531 + i 5.66550$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.75$ |         | $x = 2.8$ |         | $x = 2.85$ |         | $x = 2.9$ |         | $x = 2.95$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 7.78935    | 0.00    | 8.19192   | 0.00    | 8.61497    | 0.00    | 9.05956   | 0.00    | 9.52681    | 0.00    |
| 0.05 | 7.76534    | 0.61616 | 8.16666   | 0.64750 | 8.58841    | 0.68046 | 9.03163   | 0.71512 | 9.49744    | 0.75157 |
| 0.1  | 7.69345    | 1.22852 | 8.09106   | 1.29101 | 8.50891    | 1.35673 | 8.94802   | 1.42583 | 9.40952    | 1.49851 |
| 0.15 | 7.57413    | 1.83331 | 7.96557   | 1.92656 | 8.37694    | 2.02463 | 8.80924   | 2.12776 | 9.26358    | 2.23621 |
| 0.2  | 7.40811    | 2.42680 | 7.79098   | 2.55023 | 8.19332    | 2.68005 | 8.61616   | 2.81656 | 9.06053    | 2.96012 |
| 0.25 | 7.19642    | 3.00532 | 7.56834   | 3.15818 | 7.95919    | 3.31894 | 8.36994   | 3.48800 | 8.80162    | 3.66578 |
| 0.3  | 6.94036    | 3.56531 | 7.29905   | 3.74666 | 7.67599    | 3.93738 | 8.07213   | 4.13793 | 8.48845    | 4.34884 |
| 0.35 | 6.64151    | 4.10333 | 6.98476   | 4.31204 | 7.34547    | 4.53153 | 7.72455   | 4.76236 | 8.12294    | 5.00509 |
| 0.4  | 6.30172    | 4.61604 | 6.62740   | 4.85083 | 6.96966    | 5.09775 | 7.32934   | 5.35742 | 7.79735    | 5.63049 |
| 0.45 | 5.92307    | 5.10030 | 6.22918   | 5.35972 | 6.55088    | 5.63254 | 6.88894   | 6.91945 | 7.24424    | 6.22116 |
| 0.5  | 5.50790    | 5.55311 | 5.79256   | 5.83556 | 6.09170    | 6.13261 | 6.40608   | 6.44498 | 6.73647    | 6.77348 |
| 0.55 | 5.05878    | 5.97168 | 5.32022   | 6.27542 | 5.59498    | 5.9486  | 5.88371   | 6.93078 | 6.18717    | 7.28404 |
| 0.6  | 4.57847    | 6.35344 | 4.81509   | 6.67660 | 5.06375    | 7.01646 | 5.32508   | 7.37385 | 5.63048    | 7.74969 |
| 0.65 | 4.06993    | 6.69602 | 4.28026   | 7.03661 | 4.50131    | 7.39479 | 4.73361   | 7.77146 | 4.97774    | 8.16757 |
| 0.7  | 3.53629    | 6.99732 | 3.71905   | 7.35323 | 3.91112    | 7.72754 | 4.11295   | 8.12115 | 4.32508    | 8.53508 |
| 0.75 | 2.98086    | 7.25548 | 3.13491   | 7.62452 | 3.29681    | 8.01263 | 3.46694   | 8.42078 | 3.64575    | 8.84998 |
| 0.8  | 2.40704    | 7.46891 | 2.53144   | 7.84881 | 2.66217    | 8.24834 | 2.79956   | 8.66850 | 2.94395    | 9.11031 |
| 0.85 | 1.81839    | 7.63629 | 1.91237   | 8.02470 | 2.01112    | 8.43318 | 2.11491   | 8.86275 | 2.22399    | 9.31447 |
| 0.9  | 1.21852    | 7.75659 | 1.28150   | 8.15112 | 1.34768    | 8.56604 | 1.41723   | 9.00237 | 1.49032    | 9.46121 |
| 0.95 | 0.61115    | 7.82907 | 0.64273   | 8.22728 | 0.67592    | 8.64608 | 0.71081   | 9.08649 | 0.74747    | 9.54962 |
| 1.0  | 0.00       | 7.85328 | 0.00      | 8.25273 | 0.00       | 8.67281 | 0.00      | 9.11458 | 0.00       | 9.57915 |
| 1.05 | 0.61115    | 7.82907 | 0.64273   | 8.22728 | 0.67592    | 8.64608 | 0.71081   | 9.08649 | 0.74747    | 9.54962 |
| 1.1  | 1.21852    | 7.75659 | 1.28150   | 8.15112 | 1.34768    | 8.56604 | 1.41723   | 9.00237 | 1.49032    | 9.46121 |
| 1.15 | 1.81839    | 7.63629 | 1.91237   | 8.02470 | 2.01112    | 8.43318 | 2.11491   | 8.86275 | 2.22399    | 9.31447 |
| 1.2  | 2.40704    | 7.46891 | 2.53144   | 7.84881 | 2.66217    | 8.24834 | 2.79956   | 8.66850 | 2.94395    | 9.11031 |
| 1.25 | 2.98086    | 7.25548 | 3.13491   | 7.62452 | 3.29681    | 8.01263 | 3.46694   | 8.42078 | 3.64575    | 8.84998 |
| 1.3  | 3.53629    | 6.99732 | 3.71905   | 7.35323 | 3.91112    | 7.72754 | 4.11295   | 8.12115 | 4.32508    | 8.53508 |
| 1.35 | 4.06993    | 6.69602 | 4.28026   | 7.03661 | 4.50131    | 7.39479 | 4.73361   | 7.77146 | 4.97774    | 8.16757 |
| 1.4  | 4.57847    | 6.35344 | 4.81509   | 6.67660 | 5.06375    | 7.01646 | 5.32508   | 7.37385 | 5.63048    | 7.74969 |
| 1.45 | 5.05878    | 5.97168 | 5.32022   | 6.27542 | 5.59498    | 6.59486 | 5.88371   | 6.93078 | 6.18717    | 7.28404 |
| 1.5  | 5.50790    | 5.55311 | 5.79256   | 5.83556 | 6.09170    | 6.13261 | 6.40608   | 6.44498 | 6.73647    | 6.77348 |
| 1.55 | 5.92307    | 5.10030 | 6.22918   | 5.35972 | 6.55088    | 5.63254 | 6.88894   | 5.91945 | 7.24424    | 6.22116 |
| 1.6  | 6.30172    | 4.61604 | 6.62740   | 4.85083 | 6.96966    | 5.09775 | 7.32934   | 5.35742 | 7.70735    | 5.63049 |
| 1.65 | 6.64151    | 4.10333 | 6.98476   | 4.31204 | 7.34547    | 4.53153 | 7.72455   | 4.76236 | 8.12294    | 5.00509 |
| 1.7  | 6.94036    | 3.56531 | 7.29905   | 3.74666 | 7.67599    | 3.93738 | 8.07213   | 4.13793 | 8.48845    | 4.34884 |
| 1.75 | 7.19642    | 3.00532 | 7.56834   | 3.15818 | 7.95919    | 3.31894 | 8.36994   | 3.48800 | 8.80162    | 3.66578 |
| 1.8  | 7.40811    | 2.42680 | 7.79098   | 2.55023 | 8.19332    | 2.68005 | 8.61616   | 2.81656 | 9.06053    | 2.96012 |
| 1.85 | 7.57413    | 1.83331 | 7.96557   | 1.92656 | 8.37694    | 2.02463 | 8.80924   | 2.12776 | 9.26358    | 2.23621 |
| 1.9  | 7.69345    | 1.22852 | 8.09106   | 1.29101 | 8.50891    | 1.35673 | 8.94802   | 1.42583 | 9.40952    | 1.49851 |
| 1.95 | 7.76534    | 0.61616 | 8.16666   | 0.64750 | 8.58841    | 0.68046 | 9.03163   | 0.71512 | 9.49744    | 0.75157 |
| 2.0  | 7.78935    | 0.00    | 8.19192   | 0.00    | 8.61497    | 0.00    | 9.05956   | 0.00    | 9.52681    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(2.9 + i0.9) = 1.41723 + i9.00237$ .  
 $\sinh(2.8 + i1.4) = -4.81509 + i6.67660$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| q    | x = 3.0  |          | x = 3.05 |          | x = 3.10 |          | x = 3.15 |          | x = 3.20 |          |
|------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.0  | 10.01787 | 0.00     | 10.53399 | 0.00     | 11.07645 | 0.00     | 11.64661 | 0.00     | 12.24588 | 0.00     |
| 0.05 | 9.98699  | 0.78990  | 10.50150 | 0.83020  | 11.04230 | 0.87258  | 11.61070 | 0.91714  | 12.20810 | 0.96399  |
| 0.1  | 9.89454  | 1.57493  | 10.40430 | 1.65529  | 10.94010 | 1.73979  | 11.50320 | 1.82863  | 12.09510 | 1.92205  |
| 0.15 | 9.74108  | 2.35025  | 10.24290 | 2.47017  | 10.77040 | 2.59626  | 11.32480 | 2.72885  | 11.90750 | 2.86826  |
| 0.2  | 9.52757  | 3.11108  | 10.01840 | 3.26982  | 10.53430 | 3.43673  | 11.07660 | 3.61224  | 11.64650 | 3.79678  |
| 0.25 | 9.25531  | 3.85273  | 9.73216  | 4.04931  | 10.23330 | 4.25602  | 10.76010 | 4.47336  | 11.31370 | 4.70190  |
| 0.3  | 8.92599  | 4.57062  | 9.38586  | 4.80383  | 9.86919  | 5.04906  | 10.37720 | 5.30690  | 10.91120 | 5.57802  |
| 0.35 | 8.54164  | 5.26034  | 8.98171  | 5.52874  | 9.44423  | 5.81097  | 9.93036  | 6.10773  | 10.44130 | 6.41976  |
| 0.4  | 8.10463  | 5.91762  | 8.52218  | 6.21956  | 8.96104  | 6.53795  | 9.42230  | 6.87089  | 9.90712  | 7.22191  |
| 0.45 | 7.61765  | 6.53842  | 8.01011  | 6.87204  | 8.42260  | 7.22284  | 8.85615  | 7.59170  | 9.31184  | 7.97954  |
| 0.5  | 7.08371  | 7.11891  | 7.44866  | 7.48215  | 7.83223  | 7.86409  | 8.23539  | 8.26570  | 8.65915  | 8.68797  |
| 0.55 | 6.50609  | 7.65551  | 6.84128  | 8.04612  | 7.19358  | 8.45686  | 7.56387  | 8.88873  | 7.95306  | 9.34284  |
| 0.6  | 5.88836  | 8.14491  | 6.19173  | 8.56049  | 6.51058  | 8.99749  | 6.84570  | 9.45697  | 7.19795  | 9.94011  |
| 0.65 | 5.23433  | 8.58409  | 5.50400  | 9.02209  | 5.78743  | 9.48264  | 6.08533  | 9.96690  | 6.39846  | 10.47610 |
| 0.7  | 4.54806  | 8.97035  | 4.78233  | 9.42805  | 5.02860  | 9.90933  | 5.28745  | 10.41540 | 5.55952  | 10.94750 |
| 0.75 | 3.83368  | 9.30131  | 4.03119  | 9.77589  | 4.23878  | 10.27490 | 4.45696  | 10.79970 | 4.68630  | 11.35140 |
| 0.8  | 3.09569  | 9.57492  | 3.25518  | 10.06350 | 3.42281  | 10.57720 | 3.59900  | 11.11730 | 3.78419  | 11.68530 |
| 0.85 | 2.33863  | 9.78949  | 2.45911  | 10.28900 | 2.58575  | 10.81420 | 2.71885  | 11.36650 | 2.85874  | 11.94720 |
| 0.9  | 1.56714  | 9.94371  | 1.64788  | 10.45110 | 1.73274  | 10.98460 | 1.82193  | 11.54550 | 1.91568  | 12.13540 |
| 0.95 | 0.78599  | 10.03660 | 0.82649  | 10.54870 | 0.86905  | 11.08720 | 0.91378  | 11.65340 | 0.96080  | 12.24880 |
| 1.0  | 0.00     | 10.06766 | 0.00     | 10.58135 | 0.00     | 11.12150 | 0.00     | 11.68946 | 0.00     | 12.28665 |
| 1.05 | 0.78599  | 10.03660 | 0.82649  | 10.54870 | 0.86905  | 11.08720 | 0.91378  | 11.65340 | 0.96080  | 12.24880 |
| 1.1  | 1.56714  | 9.94371  | 1.64788  | 10.45110 | 1.73274  | 10.98460 | 1.82193  | 11.54550 | 1.91568  | 12.13540 |
| 1.15 | 2.33863  | 9.78949  | 2.45911  | 10.28900 | 2.58575  | 10.81420 | 2.71885  | 11.36650 | 2.85874  | 11.94720 |
| 1.2  | 3.09569  | 9.57492  | 3.25518  | 10.06350 | 3.42281  | 10.57720 | 3.59900  | 11.11730 | 3.78419  | 11.68530 |
| 1.25 | 3.83368  | 9.30131  | 4.03119  | 9.77589  | 4.23878  | 10.27490 | 4.45696  | 10.79970 | 4.68630  | 11.35140 |
| 1.3  | 4.54806  | 8.97035  | 4.78233  | 9.42805  | 5.02860  | 9.90933  | 5.28745  | 10.41540 | 5.55952  | 10.94750 |
| 1.35 | 5.23433  | 8.58409  | 5.50400  | 9.02209  | 5.78743  | 9.48264  | 6.08533  | 9.96690  | 6.39846  | 10.47610 |
| 1.4  | 5.88836  | 8.14491  | 6.19173  | 8.56049  | 6.51058  | 8.99749  | 6.84570  | 9.45697  | 7.19795  | 9.94011  |
| 1.45 | 6.50609  | 7.65551  | 6.84128  | 8.04612  | 7.19358  | 8.45686  | 7.56387  | 8.88873  | 7.95306  | 9.34284  |
| 1.5  | 7.08371  | 7.11891  | 7.44866  | 7.48215  | 7.83223  | 7.86409  | 8.23539  | 8.26570  | 8.65915  | 8.68797  |
| 1.55 | 7.61765  | 6.53842  | 8.01011  | 6.87204  | 8.42260  | 7.22284  | 8.85615  | 7.59170  | 9.31184  | 7.97954  |
| 1.6  | 8.10463  | 5.91762  | 8.52218  | 6.21956  | 8.96104  | 6.53795  | 9.42230  | 6.87089  | 9.90712  | 7.22191  |
| 1.65 | 8.54164  | 5.26034  | 8.98171  | 5.52874  | 9.44423  | 5.81097  | 9.93036  | 6.10773  | 10.44130 | 6.41976  |
| 1.7  | 8.92599  | 4.57062  | 9.38586  | 4.80383  | 9.86919  | 5.04906  | 10.37720 | 5.30690  | 10.91120 | 5.57802  |
| 1.75 | 9.25531  | 3.85273  | 9.73216  | 4.04931  | 10.23330 | 4.25602  | 10.76010 | 4.47336  | 11.31370 | 4.70190  |
| 1.8  | 9.52757  | 3.11108  | 10.01840 | 3.26982  | 10.53430 | 3.43673  | 11.07660 | 3.61224  | 11.64650 | 3.79678  |
| 1.85 | 9.74108  | 2.35025  | 10.24290 | 2.47017  | 10.77040 | 2.59626  | 11.32480 | 2.72885  | 11.90750 | 2.86826  |
| 1.9  | 9.89454  | 1.57493  | 10.40430 | 1.65529  | 10.94010 | 1.73979  | 11.50320 | 1.82863  | 12.09510 | 1.92205  |
| 1.95 | 9.98699  | 0.78990  | 10.50150 | 0.83020  | 11.04230 | 0.87258  | 11.61070 | 0.91714  | 12.20810 | 0.96399  |
| 2.0  | 10.01787 | 0.00     | 10.53399 | 0.00     | 11.07645 | 0.00     | 11.64661 | 0.00     | 12.24588 | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(3.0 + i \underline{0.95}) = 0.78599 + i \underline{10.03660}$ .  
 $\sinh(3.0 + i \underline{1.05}) = -0.78599 + i \underline{10.03660}$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.25$ |          | $x = 3.30$ |          | $x = 3.35$ |          | $x = 3.40$ |          | $x = 3.45$ |          |
|------|------------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| 0.0  | 12.87578   | 0.00     | 13.53788   | 0.00     | 14.23382   | 0.00     | 14.96536   | 0.00     | 15.73432   | 0.00     |
| 0.05 | 12.83610   | 1.01326  | 13.49615   | 1.06507  | 14.18950   | 1.11953  | 14.91923   | 1.17679  | 15.68581   | 1.23699  |
| 0.1  | 12.71726   | 2.02028  | 13.37120   | 2.12356  | 14.05851   | 2.23215  | 14.78111   | 2.34032  | 15.54061   | 2.46036  |
| 0.15 | 12.52002   | 3.01484  | 13.16388   | 3.16897  | 13.84054   | 3.33101  | 14.55188   | 3.50139  | 15.29959   | 3.68053  |
| 0.2  | 12.24560   | 3.99082  | 12.87530   | 4.19483  | 13.53717   | 4.40933  | 14.23291   | 4.63487  | 14.96423   | 4.87198  |
| 0.25 | 11.89566   | 4.94218  | 12.50736   | 5.19485  | 13.15033   | 5.46075  | 13.82620   | 5.73977  | 14.53662   | 6.03341  |
| 0.3  | 11.47240   | 5.86309  | 12.06234   | 6.16381  | 12.68244   | 6.47795  | 13.33423   | 6.80928  | 14.01940   | 7.15765  |
| 0.35 | 10.97841   | 6.74784  | 11.54294   | 7.09279  | 12.13633   | 7.45549  | 12.76006   | 7.83682  | 13.41572   | 8.23775  |
| 0.4  | 10.41673   | 7.59099  | 10.95235   | 7.97905  | 11.51541   | 8.38705  | 12.10723   | 8.81604  | 12.72933   | 9.26706  |
| 0.45 | 9.79082    | 8.38733  | 10.29428   | 8.81610  | 10.82349   | 9.26691  | 11.37975   | 9.74090  | 11.96447   | 10.23924 |
| 0.5  | 9.10455    | 9.13197  | 9.57273    | 9.59881  | 10.06483   | 10.08964 | 10.58212   | 10.60570 | 11.12585   | 11.14830 |
| 0.55 | 8.36217    | 9.82031  | 8.79215    | 10.32233 | 9.24413    | 10.85016 | 9.71923    | 11.40512 | 10.21863   | 11.98861 |
| 0.6  | 7.56820    | 10.44810 | 7.95737    | 10.98222 | 8.36643    | 11.54379 | 8.79642    | 12.13423 | 9.24840    | 12.75501 |
| 0.65 | 6.72758    | 11.01147 | 7.07352    | 11.57440 | 7.43715    | 12.16623 | 7.81938    | 12.78852 | 8.22116    | 13.44278 |
| 0.7  | 5.84548    | 11.50695 | 6.14670    | 12.09520 | 6.46202    | 12.71369 | 6.79414    | 13.36397 | 7.14324    | 14.04764 |
| 0.75 | 4.92735    | 11.93150 | 5.18072    | 12.54145 | 5.44705    | 13.18275 | 5.72700    | 13.85702 | 6.02127    | 14.56595 |
| 0.8  | 3.97883    | 12.28247 | 4.18343    | 12.91035 | 4.39850    | 13.57054 | 4.62455    | 14.26465 | 4.86217    | 14.99442 |
| 0.85 | 3.00579    | 12.55773 | 3.16036    | 13.19970 | 3.32282    | 13.87465 | 3.49359    | 14.58432 | 3.67311    | 15.33045 |
| 0.9  | 2.01422    | 12.75556 | 2.11780    | 13.40764 | 2.22666    | 14.09323 | 2.34110    | 14.81410 | 2.46139    | 15.57196 |
| 0.95 | 1.01022    | 12.87474 | 1.06217    | 13.53290 | 1.11677    | 14.22498 | 1.17417    | 14.95250 | 1.23450    | 15.71746 |
| 1.0  | 0.00       | 12.91456 | 0.00       | 13.57476 | 0.00       | 14.26891 | 0.00       | 14.99874 | 0.00       | 15.76607 |
| 1.05 | 1.01022    | 12.87474 | 1.06217    | 13.53290 | 1.11677    | 14.22498 | 1.17417    | 14.95250 | 1.23450    | 15.71746 |
| 1.1  | 2.01422    | 12.75556 | 2.11780    | 13.40764 | 2.22666    | 14.09323 | 2.34110    | 14.81410 | 2.46139    | 15.57196 |
| 1.15 | 3.00579    | 12.55773 | 3.16036    | 13.19970 | 3.32282    | 13.87465 | 3.49359    | 14.58432 | 3.67311    | 15.33045 |
| 1.2  | 3.97883    | 12.28247 | 4.18343    | 12.91035 | 4.39850    | 13.57054 | 4.62455    | 14.26465 | 4.86217    | 14.99442 |
| 1.25 | 4.92735    | 11.93150 | 5.18072    | 12.54145 | 5.44705    | 13.18275 | 5.72700    | 13.85702 | 6.02127    | 14.56595 |
| 1.3  | 5.84548    | 11.50695 | 6.14670    | 12.09520 | 6.46202    | 12.71369 | 6.79414    | 13.36397 | 7.14324    | 14.04764 |
| 1.35 | 6.72758    | 11.01147 | 7.07352    | 11.57440 | 7.43715    | 12.16623 | 7.81938    | 12.78852 | 8.22116    | 13.44278 |
| 1.4  | 7.56820    | 10.44810 | 7.95737    | 10.98222 | 8.36643    | 11.54379 | 8.79642    | 12.13423 | 9.24840    | 12.75501 |
| 1.45 | 8.36217    | 9.82031  | 8.79215    | 10.32233 | 9.24413    | 10.85016 | 9.71923    | 11.40512 | 10.21863   | 11.98861 |
| 1.5  | 9.10455    | 9.13197  | 9.57273    | 9.59881  | 10.06483   | 10.08964 | 10.58212   | 10.60570 | 11.12585   | 11.14830 |
| 1.55 | 9.79082    | 8.38733  | 10.29428   | 8.81610  | 10.82349   | 9.26691  | 11.37975   | 9.74090  | 11.96447   | 10.23924 |
| 1.6  | 10.41673   | 7.59099  | 10.95235   | 7.97905  | 11.51541   | 8.38705  | 12.10723   | 8.81604  | 12.72933   | 9.26706  |
| 1.65 | 10.97841   | 6.74784  | 11.54294   | 7.09279  | 12.13633   | 7.45549  | 12.76006   | 7.83682  | 13.41572   | 8.23775  |
| 1.7  | 11.47240   | 5.86309  | 12.06234   | 6.16381  | 12.68244   | 6.47795  | 13.33423   | 6.80928  | 14.01940   | 7.15765  |
| 1.75 | 11.89566   | 4.94218  | 12.50736   | 5.19485  | 13.15033   | 5.46075  | 13.82620   | 5.73977  | 14.53662   | 6.03341  |
| 1.8  | 12.24560   | 3.99082  | 12.87530   | 4.19483  | 13.53717   | 4.40933  | 14.23291   | 4.63487  | 14.96423   | 4.87198  |
| 1.85 | 12.52002   | 3.01484  | 13.16388   | 3.16897  | 13.84054   | 3.33101  | 14.55188   | 3.50139  | 15.29959   | 3.68053  |
| 1.9  | 12.71726   | 2.02028  | 13.37120   | 2.12356  | 14.05851   | 2.23215  | 14.78111   | 2.34032  | 15.54061   | 2.46036  |
| 1.95 | 12.83610   | 1.01326  | 13.49615   | 1.06507  | 14.18950   | 1.11953  | 14.91923   | 1.17679  | 15.68581   | 1.23699  |
| 2.0  | 12.87578   | 0.00     | 13.53788   | 0.00     | 14.23382   | 0.00     | 14.96536   | 0.00     | 15.73432   | 0.00     |

Note. Negative questions are in heavy type.

Examples.  $\sinh(3.40 + i0) = 14.96536 + i0$ .

$\sinh(3.45 + i1.45) = -10.21863 + i11.98861$ .

$$z = \frac{m}{\pi/2} - n4 - m2$$

$$m, n = 0, 1, 2, \dots$$

[55]

$$1 \leq 2 \leq \frac{m}{\pi/2} - n4 \leq 4$$

multiply  $(\sinh[x + i8])$  by  
-1 i.e.  $(-1)(u + iv)$

select  $n$  so that  
 $0 \leq \frac{m}{\pi/2} - n4 \leq 4$   
select  $m$  so that

$$\leq \left( \frac{m}{\pi/2} - n4 \right) - m2 \leq 0$$

ATHERFORCE



TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.50$ |          | $x = 3.55$ |          | $x = 3.60$ |          | $x = 3.65$ |          | $x = 3.70$ |          |
|------|------------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| 0.0  | 16.54263   | 0.00     | 17.39230   | 0.00     | 18.28546   | 0.00     | 19.22434   | 0.00     | 20.21129   | 0.00     |
| 0.05 | 16.49163   | 1.30029  | 17.33870   | 1.36684  | 18.22900   | 1.43680  | 19.16506   | 1.51036  | 20.14900   | 1.58770  |
| 0.1  | 16.33896   | 2.59256  | 17.17817   | 2.72525  | 18.06033   | 2.86475  | 18.98765   | 3.01142  | 19.96246   | 3.16561  |
| 0.15 | 16.08555   | 3.86885  | 16.91175   | 4.06686  | 17.78022   | 4.27503  | 18.69316   | 4.49390  | 19.65290   | 4.72401  |
| 0.2  | 15.73300   | 5.12129  | 16.54105   | 5.38339  | 17.39050   | 5.65896  | 18.28342   | 5.94868  | 19.22208   | 6.25327  |
| 0.25 | 15.28340   | 6.34215  | 16.06840   | 6.66674  | 16.89356   | 7.00800  | 17.76096   | 7.36678  | 18.67280   | 7.74399  |
| 0.3  | 14.73960   | 7.52391  | 15.49665   | 7.90898  | 16.29246   | 8.31383  | 17.12900   | 8.73946  | 18.00840   | 9.18696  |
| 0.35 | 14.10490   | 8.65928  | 14.82033   | 9.10246  | 15.59090   | 9.56840  | 16.39143   | 10.05827 | 17.23295   | 10.57330 |
| 0.4  | 13.38330   | 9.74126  | 14.07066   | 10.23982 | 14.79326   | 10.76400 | 15.55281   | 11.31505 | 16.35127   | 11.89444 |
| 0.45 | 12.57910   | 10.76320 | 13.22520   | 11.31405 | 13.90436   | 11.89320 | 14.61830   | 12.50208 | 15.36878   | 13.14223 |
| 0.5  | 11.69740   | 11.71820 | 12.29820   | 12.31852 | 12.92978   | 12.94910 | 13.59365   | 13.61203 | 14.29155   | 14.30902 |
| 0.55 | 10.74357   | 12.60210 | 11.29540   | 13.24705 | 11.87545   | 13.92515 | 12.48520   | 14.63806 | 13.12620   | 15.38760 |
| 0.6  | 9.72361    | 13.40770 | 10.22293   | 14.09390 | 10.74789   | 14.81535 | 11.29972   | 15.57383 | 11.87990   | 16.37127 |
| 0.65 | 8.64360    | 14.13065 | 9.08745    | 14.85386 | 9.55412    | 15.61421 | 10.04469   | 16.41360 | 10.56037   | 17.25404 |
| 0.7  | 7.51020    | 14.76650 | 7.89594    | 15.52225 | 8.30143    | 16.31680 | 8.72766    | 17.15216 | 9.17573    | 18.03040 |
| 0.75 | 6.33059    | 15.31130 | 6.65574    | 16.09492 | 6.99754    | 16.91880 | 7.35683    | 17.78497 | 7.73453    | 18.69565 |
| 0.8  | 5.11195    | 15.76170 | 5.37451    | 16.56840 | 5.65052    | 17.41650 | 5.94065    | 18.30814 | 6.24563    | 19.24560 |
| 0.85 | 3.86180    | 16.11491 | 4.06015    | 16.93970 | 4.26865    | 17.80680 | 4.48783    | 18.71843 | 4.71823    | 19.67600 |
| 0.9  | 2.58783    | 16.36878 | 2.72075    | 17.20653 | 2.86047    | 18.08732 | 3.00735    | 19.01331 | 3.16174    | 19.98688 |
| 0.95 | 1.29792    | 16.52173 | 1.36458    | 17.36731 | 1.43466    | 18.25632 | 1.50832    | 19.19100 | 1.58576    | 20.17362 |
| 1.0  | 0.00       | 16.57282 | 0.00       | 17.42102 | 0.00       | 18.31278 | 0.00       | 19.25033 | 0.00       | 20.23601 |
| 1.05 | 1.29792    | 16.52173 | 1.36458    | 17.36731 | 1.43466    | 18.25632 | 1.50832    | 19.19100 | 1.58576    | 20.17362 |
| 1.1  | 2.58783    | 16.36878 | 2.72075    | 17.20653 | 2.86047    | 18.08732 | 3.00735    | 19.01331 | 3.16174    | 19.98688 |
| 1.15 | 3.86180    | 16.11491 | 4.06015    | 16.93970 | 4.26865    | 17.80680 | 4.48783    | 18.71843 | 4.71823    | 19.67600 |
| 1.2  | 5.11195    | 15.76170 | 5.37451    | 16.56840 | 5.65052    | 17.41650 | 5.94065    | 18.30814 | 6.24563    | 19.24560 |
| 1.25 | 6.33059    | 15.31130 | 6.65574    | 16.09492 | 6.99754    | 16.91880 | 7.35683    | 17.78497 | 7.73453    | 18.69565 |
| 1.3  | 7.51020    | 14.76650 | 7.89594    | 15.52225 | 8.30143    | 16.31680 | 8.72766    | 17.15216 | 9.17573    | 18.03040 |
| 1.35 | 8.64360    | 14.13065 | 9.08745    | 14.85386 | 9.55412    | 15.61421 | 10.04469   | 16.41360 | 10.56037   | 17.25404 |
| 1.4  | 9.72361    | 13.40770 | 10.22293   | 14.09390 | 10.74789   | 14.81535 | 11.29972   | 15.57383 | 11.87990   | 16.37127 |
| 1.45 | 10.74357   | 12.60210 | 11.29540   | 13.24705 | 11.87545   | 13.92515 | 12.48520   | 14.63806 | 13.12620   | 15.38760 |
| 1.5  | 11.69740   | 11.71820 | 12.29820   | 12.31852 | 12.92978   | 12.94910 | 13.59365   | 13.61203 | 14.29155   | 14.30902 |
| 1.55 | 12.57910   | 10.76320 | 13.22520   | 11.31405 | 13.90436   | 11.89320 | 14.61830   | 12.50208 | 15.36878   | 13.14223 |
| 1.6  | 13.38330   | 9.74126  | 14.07066   | 10.23982 | 14.79326   | 10.76400 | 15.55281   | 11.31505 | 16.35127   | 11.89444 |
| 1.65 | 14.10490   | 8.65928  | 14.82933   | 9.10246  | 15.59090   | 9.56840  | 16.39143   | 10.05827 | 17.23295   | 10.57330 |
| 1.7  | 14.73960   | 7.52391  | 15.49665   | 7.90898  | 16.29246   | 8.31383  | 17.12900   | 8.73946  | 18.00840   | 9.18696  |
| 1.75 | 15.28340   | 6.34215  | 16.06840   | 6.66674  | 16.89356   | 7.00800  | 17.76096   | 7.36678  | 18.67280   | 7.74399  |
| 1.8  | 15.73300   | 5.12129  | 16.54105   | 5.38339  | 17.39050   | 5.65896  | 18.28342   | 5.94868  | 19.22208   | 6.25327  |
| 1.85 | 16.08555   | 3.86885  | 16.91175   | 4.06686  | 17.78022   | 4.27503  | 18.69316   | 4.49390  | 19.65290   | 4.72401  |
| 1.9  | 16.33896   | 2.59256  | 17.17817   | 2.72525  | 18.06033   | 2.86475  | 18.98765   | 3.01142  | 19.96246   | 3.16561  |
| 1.95 | 16.49163   | 1.30029  | 17.33870   | 1.36684  | 18.22900   | 1.43680  | 19.16506   | 1.51036  | 20.14900   | 1.58770  |
| 2.0  | 16.54263   | 0.00     | 17.39230   | 0.00     | 18.28546   | 0.00     | 19.22434   | 0.00     | 20.21129   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(3.70 + i0.5) = 14.29155 + i14.30902$ .  
 $\sinh(3.70 + i1.5) = -14.29155 + i14.30902$ .

TABLE VII. HYPERBOLIC SINES.  $\sinh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.75$ |          | $x = 3.80$ |          | $x = 3.85$ |          | $x = 3.90$ |          | $x = 3.95$ |          |
|------|------------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| 0.0  | 21.24878   | 0.00     | 22.33941   | 0.00     | 23.48589   | 0.00     | 24.69110   | 0.00     | 25.95806   | 0.00     |
| 0.05 | 21.18327   | 1.66900  | 22.27052   | 1.75448  | 23.41348   | 1.84435  | 24.61500   | 1.93883  | 25.87805   | 2.03816  |
| 0.1  | 20.98716   | 3.32772  | 22.06437   | 3.49815  | 23.19673   | 3.67733  | 24.38710   | 3.86571  | 25.63849   | 4.06375  |
| 0.15 | 20.66167   | 4.96592  | 21.72216   | 5.22025  | 22.83696   | 5.48764  | 24.00888   | 5.76875  | 25.24084   | 6.06429  |
| 0.2  | 20.20879   | 6.57350  | 21.24603   | 6.91017  | 22.33640   | 7.26411  | 23.48262   | 7.63623  | 24.68760   | 8.02744  |
| 0.25 | 19.63131   | 8.14055  | 20.63891   | 8.55748  | 21.69813   | 8.99580  | 22.81160   | 9.45662  | 23.98212   | 9.94109  |
| 0.3  | 18.93280   | 9.65741  | 19.90455   | 10.15203 | 20.92608   | 10.67203 | 21.99993   | 11.21871 | 23.12881   | 11.79366 |
| 0.35 | 18.11756   | 11.11473 | 19.04746   | 11.68400 | 20.02501   | 12.28246 | 21.05272   | 12.91164 | 22.13290   | 13.57315 |
| 0.4  | 17.19062   | 12.50353 | 18.07296   | 13.14392 | 19.00048   | 13.81716 | 19.97556   | 14.52497 | 21.00052   | 15.26910 |
| 0.45 | 16.15770   | 13.81524 | 16.98701   | 14.52281 | 17.85880   | 15.26668 | 18.77526   | 16.04874 | 19.73867   | 16.87098 |
| 0.5  | 15.02516   | 15.04177 | 15.79634   | 15.81216 | 16.60702   | 16.62208 | 17.45924   | 17.47355 | 18.35512   | 18.36873 |
| 0.55 | 13.79098   | 16.17556 | 14.50828   | 17.00402 | 15.25286   | 17.87500 | 16.03558   | 18.79065 | 16.85842   | 19.75331 |
| 0.6  | 12.48971   | 17.20963 | 13.13076   | 18.09105 | 13.80465   | 19.01770 | 14.51307   | 19.99190 | 15.25776   | 21.01610 |
| 0.65 | 11.10246   | 18.13760 | 11.67230   | 19.06655 | 12.27134   | 20.04315 | 12.90106   | 21.06988 | 13.56305   | 22.14931 |
| 0.7  | 9.64674    | 18.95373 | 10.14188   | 19.92448 | 10.66233   | 20.94503 | 11.20952   | 22.01797 | 11.78474   | 23.14597 |
| 0.75 | 8.13156    | 19.65301 | 8.54892    | 20.65958 | 8.98766    | 21.71778 | 9.44887    | 22.83030 | 9.93373    | 23.99991 |
| 0.8  | 6.56624    | 20.23113 | 6.90325    | 21.26731 | 7.25754    | 22.35664 | 7.62997    | 23.50188 | 8.02149    | 24.70590 |
| 0.85 | 4.96043    | 20.68452 | 5.21503    | 21.74391 | 5.48267    | 22.85766 | 5.76402    | 24.02856 | 6.05979    | 25.25957 |
| 0.9  | 3.32404    | 21.01038 | 3.49465    | 22.08646 | 3.67400    | 23.21775 | 3.86254    | 24.40710 | 4.06074    | 25.65749 |
| 0.95 | 1.66716    | 21.20670 | 1.75273    | 22.29283 | 1.84268    | 23.43470 | 1.93724    | 24.63516 | 2.03665    | 25.89724 |
| 1.0  | 0.00       | 21.27230 | 0.00       | 22.36178 | 0.00       | 23.50717 | 0.00       | 24.71135 | 0.00       | 25.97731 |
| 1.05 | 1.66716    | 21.20670 | 1.75273    | 22.29283 | 1.84268    | 23.43470 | 1.93724    | 24.63516 | 2.03665    | 25.89724 |
| 1.1  | 3.32404    | 21.01038 | 3.49465    | 22.08646 | 3.67400    | 23.21775 | 3.86254    | 24.40710 | 4.06074    | 25.65749 |
| 1.15 | 4.96043    | 20.68452 | 5.21503    | 21.74391 | 5.48267    | 22.85766 | 5.76402    | 24.02856 | 6.05979    | 25.25957 |
| 1.2  | 6.56624    | 20.23113 | 6.90325    | 21.26731 | 7.25754    | 22.35664 | 7.62997    | 23.50188 | 8.02149    | 24.70590 |
| 1.25 | 8.13156    | 19.65301 | 8.54892    | 20.65958 | 8.98766    | 21.71778 | 9.44887    | 22.83030 | 9.93373    | 23.99991 |
| 1.3  | 9.64674    | 18.95373 | 10.14188   | 19.92448 | 10.66233   | 20.94503 | 11.20952   | 22.01797 | 11.78474   | 23.14597 |
| 1.35 | 11.10246   | 18.13760 | 11.67230   | 19.06655 | 12.27134   | 20.04315 | 12.90106   | 21.06988 | 13.56305   | 22.14931 |
| 1.4  | 12.48971   | 17.20963 | 13.13076   | 18.09105 | 13.80465   | 19.01770 | 14.51307   | 19.99190 | 15.25776   | 21.01610 |
| 1.45 | 13.79098   | 16.17556 | 14.50828   | 17.00402 | 15.25286   | 17.87500 | 16.03558   | 18.79065 | 16.85842   | 19.75331 |
| 1.5  | 15.02516   | 15.04177 | 15.79634   | 15.81216 | 16.60702   | 16.62208 | 17.45924   | 17.47355 | 18.35512   | 18.36873 |
| 1.55 | 16.15770   | 13.81524 | 16.98701   | 14.52281 | 17.85880   | 15.26668 | 18.77526   | 16.04874 | 19.73867   | 16.87098 |
| 1.6  | 17.19062   | 12.50353 | 18.07296   | 13.14392 | 19.00048   | 13.81716 | 19.97556   | 14.52497 | 21.00052   | 15.26910 |
| 1.65 | 18.11756   | 11.11473 | 19.04746   | 11.68400 | 20.02501   | 12.28246 | 21.05272   | 12.91164 | 22.13290   | 13.57315 |
| 1.7  | 18.93280   | 9.65741  | 19.90455   | 10.15203 | 20.92608   | 10.67203 | 21.99993   | 11.21871 | 23.12881   | 11.79366 |
| 1.75 | 19.63131   | 8.14055  | 20.63891   | 8.55748  | 21.69813   | 8.99580  | 22.81160   | 9.45662  | 23.98212   | 9.94109  |
| 1.8  | 20.20879   | 6.57350  | 21.24603   | 6.91017  | 22.33640   | 7.26411  | 23.48262   | 7.63623  | 24.68760   | 8.02744  |
| 1.85 | 20.66167   | 4.96592  | 21.72216   | 5.22025  | 22.83696   | 5.48764  | 24.00888   | 5.76875  | 25.24084   | 6.06429  |
| 1.9  | 20.98716   | 3.32772  | 22.06437   | 3.49815  | 23.19673   | 3.67733  | 24.38710   | 3.86571  | 25.63849   | 4.06375  |
| 1.95 | 21.18327   | 1.66900  | 22.27052   | 1.75448  | 23.41348   | 1.84435  | 24.61500   | 1.93883  | 25.87805   | 2.03816  |
| 2.0  | 21.24878   | 0.00     | 22.33941   | 0.00     | 23.48589   | 0.00     | 24.69110   | 0.00     | 25.95806   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(3.90 + i 0.75) = 9.44887 + i 22.83030$ .  
 $\sinh(3.95 + i 1.95) = -25.87805 + i 2.03816$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ 

| $q$  | $x = 0$ |      | $x = 0.05$ |         | $x = 0.1$ |         | $x = 0.15$ |         | $x = 0.2$ |         |
|------|---------|------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 1.0000  | 0.00 | 1.00125    | 0.00    | 1.00500   | 0.00    | 1.01127    | 0.00    | 1.02007   | 0.00    |
| 0.05 | 0.99692 | 0.00 | 0.99816    | 0.00392 | 1.00191   | 0.00786 | 1.00815    | 0.01181 | 1.01692   | 0.01580 |
| 0.1  | 0.98769 | 0.00 | 0.98892    | 0.00783 | 0.99263   | 0.01567 | 0.99882    | 0.02355 | 1.00751   | 0.03150 |
| 0.15 | 0.97237 | 0.00 | 0.97385    | 0.01168 | 0.97724   | 0.02338 | 0.98333    | 0.03515 | 0.99188   | 0.04700 |
| 0.2  | 0.95106 | 0.00 | 0.95225    | 0.01546 | 0.95582   | 0.03095 | 0.96178    | 0.04653 | 0.97014   | 0.06222 |
| 0.25 | 0.92388 | 0.00 | 0.92503    | 0.01914 | 0.92850   | 0.03833 | 0.93429    | 0.05762 | 0.94242   | 0.07705 |
| 0.3  | 0.89101 | 0.00 | 0.89208    | 0.02271 | 0.89547   | 0.04547 | 0.90105    | 0.06835 | 0.90889   | 0.09141 |
| 0.35 | 0.85264 | 0.00 | 0.85367    | 0.02614 | 0.85691   | 0.05234 | 0.86225    | 0.07867 | 0.86975   | 0.10520 |
| 0.4  | 0.80902 | 0.00 | 0.81000    | 0.02940 | 0.81307   | 0.05888 | 0.81814    | 0.08850 | 0.82525   | 0.11834 |
| 0.45 | 0.76041 | 0.00 | 0.76133    | 0.03249 | 0.76421   | 0.06505 | 0.76898    | 0.09778 | 0.77507   | 0.13076 |
| 0.5  | 0.70711 | 0.00 | 0.70796    | 0.03537 | 0.71065   | 0.07083 | 0.71508    | 0.10646 | 0.72130   | 0.14237 |
| 0.55 | 0.64945 | 0.00 | 0.65023    | 0.03804 | 0.65270   | 0.07617 | 0.65677    | 0.11449 | 0.66248   | 0.15310 |
| 0.6  | 0.58779 | 0.00 | 0.58850    | 0.04047 | 0.59027   | 0.08104 | 0.59441    | 0.12181 | 0.59958   | 0.16288 |
| 0.65 | 0.52250 | 0.00 | 0.52313    | 0.04265 | 0.52511   | 0.08541 | 0.52839    | 0.12838 | 0.53298   | 0.17167 |
| 0.7  | 0.45399 | 0.00 | 0.45439    | 0.04457 | 0.45626   | 0.08925 | 0.45911    | 0.13415 | 0.46310   | 0.17939 |
| 0.75 | 0.38268 | 0.00 | 0.38316    | 0.04621 | 0.38460   | 0.09254 | 0.38700    | 0.13910 | 0.39036   | 0.18601 |
| 0.8  | 0.30902 | 0.00 | 0.30940    | 0.04757 | 0.31056   | 0.09526 | 0.31250    | 0.14319 | 0.31522   | 0.19148 |
| 0.85 | 0.23345 | 0.00 | 0.23374    | 0.04864 | 0.23461   | 0.09740 | 0.23608    | 0.14640 | 0.23813   | 0.19577 |
| 0.9  | 0.15643 | 0.00 | 0.15663    | 0.04941 | 0.15722   | 0.09893 | 0.15820    | 0.14871 | 0.15957   | 0.19886 |
| 0.95 | 0.07846 | 0.00 | 0.07856    | 0.04987 | 0.07885   | 0.09986 | 0.07934    | 0.15010 | 0.08003   | 0.20072 |
| 1.0  | 0.00    | 0.00 | 0.00       | 0.05002 | 0.00      | 0.10017 | 0.00       | 0.15056 | 0.00      | 0.20134 |
| 1.05 | 0.07846 | 0.00 | 0.07856    | 0.04987 | 0.07885   | 0.09986 | 0.07934    | 0.15010 | 0.08003   | 0.20072 |
| 1.1  | 0.15643 | 0.00 | 0.15663    | 0.04941 | 0.15722   | 0.09893 | 0.15820    | 0.14871 | 0.15957   | 0.19886 |
| 1.15 | 0.23345 | 0.00 | 0.23374    | 0.04864 | 0.23461   | 0.09740 | 0.23608    | 0.14640 | 0.23813   | 0.19577 |
| 1.2  | 0.30902 | 0.00 | 0.30940    | 0.04757 | 0.31056   | 0.09526 | 0.31250    | 0.14319 | 0.31522   | 0.19148 |
| 1.25 | 0.38268 | 0.00 | 0.38316    | 0.04621 | 0.38460   | 0.09254 | 0.38700    | 0.13910 | 0.39036   | 0.18601 |
| 1.3  | 0.45399 | 0.00 | 0.45439    | 0.04457 | 0.45626   | 0.08925 | 0.45911    | 0.13415 | 0.46310   | 0.17939 |
| 1.35 | 0.52250 | 0.00 | 0.52313    | 0.04265 | 0.52511   | 0.08541 | 0.52839    | 0.12838 | 0.53298   | 0.17167 |
| 1.4  | 0.58779 | 0.00 | 0.58850    | 0.04047 | 0.59027   | 0.08104 | 0.59441    | 0.12181 | 0.59958   | 0.16288 |
| 1.45 | 0.64945 | 0.00 | 0.65023    | 0.03804 | 0.65270   | 0.07617 | 0.65677    | 0.11449 | 0.66248   | 0.15310 |
| 1.5  | 0.70711 | 0.00 | 0.70796    | 0.03537 | 0.71065   | 0.07083 | 0.71508    | 0.10646 | 0.72130   | 0.14237 |
| 1.55 | 0.76041 | 0.00 | 0.76133    | 0.03249 | 0.76421   | 0.06505 | 0.76898    | 0.09778 | 0.77507   | 0.13076 |
| 1.6  | 0.80902 | 0.00 | 0.81000    | 0.02940 | 0.81307   | 0.05888 | 0.81814    | 0.08850 | 0.82525   | 0.11834 |
| 1.65 | 0.85264 | 0.00 | 0.85367    | 0.02614 | 0.85691   | 0.05234 | 0.86225    | 0.07867 | 0.86975   | 0.10520 |
| 1.7  | 0.89101 | 0.00 | 0.89208    | 0.02271 | 0.89547   | 0.04547 | 0.90105    | 0.06835 | 0.90889   | 0.09141 |
| 1.75 | 0.92388 | 0.00 | 0.92503    | 0.01914 | 0.92850   | 0.03833 | 0.93429    | 0.05762 | 0.94242   | 0.07705 |
| 1.8  | 0.95106 | 0.00 | 0.95225    | 0.01546 | 0.95582   | 0.03095 | 0.96178    | 0.04653 | 0.97014   | 0.06222 |
| 1.85 | 0.97237 | 0.00 | 0.97385    | 0.01168 | 0.97724   | 0.02338 | 0.98333    | 0.03515 | 0.99188   | 0.04700 |
| 1.9  | 0.98769 | 0.00 | 0.98892    | 0.00783 | 0.99263   | 0.01567 | 0.99882    | 0.02355 | 1.00751   | 0.03150 |
| 1.95 | 0.99692 | 0.00 | 0.99816    | 0.00392 | 1.00191   | 0.00786 | 1.00815    | 0.01181 | 1.01692   | 0.01580 |
| 2.0  | 1.0000  | 0.00 | 1.00125    | 0.00    | 1.00500   | 0.00    | 1.01127    | 0.00    | 1.02007   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(0 + i\mathbf{0.75}) = 0.38268 + i\mathbf{0.}$

$\cosh(0.2 + i\mathbf{1.5}) = -0.72130 + i\mathbf{0.14237}$

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 0.25$ |         | $x = 0.3$ |         | $x = 0.35$ |         | $x = 0.4$ |         | $x = 0.45$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 1.03141    | 0.00    | 1.04534   | 0.00    | 1.06188    | 0.00    | 1.08107   | 0.00    | 1.10297    | 0.00    |
| 0.05 | 1.02823    | 0.01982 | 1.04212   | 0.02389 | 1.05860    | 0.02803 | 1.07774   | 0.03223 | 1.09957    | 0.03651 |
| 0.1  | 1.01871    | 0.03952 | 1.03247   | 0.04764 | 1.04880    | 0.05588 | 1.06776   | 0.06426 | 1.08939    | 0.07280 |
| 0.15 | 1.00292    | 0.05897 | 1.01646   | 0.07109 | 1.03254    | 0.08338 | 1.05120   | 0.09589 | 1.07250    | 0.10863 |
| 0.2  | 0.98093    | 0.07806 | 0.99418   | 0.09410 | 1.00991    | 0.11038 | 1.02816   | 0.12693 | 1.04899    | 0.14380 |
| 0.25 | 0.95290    | 0.09667 | 0.96577   | 0.11654 | 0.98105    | 0.13669 | 0.99878   | 0.15719 | 1.01901    | 0.17808 |
| 0.3  | 0.91900    | 0.11468 | 0.93140   | 0.13825 | 0.94614    | 0.16216 | 0.96324   | 0.18648 | 0.98275    | 0.21126 |
| 0.35 | 0.87942    | 0.13199 | 0.89130   | 0.15911 | 0.90540    | 0.18663 | 0.92177   | 0.21462 | 0.94044    | 0.24314 |
| 0.4  | 0.83443    | 0.14848 | 0.84570   | 0.17899 | 0.85908    | 0.20995 | 0.87461   | 0.24143 | 0.89232    | 0.27352 |
| 0.45 | 0.78429    | 0.16406 | 0.79488   | 0.19777 | 0.80746    | 0.23198 | 0.82206   | 0.26676 | 0.83871    | 0.30222 |
| 0.5  | 0.72932    | 0.17862 | 0.73917   | 0.21533 | 0.75086    | 0.25257 | 0.76443   | 0.29045 | 0.77992    | 0.32905 |
| 0.55 | 0.66985    | 0.19208 | 0.67889   | 0.23156 | 0.68964    | 0.27161 | 0.70210   | 0.31234 | 0.71632    | 0.35385 |
| 0.6  | 0.60625    | 0.20437 | 0.61444   | 0.24636 | 0.62416    | 0.28897 | 0.63544   | 0.33231 | 0.64831    | 0.37047 |
| 0.65 | 0.53891    | 0.21539 | 0.54619   | 0.25965 | 0.55483    | 0.30455 | 0.56486   | 0.35022 | 0.57630    | 0.39677 |
| 0.7  | 0.46825    | 0.22508 | 0.47457   | 0.27133 | 0.48208    | 0.31826 | 0.49080   | 0.36598 | 0.50074    | 0.41462 |
| 0.75 | 0.39471    | 0.23338 | 0.40003   | 0.28134 | 0.40636    | 0.33000 | 0.41371   | 0.37949 | 0.42209    | 0.42992 |
| 0.8  | 0.31872    | 0.24025 | 0.32303   | 0.28962 | 0.32814    | 0.33971 | 0.33497   | 0.39065 | 0.34084    | 0.44257 |
| 0.85 | 0.24078    | 0.24563 | 0.24403   | 0.29611 | 0.24789    | 0.34732 | 0.25237   | 0.39940 | 0.25748    | 0.45249 |
| 0.9  | 0.16135    | 0.24950 | 0.16353   | 0.30077 | 0.16611    | 0.35279 | 0.16912   | 0.40570 | 0.17254    | 0.45961 |
| 0.95 | 0.08092    | 0.25183 | 0.08202   | 0.30358 | 0.08331    | 0.35609 | 0.08482   | 0.40949 | 0.08654    | 0.46391 |
| 1.0  | 0.00       | 0.25261 | 0.00      | 0.30452 | 0.00       | 0.35719 | 0.00      | 0.41075 | 0.00       | 0.46534 |
| 1.05 | 0.08092    | 0.25183 | 0.08202   | 0.30358 | 0.08331    | 0.35609 | 0.08482   | 0.40949 | 0.08654    | 0.46391 |
| 1.1  | 0.16135    | 0.24950 | 0.16353   | 0.30077 | 0.16611    | 0.35279 | 0.16912   | 0.40570 | 0.17254    | 0.45961 |
| 1.15 | 0.24078    | 0.24563 | 0.24403   | 0.29611 | 0.24789    | 0.34732 | 0.25237   | 0.39940 | 0.25748    | 0.45249 |
| 1.2  | 0.31872    | 0.24025 | 0.32303   | 0.28962 | 0.32814    | 0.33971 | 0.33407   | 0.39065 | 0.34084    | 0.44257 |
| 1.25 | 0.39471    | 0.23338 | 0.40003   | 0.28134 | 0.40636    | 0.33000 | 0.41371   | 0.37949 | 0.42209    | 0.42992 |
| 1.3  | 0.46825    | 0.22508 | 0.47457   | 0.27133 | 0.48208    | 0.31826 | 0.49080   | 0.36598 | 0.50074    | 0.41462 |
| 1.35 | 0.53891    | 0.21539 | 0.54619   | 0.25965 | 0.55483    | 0.30455 | 0.56486   | 0.35022 | 0.57630    | 0.39677 |
| 1.4  | 0.60625    | 0.20437 | 0.61444   | 0.24636 | 0.62416    | 0.28897 | 0.63544   | 0.33231 | 0.64831    | 0.37047 |
| 1.45 | 0.66985    | 0.19208 | 0.67889   | 0.23156 | 0.68964    | 0.27161 | 0.70210   | 0.31234 | 0.71632    | 0.35385 |
| 1.5  | 0.72932    | 0.17862 | 0.73917   | 0.21533 | 0.75086    | 0.25257 | 0.76443   | 0.29045 | 0.77992    | 0.32905 |
| 1.55 | 0.78429    | 0.16406 | 0.79488   | 0.19777 | 0.80746    | 0.23198 | 0.82206   | 0.26676 | 0.83871    | 0.30222 |
| 1.6  | 0.83443    | 0.14848 | 0.84570   | 0.17899 | 0.85908    | 0.20995 | 0.87461   | 0.24143 | 0.89232    | 0.27352 |
| 1.65 | 0.87942    | 0.13199 | 0.89130   | 0.15911 | 0.90540    | 0.18663 | 0.92177   | 0.21462 | 0.94044    | 0.24314 |
| 1.7  | 0.91900    | 0.11468 | 0.93140   | 0.13825 | 0.94614    | 0.16216 | 0.96324   | 0.18648 | 0.98275    | 0.21126 |
| 1.75 | 0.95290    | 0.09667 | 0.96577   | 0.11654 | 0.98105    | 0.13669 | 0.99878   | 0.15719 | 1.01901    | 0.17808 |
| 1.8  | 0.98093    | 0.07806 | 0.99418   | 0.09410 | 1.00991    | 0.11038 | 1.02816   | 0.12693 | 1.04899    | 0.14380 |
| 1.85 | 1.00292    | 0.05897 | 1.01646   | 0.07109 | 1.03254    | 0.08338 | 1.05120   | 0.09589 | 1.07250    | 0.10863 |
| 1.9  | 1.01871    | 0.03952 | 1.03247   | 0.04764 | 1.04880    | 0.05588 | 1.06776   | 0.06426 | 1.08939    | 0.07280 |
| 1.95 | 1.02823    | 0.01982 | 1.04212   | 0.02389 | 1.05860    | 0.02803 | 1.07774   | 0.03223 | 1.09957    | 0.03651 |
| 2.0  | 1.03141    | 0.00    | 1.04534   | 0.00    | 1.06188    | 0.00    | 1.08107   | 0.00    | 1.10297    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(0.3 + i0.9) = 0.16353 + i0.30077$ .  
 $\cosh(0.45 + i1.7) = -0.08275 + i0.21126$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 0.5$       | $x = 0.55$      | $x = 0.6$       | $x = 0.65$      | $x = 0.7$       |
|------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0    | 1.12763 0.00    | 1.15510 0.00    | 1.18547 0.00    | 1.21879 0.00    | 1.25517 0.00    |
| 0.05 | 1.12415 0.04088 | 1.15154 0.04536 | 1.18181 0.04995 | 1.21504 0.05467 | 1.25130 0.05952 |
| 0.1  | 1.11374 0.08152 | 1.14088 0.09044 | 1.17087 0.09959 | 1.20379 0.10900 | 1.23972 0.11867 |
| 0.15 | 1.09647 0.12165 | 1.12319 0.13497 | 1.15271 0.14862 | 1.18512 0.16265 | 1.22049 0.17709 |
| 0.2  | 1.07244 0.16103 | 1.09857 0.17866 | 1.12744 0.19674 | 1.15912 0.21531 | 1.19374 0.23442 |
| 0.25 | 1.04179 0.19942 | 1.06717 0.22125 | 1.09523 0.24364 | 1.12602 0.26663 | 1.15962 0.29030 |
| 0.3  | 1.00472 0.23657 | 1.02920 0.26248 | 1.05626 0.28904 | 1.08595 0.31632 | 1.11836 0.34439 |
| 0.35 | 0.96146 0.27227 | 0.98489 0.30208 | 1.01078 0.33265 | 1.03919 0.36405 | 1.07021 0.39636 |
| 0.4  | 0.91227 0.30629 | 0.93450 0.33983 | 0.95906 0.37422 | 0.98602 0.40954 | 1.01545 0.44589 |
| 0.45 | 0.85745 0.33842 | 0.87835 0.37548 | 0.90144 0.41347 | 0.92678 0.45250 | 0.95444 0.49266 |
| 0.5  | 0.79735 0.36847 | 0.81678 0.40882 | 0.83825 0.45018 | 0.86182 0.49268 | 0.88754 0.53640 |
| 0.55 | 0.73233 0.39624 | 0.75018 0.43903 | 0.76990 0.48412 | 0.79154 0.52981 | 0.81517 0.57683 |
| 0.6  | 0.66280 0.42158 | 0.67895 0.46773 | 0.69680 0.51506 | 0.71639 0.56368 | 0.73777 0.61371 |
| 0.65 | 0.58918 0.44431 | 0.60354 0.49296 | 0.61940 0.54284 | 0.63682 0.59408 | 0.65582 0.64680 |
| 0.7  | 0.51193 0.46430 | 0.52441 0.51514 | 0.53819 0.56726 | 0.55332 0.62081 | 0.56984 0.67590 |
| 0.75 | 0.43152 0.48143 | 0.44204 0.53414 | 0.45366 0.58819 | 0.46641 0.64371 | 0.48033 0.70084 |
| 0.8  | 0.34846 0.49559 | 0.35695 0.54986 | 0.36633 0.60549 | 0.37663 0.66205 | 0.38787 0.72146 |
| 0.85 | 0.26324 0.50670 | 0.26965 0.56218 | 0.27674 0.61906 | 0.28452 0.67750 | 0.29301 0.73763 |
| 0.9  | 0.17640 0.51468 | 0.18070 0.57103 | 0.18545 0.62882 | 0.19066 0.68817 | 0.19635 0.74925 |
| 0.95 | 0.08847 0.51949 | 0.09063 0.57637 | 0.09301 0.63469 | 0.09563 0.69460 | 0.09848 0.75625 |
| 1.0  | 0.00 0.52110    | 0.00 0.57815    | 0.00 0.63665    | 0.00 0.69675    | 0.00 0.75858    |
| 1.05 | 0.08847 0.51949 | 0.09063 0.57637 | 0.09301 0.63469 | 0.09563 0.69460 | 0.09848 0.75625 |
| 1.1  | 0.17640 0.51468 | 0.18070 0.57103 | 0.18545 0.62882 | 0.19066 0.68817 | 0.19635 0.74925 |
| 1.15 | 0.26324 0.50670 | 0.26965 0.56218 | 0.27674 0.61906 | 0.28452 0.67750 | 0.29301 0.73763 |
| 1.2  | 0.34846 0.49559 | 0.35695 0.54986 | 0.36633 0.60549 | 0.37663 0.66205 | 0.38787 0.72146 |
| 1.25 | 0.43152 0.48143 | 0.44204 0.53414 | 0.45366 0.58819 | 0.46641 0.64371 | 0.48033 0.70084 |
| 1.3  | 0.51193 0.46430 | 0.52441 0.51514 | 0.53819 0.56726 | 0.55332 0.62081 | 0.56984 0.67590 |
| 1.35 | 0.58918 0.44431 | 0.60354 0.49296 | 0.61940 0.54284 | 0.63682 0.59408 | 0.65582 0.64680 |
| 1.4  | 0.66280 0.42158 | 0.67895 0.46773 | 0.69680 0.51506 | 0.71639 0.56368 | 0.73777 0.61371 |
| 1.45 | 0.73233 0.39624 | 0.75018 0.43903 | 0.76990 0.48412 | 0.79154 0.52981 | 0.81517 0.57683 |
| 1.5  | 0.79735 0.36847 | 0.81678 0.40882 | 0.83825 0.45018 | 0.86182 0.49268 | 0.88754 0.53640 |
| 1.55 | 0.85745 0.33842 | 0.87835 0.37548 | 0.90144 0.41347 | 0.92678 0.45250 | 0.95444 0.49266 |
| 1.6  | 0.91227 0.30629 | 0.93450 0.33983 | 0.95906 0.37422 | 0.98602 0.40954 | 1.01545 0.44589 |
| 1.65 | 0.96146 0.27227 | 0.98489 0.30208 | 1.01078 0.33265 | 1.03919 0.36405 | 1.07021 0.39636 |
| 1.7  | 1.00472 0.23657 | 1.02920 0.26248 | 1.05626 0.28904 | 1.08595 0.31632 | 1.11836 0.34439 |
| 1.75 | 1.04179 0.19942 | 1.06717 0.22125 | 1.09523 0.24364 | 1.12602 0.26663 | 1.15962 0.29030 |
| 1.8  | 1.07244 0.16103 | 1.09857 0.17866 | 1.12744 0.19674 | 1.15912 0.21531 | 1.19374 0.23442 |
| 1.85 | 1.09647 0.12165 | 1.12319 0.13497 | 1.15271 0.14862 | 1.18512 0.16265 | 1.22049 0.17709 |
| 1.9  | 1.11374 0.08152 | 1.14088 0.09044 | 1.17087 0.09959 | 1.20379 0.10900 | 1.23972 0.11867 |
| 1.95 | 1.12415 0.04088 | 1.15154 0.04536 | 1.18181 0.04995 | 1.21504 0.05467 | 1.25130 0.05952 |
| 2.0  | 1.12763 0.00    | 1.15510 0.00    | 1.18547 0.00    | 1.21879 0.00    | 1.25517 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(0.6 + i \underline{0.95}) = 0.09301 + i 0.63469$ .  
 $\cosh(0.6 + i \underline{1.05}) = -0.09301 + i 0.63469$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh (x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 0.75$ |         | $x = 0.8$ |         | $x = 0.85$ |         | $x = 0.9$ |         | $x = 0.95$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 1.29468    | 0.00    | 1.33743   | 0.00    | 1.38353    | 0.00    | 1.43309   | 0.00    | 1.48623    | 0.00    |
| 0.05 | 1.29009    | 0.06452 | 1.33331   | 0.06968 | 1.37927    | 0.07502 | 1.42867   | 0.08054 | 1.48164    | 0.08627 |
| 0.1  | 1.27874    | 0.12864 | 1.32097   | 0.13893 | 1.36650    | 0.14957 | 1.41544   | 0.16058 | 1.46793    | 0.17200 |
| 0.15 | 1.25891    | 0.19197 | 1.30048   | 0.20732 | 1.34530    | 0.22320 | 1.39349   | 0.23964 | 1.44516    | 0.25667 |
| 0.2  | 1.23132    | 0.25411 | 1.27198   | 0.27444 | 1.31582    | 0.29546 | 1.36294   | 0.31721 | 1.41348    | 0.33976 |
| 0.25 | 1.19613    | 0.31469 | 1.23563   | 0.33986 | 1.27822    | 0.36589 | 1.32400   | 0.39283 | 1.37309    | 0.42076 |
| 0.3  | 1.15356    | 0.37332 | 1.19166   | 0.40319 | 1.23274    | 0.43407 | 1.27689   | 0.46603 | 1.32425    | 0.49916 |
| 0.35 | 1.10390    | 0.42966 | 1.14035   | 0.46403 | 1.17965    | 0.49957 | 1.22194   | 0.53635 | 1.26722    | 0.57448 |
| 0.4  | 1.04742    | 0.48335 | 1.08201   | 0.52202 | 1.11930    | 0.56199 | 1.15939   | 0.60337 | 1.20238    | 0.64626 |
| 0.45 | 0.98449    | 0.53405 | 1.01700   | 0.57678 | 1.05205    | 0.62095 | 1.08973   | 0.66667 | 1.13013    | 0.71406 |
| 0.5  | 0.91548    | 0.58147 | 0.94571   | 0.62799 | 0.97830    | 0.67608 | 1.01334   | 0.72586 | 1.05092    | 0.77745 |
| 0.55 | 0.84083    | 0.62529 | 0.86859   | 0.67532 | 0.89853    | 0.72704 | 0.93071   | 0.78057 | 0.96523    | 0.83605 |
| 0.6  | 0.76100    | 0.66527 | 0.78613   | 0.71849 | 0.81322    | 0.77351 | 0.84235   | 0.83047 | 0.87358    | 0.88950 |
| 0.65 | 0.67647    | 0.70114 | 0.69881   | 0.75724 | 0.72290    | 0.81522 | 0.74879   | 0.87525 | 0.77655    | 0.93747 |
| 0.7  | 0.58777    | 0.73269 | 0.60718   | 0.79131 | 0.62811    | 0.85191 | 0.65061   | 0.91463 | 0.67473    | 0.97965 |
| 0.75 | 0.49545    | 0.75972 | 0.51182   | 0.82050 | 0.52945    | 0.88334 | 0.54842   | 0.94838 | 0.56875    | 1.01579 |
| 0.8  | 0.40008    | 0.78207 | 0.41329   | 0.84464 | 0.42753    | 0.90932 | 0.44285   | 0.97628 | 0.45927    | 1.04567 |
| 0.85 | 0.30224    | 0.79960 | 0.31222   | 0.86357 | 0.32298    | 0.92970 | 0.33455   | 0.99816 | 0.34695    | 1.06911 |
| 0.9  | 0.20253    | 0.81219 | 0.20922   | 0.87717 | 0.21643    | 0.94435 | 0.22418   | 1.01388 | 0.23250    | 1.08595 |
| 0.95 | 0.10158    | 0.81978 | 0.10493   | 0.88537 | 0.10855    | 0.95317 | 0.11244   | 1.02335 | 0.11661    | 1.09610 |
| 1.0  | 0.00       | 0.82232 | 0.00      | 0.88811 | 0.00       | 0.95612 | 0.00      | 1.02652 | 0.00       | 1.09948 |
| 1.05 | 0.10158    | 0.81978 | 0.10493   | 0.88537 | 0.10855    | 0.95317 | 0.11244   | 1.02335 | 0.11661    | 1.09610 |
| 1.1  | 0.20253    | 0.81219 | 0.20922   | 0.87717 | 0.21643    | 0.94435 | 0.22418   | 1.01388 | 0.23250    | 1.08595 |
| 1.15 | 0.30224    | 0.79960 | 0.31222   | 0.86357 | 0.32298    | 0.92970 | 0.33455   | 0.99816 | 0.34695    | 1.06911 |
| 1.2  | 0.40008    | 0.78207 | 0.41329   | 0.84464 | 0.42753    | 0.90932 | 0.44285   | 0.97628 | 0.45927    | 1.04567 |
| 1.25 | 0.49545    | 0.75972 | 0.51182   | 0.82050 | 0.52945    | 0.88334 | 0.54842   | 0.94838 | 0.56875    | 1.01579 |
| 1.3  | 0.58777    | 0.73269 | 0.60718   | 0.79131 | 0.62811    | 0.85191 | 0.65061   | 0.91463 | 0.67473    | 0.97965 |
| 1.35 | 0.67647    | 0.70114 | 0.69881   | 0.75724 | 0.72290    | 0.81522 | 0.74879   | 0.87525 | 0.77655    | 0.93747 |
| 1.4  | 0.76100    | 0.66527 | 0.78613   | 0.71849 | 0.81322    | 0.77351 | 0.84235   | 0.83047 | 0.87358    | 0.88950 |
| 1.45 | 0.84083    | 0.62529 | 0.86859   | 0.67532 | 0.89853    | 0.72704 | 0.93071   | 0.78057 | 0.96523    | 0.83605 |
| 1.5  | 0.91548    | 0.58147 | 0.94571   | 0.62799 | 0.97830    | 0.67608 | 1.01334   | 0.72586 | 1.05092    | 0.77745 |
| 1.55 | 0.98449    | 0.53405 | 1.01700   | 0.57678 | 1.05205    | 0.62095 | 1.08973   | 0.66667 | 1.13013    | 0.71406 |
| 1.6  | 1.04742    | 0.48335 | 1.08201   | 0.52202 | 1.11930    | 0.56199 | 1.15939   | 0.60337 | 1.20238    | 0.64626 |
| 1.65 | 1.10390    | 0.42966 | 1.14035   | 0.46403 | 1.17965    | 0.49957 | 1.22194   | 0.53635 | 1.26722    | 0.57448 |
| 1.7  | 1.15356    | 0.37332 | 1.19166   | 0.40319 | 1.23274    | 0.43407 | 1.27689   | 0.46603 | 1.32425    | 0.49916 |
| 1.75 | 1.19613    | 0.31469 | 1.23563   | 0.33986 | 1.27822    | 0.36589 | 1.32400   | 0.39283 | 1.37309    | 0.42076 |
| 1.8  | 1.23132    | 0.25411 | 1.27198   | 0.27444 | 1.31582    | 0.29546 | 1.36294   | 0.31721 | 1.41348    | 0.33976 |
| 1.85 | 1.25891    | 0.19197 | 1.30048   | 0.20732 | 1.34530    | 0.22320 | 1.39349   | 0.23964 | 1.44516    | 0.25667 |
| 1.9  | 1.27874    | 0.12864 | 1.32097   | 0.13893 | 1.36650    | 0.14957 | 1.41544   | 0.16058 | 1.46793    | 0.17200 |
| 1.95 | 1.29069    | 0.06452 | 1.33331   | 0.06968 | 1.37927    | 0.07502 | 1.42867   | 0.08054 | 1.48164    | 0.08627 |
| 2.0  | 1.29468    | 0.00    | 1.33743   | 0.00    | 1.38353    | 0.00    | 1.43309   | 0.00    | 1.48623    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh (0.9 + i \underline{1.0}) = 0 + i \underline{1.02652}$ .

$\cosh (0.9 + i \underline{1.10}) = -0.22418 + i \underline{1.01388}$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.0$ |         | $x = 1.05$ |         | $x = 1.1$ |         | $x = 1.15$ |         | $x = 1.2$ |         |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 1.54308   | 0.00    | 1.60379    | 0.00    | 1.66852   | 0.00    | 1.73741    | 0.00    | 1.81066   | 0.00    |
| 0.05 | 1.53832   | 0.09221 | 1.59885    | 0.09838 | 1.66337   | 0.10479 | 1.73206    | 0.11147 | 1.80507   | 0.11843 |
| 0.1  | 1.52408   | 0.18384 | 1.58405    | 0.19615 | 1.64798   | 0.20894 | 1.71602    | 0.22226 | 1.78836   | 0.23613 |
| 0.15 | 1.50045   | 0.27435 | 1.55948    | 0.29271 | 1.62242   | 0.31180 | 1.68941    | 0.33167 | 1.76063   | 0.35238 |
| 0.2  | 1.46756   | 0.36316 | 1.52530    | 0.38746 | 1.58685   | 0.41274 | 1.65238    | 0.43904 | 1.72204   | 0.46645 |
| 0.25 | 1.42562   | 0.44973 | 1.48171    | 0.47983 | 1.54151   | 0.51113 | 1.60517    | 0.54371 | 1.67283   | 0.57765 |
| 0.3  | 1.37490   | 0.53353 | 1.42899    | 0.56924 | 1.48666   | 0.60637 | 1.54805    | 0.64502 | 1.61331   | 0.68528 |
| 0.35 | 1.31569   | 0.61404 | 1.36746    | 0.65514 | 1.42265   | 0.69787 | 1.48139    | 0.74235 | 1.54384   | 0.78869 |
| 0.4  | 1.24838   | 0.69077 | 1.29750    | 0.73700 | 1.34986   | 0.78508 | 1.40560    | 0.83511 | 1.46485   | 0.88724 |
| 0.45 | 1.17337   | 0.76323 | 1.21953    | 0.81432 | 1.26875   | 0.86743 | 1.32114    | 0.92272 | 1.37684   | 0.98032 |
| 0.5  | 1.09112   | 0.83099 | 1.13405    | 0.88661 | 1.17082   | 0.94445 | 1.22854    | 1.00464 | 1.28033   | 1.06735 |
| 0.55 | 1.00215   | 0.89303 | 1.04158    | 0.95344 | 1.08362   | 1.01564 | 1.12836    | 1.08037 | 1.17593   | 1.14781 |
| 0.6  | 0.90700   | 0.95076 | 0.94269    | 1.01439 | 0.98073   | 1.08056 | 1.02123    | 1.14943 | 1.06428   | 1.22118 |
| 0.65 | 0.80626   | 1.00202 | 0.83798    | 1.06909 | 0.87180   | 1.13883 | 0.90780    | 1.21141 | 0.94607   | 1.28703 |
| 0.7  | 0.70055   | 1.04711 | 0.72811    | 1.11719 | 0.75749   | 1.19007 | 0.78877    | 1.26592 | 0.82202   | 1.34494 |
| 0.75 | 0.59051   | 1.08574 | 0.61375    | 1.15841 | 0.63852   | 1.23398 | 0.66488    | 1.31263 | 0.69291   | 1.39456 |
| 0.8  | 0.47684   | 1.11768 | 0.49560    | 1.19249 | 0.51560   | 1.27028 | 0.53689    | 1.35124 | 0.55952   | 1.43558 |
| 0.85 | 0.36023   | 1.14273 | 0.37440    | 1.21921 | 0.38951   | 1.29875 | 0.40559    | 1.38152 | 0.42269   | 1.46776 |
| 0.9  | 0.24139   | 1.16073 | 0.25089    | 1.23842 | 0.26101   | 1.31920 | 0.27179    | 1.40329 | 0.28325   | 1.49088 |
| 0.95 | 0.12107   | 1.17158 | 0.12583    | 1.24999 | 0.13091   | 1.33153 | 0.13632    | 1.41640 | 0.14206   | 1.50481 |
| 1.0  | 0.00      | 1.17520 | 0.00       | 1.25386 | 0.00      | 1.33565 | 0.00       | 1.42078 | 0.00      | 1.50946 |
| 1.05 | 0.12107   | 1.17158 | 0.12583    | 1.24999 | 0.13091   | 1.33153 | 0.13632    | 1.41640 | 0.14206   | 1.50481 |
| 1.1  | 0.24139   | 1.16073 | 0.25089    | 1.23842 | 0.26101   | 1.31920 | 0.27179    | 1.40329 | 0.28325   | 1.49088 |
| 1.15 | 0.36023   | 1.14273 | 0.37440    | 1.21921 | 0.38951   | 1.29875 | 0.40559    | 1.38152 | 0.42269   | 1.46776 |
| 1.2  | 0.47684   | 1.11768 | 0.49560    | 1.19249 | 0.51560   | 1.27028 | 0.53689    | 1.35124 | 0.55952   | 1.43558 |
| 1.25 | 0.59051   | 1.08574 | 0.61375    | 1.15841 | 0.63852   | 1.23398 | 0.66488    | 1.31263 | 0.69291   | 1.39456 |
| 1.3  | 0.70055   | 1.04711 | 0.72811    | 1.11719 | 0.75749   | 1.19007 | 0.78877    | 1.26592 | 0.82202   | 1.34494 |
| 1.35 | 0.80626   | 1.00202 | 0.83798    | 1.06909 | 0.87180   | 1.13883 | 0.90780    | 1.21141 | 0.94607   | 1.28703 |
| 1.4  | 0.90700   | 0.95076 | 0.94269    | 1.01439 | 0.98073   | 1.08056 | 1.02123    | 1.14943 | 1.06428   | 1.22118 |
| 1.45 | 1.00215   | 0.89303 | 1.04158    | 0.95344 | 1.08362   | 1.01564 | 1.12836    | 1.08037 | 1.17593   | 1.14781 |
| 1.5  | 1.09112   | 0.83099 | 1.13405    | 0.88661 | 1.17982   | 0.94445 | 1.22854    | 1.00464 | 1.28033   | 1.06735 |
| 1.55 | 1.17337   | 0.76323 | 1.21953    | 0.81432 | 1.26875   | 0.86743 | 1.32114    | 0.92272 | 1.37684   | 0.98032 |
| 1.6  | 1.24838   | 0.69077 | 1.29750    | 0.73700 | 1.34986   | 0.78508 | 1.40560    | 0.83511 | 1.46485   | 0.88724 |
| 1.65 | 1.31569   | 0.61404 | 1.36746    | 0.65514 | 1.42265   | 0.69787 | 1.48139    | 0.74235 | 1.54384   | 0.78869 |
| 1.7  | 1.37490   | 0.53353 | 1.42899    | 0.56924 | 1.48666   | 0.60637 | 1.54805    | 0.64502 | 1.61331   | 0.68528 |
| 1.75 | 1.42562   | 0.44973 | 1.48171    | 0.47983 | 1.54151   | 0.51113 | 1.60517    | 0.54371 | 1.67283   | 0.57765 |
| 1.8  | 1.46756   | 0.36316 | 1.52530    | 0.38746 | 1.58685   | 0.41274 | 1.65238    | 0.43904 | 1.72204   | 0.46645 |
| 1.85 | 1.50045   | 0.27435 | 1.55948    | 0.29271 | 1.62242   | 0.31180 | 1.68941    | 0.33167 | 1.76063   | 0.35238 |
| 1.9  | 1.52408   | 0.18384 | 1.58405    | 0.19615 | 1.64798   | 0.20894 | 1.71602    | 0.22226 | 1.78836   | 0.23613 |
| 1.95 | 1.53832   | 0.09221 | 1.59885    | 0.09838 | 1.66337   | 0.10479 | 1.73206    | 0.11147 | 1.80507   | 0.11843 |
| 2.0  | 1.54308   | 0.00    | 1.60379    | 0.00    | 1.66852   | 0.00    | 1.73741    | 0.00    | 1.81066   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(1.2 + i0) = 1.81066 + i0$ .  
 $\cosh(1.1 + i1.1) = -0.26101 + i1.31920$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.25$ |         | $x = 1.3$ |         | $x = 1.35$ |         | $x = 1.4$ |         | $x = 1.45$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 1.88842    | 0.00    | 1.97091   | 0.00    | 2.05833    | 0.00    | 2.15090   | 0.00    | 2.24884    | 0.00    |
| 0.05 | 1.88260    | 0.12569 | 1.96484   | 1.13325 | 2.05200    | 0.14116 | 2.14427   | 0.14941 | 2.24191    | 0.15804 |
| 0.1  | 1.86517    | 0.25060 | 1.94665   | 0.26569 | 2.03299    | 0.28144 | 2.12442   | 0.29790 | 2.22115    | 0.31510 |
| 0.15 | 1.83624    | 0.37396 | 1.91646   | 0.39648 | 2.00146    | 0.41999 | 2.09147   | 0.44455 | 2.18671    | 0.47022 |
| 0.2  | 1.79600    | 0.49502 | 1.87445   | 0.52483 | 1.95759    | 0.55595 | 2.04562   | 0.58846 | 2.13878    | 0.62244 |
| 0.25 | 1.74467    | 0.61303 | 1.82089   | 0.64994 | 1.90165    | 0.68848 | 1.98717   | 0.72875 | 2.07766    | 0.77083 |
| 0.3  | 1.68260    | 0.72726 | 1.75610   | 0.77105 | 1.83399    | 0.81677 | 1.91646   | 0.86454 | 2.00373    | 0.91446 |
| 0.35 | 1.61014    | 0.83700 | 1.68048   | 0.88740 | 1.75502    | 0.94002 | 1.83394   | 0.99500 | 1.91745    | 1.05245 |
| 0.4  | 1.52777    | 0.94158 | 1.59451   | 0.99829 | 1.66523    | 1.05748 | 1.74012   | 1.11932 | 1.81935    | 1.18306 |
| 0.45 | 1.43597    | 1.04036 | 1.49870   | 1.10301 | 1.56517    | 1.16842 | 1.63556   | 1.23674 | 1.71007    | 1.30817 |
| 0.5  | 1.33532    | 1.13273 | 1.39365   | 1.20094 | 1.45546    | 1.27215 | 1.52092   | 1.34655 | 1.59017    | 1.42431 |
| 0.55 | 1.22643    | 1.21811 | 1.28001   | 1.29146 | 1.33678    | 1.36804 | 1.39690   | 1.44804 | 1.46051    | 1.53166 |
| 0.6  | 1.10999    | 1.29598 | 1.15848   | 1.37402 | 1.20986    | 1.45550 | 1.26427   | 1.54061 | 1.32184    | 1.62958 |
| 0.65 | 0.98670    | 1.36586 | 1.02980   | 1.44811 | 1.07548    | 1.53398 | 1.12384   | 1.62369 | 1.17502    | 1.71745 |
| 0.7  | 0.85733    | 1.42732 | 0.89478   | 1.51327 | 0.93446    | 1.60300 | 0.97649   | 1.69675 | 1.02095    | 1.79473 |
| 0.75 | 0.72267    | 1.47998 | 0.75424   | 1.56910 | 0.78769    | 1.66215 | 0.82311   | 1.75934 | 0.86060    | 1.86094 |
| 0.8  | 0.58355    | 1.52352 | 0.60905   | 1.61526 | 0.63606    | 1.71104 | 0.66466   | 1.81110 | 0.69493    | 1.91569 |
| 0.85 | 0.44084    | 1.55766 | 0.46010   | 1.65146 | 0.48051    | 1.74938 | 0.50212   | 1.85169 | 0.52498    | 1.95862 |
| 0.9  | 0.29541    | 1.58220 | 0.30832   | 1.67747 | 0.32199    | 1.77694 | 0.33647   | 1.88086 | 0.35180    | 1.98947 |
| 0.95 | 0.14816    | 1.59698 | 0.15464   | 1.69315 | 0.16150    | 1.79354 | 0.16876   | 1.89843 | 0.17644    | 2.00806 |
| 1.0  | 0.00       | 1.60192 | 0.00      | 1.69838 | 0.00       | 1.79909 | 0.00      | 1.90430 | 0.00       | 2.01427 |
| 1.05 | 0.14816    | 1.59698 | 0.15464   | 1.69315 | 0.16150    | 1.79354 | 0.16876   | 1.89843 | 0.17644    | 2.00806 |
| 1.1  | 0.29541    | 1.58220 | 0.30832   | 1.67747 | 0.32199    | 1.77694 | 0.33647   | 1.88086 | 0.35180    | 1.98947 |
| 1.15 | 0.44084    | 1.55766 | 0.46010   | 1.65146 | 0.48051    | 1.74938 | 0.50212   | 1.85169 | 0.52498    | 1.95862 |
| 1.2  | 0.58355    | 1.52352 | 0.60905   | 1.61526 | 0.63606    | 1.71104 | 0.66466   | 1.81110 | 0.69493    | 1.91569 |
| 1.25 | 0.72267    | 1.47998 | 0.75424   | 1.56910 | 0.78769    | 1.66215 | 0.82311   | 1.75934 | 0.86060    | 1.86094 |
| 1.3  | 0.85733    | 1.42732 | 0.89478   | 1.51327 | 0.93446    | 1.60300 | 0.97649   | 1.69675 | 1.02095    | 1.79473 |
| 1.35 | 0.98670    | 1.36586 | 1.02980   | 1.44811 | 1.07548    | 1.53398 | 1.12384   | 1.62369 | 1.17502    | 1.71745 |
| 1.4  | 1.10999    | 1.29598 | 1.15848   | 1.37402 | 1.20986    | 1.45550 | 1.26427   | 1.54061 | 1.32184    | 1.62958 |
| 1.45 | 1.22643    | 1.21811 | 1.28001   | 1.29146 | 1.33678    | 1.36804 | 1.39690   | 1.44804 | 1.46051    | 1.53166 |
| 1.5  | 1.33532    | 1.13273 | 1.39365   | 1.20094 | 1.45546    | 1.27215 | 1.52092   | 1.34655 | 1.59017    | 1.42431 |
| 1.55 | 1.43597    | 1.04036 | 1.49870   | 1.10301 | 1.56517    | 1.16842 | 1.63556   | 1.23674 | 1.71007    | 1.30817 |
| 1.6  | 1.52777    | 0.94158 | 1.59451   | 0.99829 | 1.66523    | 1.05748 | 1.74012   | 1.11932 | 1.81935    | 1.18306 |
| 1.65 | 1.61014    | 0.83700 | 1.68048   | 0.88740 | 1.75502    | 0.94002 | 1.83394   | 0.99500 | 1.91745    | 1.05245 |
| 1.7  | 1.68260    | 0.72726 | 1.75610   | 0.77105 | 1.83399    | 0.81677 | 1.91646   | 0.86454 | 2.00373    | 0.91446 |
| 1.75 | 1.74467    | 0.61303 | 1.82089   | 0.64994 | 1.90165    | 0.68848 | 1.98717   | 0.72875 | 2.07766    | 0.77083 |
| 1.8  | 1.79600    | 0.49502 | 1.87445   | 0.52483 | 1.95759    | 0.55595 | 2.04562   | 0.58846 | 2.13878    | 0.62244 |
| 1.85 | 1.83624    | 0.37396 | 1.91646   | 0.39648 | 2.00146    | 0.41999 | 2.09147   | 0.44455 | 2.18671    | 0.47022 |
| 1.9  | 1.86517    | 0.25060 | 1.94665   | 0.26569 | 2.03299    | 0.28144 | 2.12442   | 0.29790 | 2.22115    | 0.31510 |
| 1.95 | 1.88260    | 0.12569 | 1.96484   | 0.13325 | 2.05200    | 0.14116 | 2.14427   | 0.14941 | 2.24191    | 0.15804 |
| 2.0  | 1.88842    | 0.00    | 1.97091   | 0.00    | 2.05833    | 0.00    | 2.15090   | 0.00    | 2.24884    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(1.4 + i \underline{1.9}) = -2.12442 + i \underline{0.29790}$ ,  
 $\cosh(1.4 + i \underline{1.4}) = -1.26427 + i \underline{1.54061}$ .



TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.5$ |         | $x = 1.55$ |         | $x = 1.6$ |         | $x = 1.65$ |         | $x = 1.7$ |         |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 2.35241   | 0.00    | 2.46186    | 0.00    | 2.57746   | 0.00    | 2.69952    | 0.00    | 2.82832   | 0.00    |
| 0.05 | 2.34516   | 0.16706 | 2.45427    | 0.17650 | 2.56952   | 0.18639 | 2.69121    | 0.19673 | 2.81960   | 0.20757 |
| 0.1  | 2.32345   | 0.33309 | 2.43155    | 0.35192 | 2.54573   | 0.37162 | 2.66629    | 0.39225 | 2.79350   | 0.41387 |
| 0.15 | 2.28742   | 0.49707 | 2.39384    | 0.52516 | 2.50625   | 0.55456 | 2.62494    | 0.58536 | 2.75017   | 0.61761 |
| 0.2  | 2.23727   | 0.65798 | 2.34137    | 0.69517 | 2.45131   | 0.73409 | 2.56740    | 0.77485 | 2.68989   | 0.81754 |
| 0.25 | 2.17334   | 0.81484 | 2.27446    | 0.86089 | 2.38127   | 0.90909 | 2.49404    | 0.95957 | 2.61302   | 1.01244 |
| 0.3  | 2.09601   | 0.96667 | 2.19353    | 1.02130 | 2.29654   | 1.07848 | 2.40529    | 1.13837 | 2.52005   | 1.20109 |
| 0.35 | 2.00576   | 1.11255 | 2.09908    | 1.17542 | 2.19765   | 1.24123 | 2.30173    | 1.31015 | 2.41154   | 1.38234 |
| 0.4  | 1.90314   | 1.25156 | 1.99169    | 1.32229 | 2.08522   | 1.39632 | 2.18396    | 1.47385 | 2.28816   | 1.55506 |
| 0.45 | 1.78879   | 1.38286 | 1.87201    | 1.46101 | 1.95992   | 1.54281 | 2.05274    | 1.62847 | 2.15067   | 1.71820 |
| 0.5  | 1.66341   | 1.50563 | 1.74080    | 1.59071 | 1.82254   | 1.67978 | 1.90885    | 1.77305 | 1.99992   | 1.87074 |
| 0.55 | 1.52777   | 1.61912 | 1.59885    | 1.71062 | 1.67393   | 1.80640 | 1.75320    | 1.90669 | 1.83684   | 2.01175 |
| 0.6  | 1.38271   | 1.72263 | 1.44704    | 1.81997 | 1.51500   | 1.92187 | 1.58674    | 2.02858 | 1.66244   | 2.14036 |
| 0.65 | 1.22913   | 1.81551 | 1.28632    | 1.91811 | 1.34672   | 2.02550 | 1.41050    | 2.13797 | 1.47779   | 2.25577 |
| 0.7  | 1.06797   | 1.89720 | 1.11766    | 2.00442 | 1.17015   | 2.11664 | 1.22556    | 2.23417 | 1.28403   | 2.35727 |
| 0.75 | 0.90023   | 1.96720 | 0.94211    | 2.07837 | 0.98636   | 2.19473 | 1.03306    | 2.31660 | 1.08235   | 2.44424 |
| 0.8  | 0.72693   | 2.02507 | 0.76076    | 2.13951 | 0.79648   | 2.25930 | 0.83420    | 2.38474 | 0.87400   | 2.51614 |
| 0.85 | 0.54916   | 2.07045 | 0.57471    | 2.18745 | 0.60170   | 2.30993 | 0.63019    | 2.43818 | 0.66026   | 2.57253 |
| 0.9  | 0.36800   | 2.10307 | 0.38512    | 2.22191 | 0.40320   | 2.34632 | 0.42230    | 2.47659 | 0.44245   | 2.61306 |
| 0.95 | 0.18457   | 2.12272 | 0.19316    | 2.24268 | 0.20223   | 2.36824 | 0.21180    | 2.49973 | 0.22191   | 2.63747 |
| 1.0  | 0.00      | 2.12928 | 0.00       | 2.24961 | 0.00      | 2.37557 | 0.00       | 2.50747 | 0.00      | 2.64563 |
| 1.05 | 0.18457   | 2.12272 | 0.19316    | 2.24268 | 0.20223   | 2.36824 | 0.21180    | 2.49973 | 0.22191   | 2.63747 |
| 1.1  | 0.36800   | 2.10307 | 0.38512    | 2.22191 | 0.40320   | 2.34632 | 0.42230    | 2.47659 | 0.44245   | 2.61306 |
| 1.15 | 0.54916   | 2.07045 | 0.57471    | 2.18745 | 0.60170   | 2.30993 | 0.63019    | 2.43818 | 0.66026   | 2.57253 |
| 1.2  | 0.72693   | 2.02507 | 0.76076    | 2.13951 | 0.79648   | 2.25930 | 0.83420    | 2.38474 | 0.87400   | 2.51614 |
| 1.25 | 0.90023   | 1.96720 | 0.94211    | 2.07837 | 0.98636   | 2.19473 | 1.03306    | 2.31660 | 1.08235   | 2.44424 |
| 1.3  | 1.06797   | 1.89720 | 1.11766    | 2.00442 | 1.17015   | 2.11664 | 1.22556    | 2.23417 | 1.28403   | 2.35727 |
| 1.35 | 1.22913   | 1.81551 | 1.28632    | 1.91811 | 1.34672   | 2.02550 | 1.41050    | 2.13797 | 1.47779   | 2.25577 |
| 1.4  | 1.38271   | 1.72263 | 1.44704    | 1.81997 | 1.51500   | 1.92187 | 1.58674    | 2.02858 | 1.66244   | 2.14036 |
| 1.45 | 1.52777   | 1.61912 | 1.59885    | 1.71062 | 1.67393   | 1.80640 | 1.75320    | 1.90669 | 1.83684   | 2.01175 |
| 1.5  | 1.66341   | 1.50563 | 1.74080    | 1.59071 | 1.82254   | 1.67978 | 1.90885    | 1.77305 | 1.99992   | 1.87074 |
| 1.55 | 1.78879   | 1.38286 | 1.87201    | 1.46101 | 1.95992   | 1.54281 | 2.05274    | 1.62847 | 2.15067   | 1.71820 |
| 1.6  | 1.90314   | 1.25156 | 1.99169    | 1.32229 | 2.08522   | 1.39632 | 2.18396    | 1.47385 | 2.28816   | 1.55506 |
| 1.65 | 2.00576   | 1.11255 | 2.09908    | 1.17542 | 2.19765   | 1.24123 | 2.30173    | 1.31015 | 2.41154   | 1.38234 |
| 1.7  | 2.09601   | 0.96667 | 2.19353    | 1.02130 | 2.29654   | 1.07848 | 2.40529    | 1.13837 | 2.52005   | 1.20109 |
| 1.75 | 2.17334   | 0.81484 | 2.27446    | 0.86089 | 2.38127   | 0.90909 | 2.49404    | 0.95957 | 2.61302   | 1.01244 |
| 1.8  | 2.23727   | 0.65798 | 2.34137    | 0.69517 | 2.45131   | 0.73409 | 2.56740    | 0.77485 | 2.68989   | 0.81754 |
| 1.85 | 2.28742   | 0.49707 | 2.39384    | 0.52516 | 2.50625   | 0.55456 | 2.62494    | 0.58536 | 2.75017   | 0.61761 |
| 1.9  | 2.32345   | 0.33309 | 2.43155    | 0.35192 | 2.54573   | 0.37162 | 2.66629    | 0.39225 | 2.79350   | 0.41387 |
| 1.95 | 2.34516   | 0.16706 | 2.45427    | 0.17650 | 2.56952   | 0.18639 | 2.69121    | 0.19673 | 2.81960   | 0.20757 |
| 2.0  | 2.35241   | 0.00    | 2.46186    | 0.00    | 2.57746   | 0.00    | 2.69952    | 0.00    | 2.82832   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(1.6 + i\mathbf{0.4}) = 2.08522 + i\mathbf{1.39632}$ .  
 $\cosh(1.7 + i\mathbf{1.2}) = -\mathbf{0.87400} + i\mathbf{2.51614}$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.75$ |         | $x = 1.8$ |         | $x = 1.85$ |         | $x = 1.9$ |         | $x = 1.95$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 2.96419    | 0.00    | 3.10747   | 0.00    | 3.25853    | 0.00    | 3.41773   | 0.00    | 3.58548    | 0.00    |
| 0.05 | 2.95505    | 0.21893 | 3.09789   | 0.23084 | 3.24848    | 0.24332 | 3.40719   | 0.25642 | 3.57443    | 0.27015 |
| 0.1  | 2.92769    | 0.43652 | 3.06921   | 0.46026 | 3.21841    | 0.48627 | 3.37565   | 0.51125 | 3.54134    | 0.53864 |
| 0.15 | 2.88229    | 0.65141 | 3.02161   | 0.68684 | 3.16850    | 0.72398 | 3.32330   | 0.76294 | 3.48641    | 0.80380 |
| 0.2  | 2.81911    | 0.86229 | 2.95538   | 0.90918 | 3.09904    | 0.95835 | 3.25045   | 1.00992 | 3.41000    | 1.06401 |
| 0.25 | 2.73855    | 1.06784 | 2.87093   | 1.12592 | 3.01049    | 1.18681 | 3.15757   | 1.25067 | 3.31255    | 1.31766 |
| 0.3  | 2.64111    | 1.26682 | 2.76878   | 1.33572 | 2.90337    | 1.40796 | 3.04522   | 1.48372 | 3.19469    | 1.56318 |
| 0.35 | 2.52739    | 1.45799 | 2.64956   | 1.53728 | 2.77835    | 1.62042 | 2.91409   | 1.60761 | 3.05713    | 1.79907 |
| 0.4  | 2.39808    | 1.64016 | 2.51400   | 1.72937 | 2.63620    | 1.82289 | 2.76501   | 1.92098 | 2.90071    | 2.02387 |
| 0.45 | 2.25399    | 1.81223 | 2.36294   | 1.91079 | 2.47780    | 2.01413 | 2.59887   | 2.12250 | 2.72642    | 2.23618 |
| 0.5  | 2.09600    | 1.97312 | 2.19731   | 2.08043 | 2.30413    | 2.19294 | 2.41670   | 2.31094 | 2.53532    | 2.43471 |
| 0.55 | 1.92509    | 2.12185 | 2.01814   | 2.23725 | 2.11624    | 2.35824 | 2.21064   | 2.48513 | 2.32858    | 2.61823 |
| 0.6  | 1.74231    | 2.25749 | 1.82653   | 2.38027 | 1.91531    | 2.50900 | 2.00889   | 2.64400 | 2.10749    | 2.78561 |
| 0.65 | 1.54878    | 2.37922 | 1.62365   | 2.50862 | 1.70258    | 2.64429 | 1.78576   | 2.78657 | 1.87341    | 2.93582 |
| 0.7  | 1.34571    | 2.48627 | 1.41076   | 2.62149 | 1.47934    | 2.76327 | 1.55162   | 2.91196 | 1.62777    | 3.06792 |
| 0.75 | 1.13435    | 2.57800 | 1.18918   | 2.71821 | 1.24698    | 2.86522 | 1.30791   | 3.01939 | 1.37210    | 3.18111 |
| 0.8  | 0.91599    | 2.65384 | 0.96026   | 2.79817 | 1.00694    | 2.94950 | 1.05614   | 3.10821 | 1.10797    | 3.27468 |
| 0.85 | 0.69198    | 2.71331 | 0.72542   | 2.86088 | 0.76069    | 3.01560 | 0.79785   | 3.17787 | 0.83701    | 3.34807 |
| 0.9  | 0.46370    | 2.75606 | 0.48612   | 2.90595 | 0.50975    | 3.06311 | 0.53465   | 3.22793 | 0.56089    | 3.40081 |
| 0.95 | 0.23257    | 2.78181 | 0.24381   | 2.93310 | 0.25566    | 3.09173 | 0.26815   | 3.25809 | 0.28131    | 3.43259 |
| 1.0  | 0.00       | 2.79041 | 0.00      | 2.94217 | 0.00       | 3.10129 | 0.00      | 3.26816 | 0.00       | 3.44321 |
| 1.05 | 0.23257    | 2.78181 | 0.24381   | 2.93310 | 0.25566    | 3.09173 | 0.26815   | 3.25809 | 0.28131    | 3.43259 |
| 1.1  | 0.46370    | 2.75606 | 0.48612   | 2.90595 | 0.50975    | 3.06311 | 0.53465   | 3.22793 | 0.56089    | 3.40081 |
| 1.15 | 0.69198    | 2.71331 | 0.72542   | 2.86088 | 0.76069    | 3.01560 | 0.79785   | 3.17787 | 0.83701    | 3.34807 |
| 1.2  | 0.91599    | 2.65384 | 0.96026   | 2.79817 | 1.00694    | 2.94950 | 1.05614   | 3.10821 | 1.10797    | 3.27468 |
| 1.25 | 1.13435    | 2.57800 | 1.18918   | 2.71821 | 1.24698    | 2.86522 | 1.30791   | 3.01939 | 1.37210    | 3.18111 |
| 1.3  | 1.34571    | 2.48627 | 1.41076   | 2.62149 | 1.47934    | 2.76327 | 1.55162   | 2.91196 | 1.62777    | 3.06792 |
| 1.35 | 1.54878    | 2.37922 | 1.62365   | 2.50862 | 1.70258    | 2.64429 | 1.78576   | 2.78657 | 1.87341    | 2.93582 |
| 1.4  | 1.74231    | 2.25749 | 1.82653   | 2.38027 | 1.91531    | 2.50900 | 2.00889   | 2.64400 | 2.10749    | 2.78561 |
| 1.45 | 1.92509    | 2.12185 | 2.01814   | 2.23725 | 2.11624    | 2.35824 | 2.21964   | 2.48513 | 2.32858    | 2.61823 |
| 1.5  | 2.09600    | 1.97312 | 2.19731   | 2.08043 | 2.30413    | 2.19294 | 2.41670   | 2.31094 | 2.53532    | 2.43471 |
| 1.55 | 2.25399    | 1.81223 | 2.36294   | 1.91079 | 2.47780    | 2.01413 | 2.59887   | 2.12250 | 2.72642    | 2.23618 |
| 1.6  | 2.39808    | 1.64016 | 2.51400   | 1.72937 | 2.63620    | 1.82289 | 2.76501   | 1.92098 | 2.90071    | 2.02387 |
| 1.65 | 2.52739    | 1.45799 | 2.64956   | 1.53728 | 2.77835    | 1.62042 | 2.91409   | 1.60761 | 3.05713    | 1.79907 |
| 1.7  | 2.64111    | 1.26682 | 2.76878   | 1.33572 | 2.90337    | 1.40796 | 3.04522   | 1.48372 | 3.19469    | 1.56318 |
| 1.75 | 2.73855    | 1.06784 | 2.87093   | 1.12592 | 3.01049    | 1.18681 | 3.15757   | 1.25067 | 3.31255    | 1.31766 |
| 1.8  | 2.81911    | 0.86229 | 2.95538   | 0.90918 | 3.09904    | 0.95835 | 3.25045   | 1.00992 | 3.41000    | 1.06401 |
| 1.85 | 2.88229    | 0.65141 | 3.02161   | 0.68684 | 3.16850    | 0.72398 | 3.32330   | 0.76294 | 3.48641    | 0.80380 |
| 1.9  | 2.92769    | 0.43652 | 3.06921   | 0.46026 | 3.21841    | 0.48627 | 3.37565   | 0.51125 | 3.54134    | 0.53864 |
| 1.95 | 2.95505    | 0.21893 | 3.09789   | 0.23084 | 3.24848    | 0.24332 | 3.40719   | 0.25642 | 3.57443    | 0.27015 |
| 2.0  | 2.96419    | 0.00    | 3.10747   | 0.00    | 3.25853    | 0.00    | 3.41773   | 0.00    | 3.58548    | 0.00    |

• Note. Negative quantities are in heavy type.

Examples.  $\cosh(1.8 + i 0.2) = 2.95538 + i 0.90918$ .  
 $\cosh(1.8 + i 2.0) = -3.10747 + i 0$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.0$ |         | $x = 2.05$ |         | $x = 2.1$ |         | $x = 2.15$ |         | $x = 2.2$ |         |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 3.76220   | 0.00    | 3.94832    | 0.00    | 4.14431   | 0.00    | 4.35067    | 0.00    | 4.56791   | 0.00    |
| 0.05 | 3.75059   | 0.28456 | 3.93615    | 0.29968 | 4.13154   | 0.31555 | 4.33726    | 0.33221 | 4.55382   | 0.34970 |
| 0.1  | 3.71587   | 0.56737 | 3.89971    | 0.59751 | 4.09329   | 0.62916 | 4.29711    | 0.66237 | 4.51167   | 0.69724 |
| 0.15 | 3.65825   | 0.84667 | 3.83923    | 0.89166 | 4.02981   | 0.93888 | 4.23046    | 0.98845 | 4.44170   | 1.04049 |
| 0.2  | 3.57806   | 1.12076 | 3.75507    | 1.18032 | 3.94148   | 1.24282 | 4.13773    | 1.30844 | 4.34433   | 1.37732 |
| 0.25 | 3.47581   | 1.38794 | 3.64777    | 1.46169 | 3.82885   | 1.53910 | 4.01950    | 1.62035 | 4.22019   | 1.70566 |
| 0.3  | 3.35214   | 1.64656 | 3.51798    | 1.73405 | 3.69261   | 1.82589 | 3.87647    | 1.92228 | 4.07003   | 2.02349 |
| 0.35 | 3.20780   | 1.89503 | 3.36649    | 1.99573 | 3.53361   | 2.10142 | 3.70956    | 2.21236 | 3.89478   | 2.32883 |
| 0.4  | 3.04368   | 2.13182 | 3.19426    | 2.24509 | 3.35283   | 2.36399 | 3.51977    | 2.48879 | 3.69552   | 2.61982 |
| 0.45 | 2.86080   | 2.35546 | 3.00232    | 2.48062 | 3.15137   | 2.61199 | 3.30828    | 2.74988 | 3.47347   | 2.89466 |
| 0.5  | 2.66027   | 2.56458 | 2.79188    | 2.70085 | 2.93048   | 2.84389 | 3.07639    | 2.99402 | 3.23000   | 3.15165 |
| 0.55 | 2.44335   | 2.75789 | 2.56423    | 2.90443 | 2.69152   | 3.05825 | 2.82553    | 3.21970 | 2.96661   | 3.38921 |
| 0.6  | 2.21136   | 2.93420 | 2.32076    | 3.09011 | 2.43597   | 3.25376 | 2.55726    | 3.42553 | 2.68495   | 3.60588 |
| 0.65 | 1.96574   | 3.09241 | 2.06299    | 3.2673  | 2.16540   | 3.42920 | 2.27322    | 3.61024 | 2.38672   | 3.80031 |
| 0.7  | 1.70800   | 3.23156 | 1.79250    | 3.40327 | 1.88148   | 3.58351 | 1.97516    | 3.77269 | 2.07379   | 3.97131 |
| 0.75 | 1.43973   | 3.35078 | 1.51096    | 3.52883 | 1.58596   | 3.71571 | 1.66493    | 3.91188 | 1.74806   | 4.11783 |
| 0.8  | 1.16258   | 3.44935 | 1.22010    | 3.63264 | 1.28066   | 3.82501 | 1.34443    | 4.02695 | 1.41156   | 4.23896 |
| 0.85 | 0.87827   | 3.52666 | 0.92172    | 3.71404 | 0.96747   | 3.91074 | 1.01564    | 4.11720 | 1.06636   | 4.33396 |
| 0.9  | 0.58854   | 3.58221 | 0.61765    | 3.77256 | 0.64831   | 3.97235 | 0.68059    | 4.18206 | 0.71458   | 4.40223 |
| 0.95 | 0.29518   | 3.61568 | 0.30978    | 3.80781 | 0.32516   | 4.00946 | 0.34135    | 4.22113 | 0.35839   | 4.44337 |
| 1.0  | 0.00      | 3.62686 | 0.00       | 3.81958 | 0.00      | 4.02186 | 0.00       | 4.23419 | 0.00      | 4.45711 |
| 1.05 | 0.29518   | 3.61568 | 0.30978    | 3.80781 | 0.32516   | 4.00946 | 0.34135    | 4.22113 | 0.35839   | 4.44337 |
| 1.1  | 0.58854   | 3.58221 | 0.61765    | 3.77256 | 0.64831   | 3.97235 | 0.68059    | 4.18206 | 0.71458   | 4.40223 |
| 1.15 | 0.87827   | 3.52666 | 0.92172    | 3.71404 | 0.96747   | 3.91074 | 1.01564    | 4.11720 | 1.06636   | 4.33396 |
| 1.2  | 1.16258   | 3.44935 | 1.22010    | 3.63264 | 1.28066   | 3.82501 | 1.34443    | 4.02695 | 1.41156   | 4.23896 |
| 1.25 | 1.43973   | 3.35078 | 1.51096    | 3.52883 | 1.58596   | 3.71571 | 1.66493    | 3.91188 | 1.74806   | 4.11783 |
| 1.3  | 1.70800   | 3.23156 | 1.79250    | 3.40327 | 1.88148   | 3.58351 | 1.97516    | 3.77269 | 2.07379   | 3.97131 |
| 1.35 | 1.96574   | 3.09241 | 2.06299    | 3.2673  | 2.16540   | 3.42920 | 2.27322    | 3.61024 | 2.38672   | 3.80031 |
| 1.4  | 2.21136   | 2.93420 | 2.32076    | 3.09011 | 2.43597   | 3.25376 | 2.55726    | 3.42553 | 2.68495   | 3.60588 |
| 1.45 | 2.44335   | 2.75789 | 2.56423    | 2.90443 | 2.69152   | 3.05825 | 2.82553    | 3.21970 | 2.96661   | 3.38921 |
| 1.5  | 2.66027   | 2.56458 | 2.79188    | 2.70085 | 2.93048   | 2.84389 | 3.07639    | 2.99402 | 3.23000   | 3.15165 |
| 1.55 | 2.86080   | 2.35546 | 3.00232    | 2.48062 | 3.15137   | 2.61199 | 3.30828    | 2.74988 | 3.47347   | 2.89466 |
| 1.6  | 3.04368   | 2.13182 | 3.19426    | 2.24509 | 3.35283   | 2.36399 | 3.51977    | 2.48879 | 3.69552   | 2.61982 |
| 1.65 | 3.20780   | 1.89503 | 3.36649    | 1.99573 | 3.53361   | 2.10142 | 3.70956    | 2.21236 | 3.89478   | 2.32883 |
| 1.7  | 3.35214   | 1.64656 | 3.51798    | 1.73405 | 3.69261   | 1.82589 | 3.87647    | 1.92228 | 4.07003   | 2.02349 |
| 1.75 | 3.47581   | 1.38794 | 3.64777    | 1.46169 | 3.82885   | 1.53910 | 4.01950    | 1.62035 | 4.22019   | 1.70566 |
| 1.8  | 3.57806   | 1.12076 | 3.75507    | 1.18032 | 3.94148   | 1.24282 | 4.13773    | 1.30844 | 4.34433   | 1.37732 |
| 1.85 | 3.65825   | 0.84667 | 3.83923    | 0.89166 | 4.02981   | 0.93888 | 4.23046    | 0.98845 | 4.44170   | 1.04049 |
| 1.9  | 3.71587   | 0.56737 | 3.89971    | 0.59751 | 4.09329   | 0.62916 | 4.29711    | 0.66237 | 4.51167   | 0.69724 |
| 1.95 | 3.75059   | 0.28456 | 3.93615    | 0.29968 | 4.13154   | 0.31555 | 4.33726    | 0.33221 | 4.55382   | 0.34970 |
| 2.0  | 3.76220   | 0.00    | 3.94832    | 0.00    | 4.14431   | 0.00    | 4.35067    | 0.00    | 4.56791   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(2.1 + i \underline{0.8}) = 1.28066 + i \underline{3.82501}$ .  
 $\cosh(2.2 + i \underline{1.25}) = -1.74806 + i \underline{4.11783}$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.25$ |         | $x = 2.3$ |         | $x = 2.35$ |         | $x = 2.4$ |         | $x = 2.45$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 4.70657    | 0.00    | 5.03722   | 0.00    | 5.29047    | 0.00    | 5.55695   | 0.00    | 5.83732    | 0.00    |
| 0.05 | 4.78178    | 0.36807 | 5.02169   | 0.38735 | 5.27416    | 0.40760 | 5.53981   | 0.42888 | 5.81933    | 0.45122 |
| 0.1  | 4.73751    | 0.73386 | 4.97521   | 0.77231 | 5.22533    | 0.81269 | 5.48853   | 0.85511 | 5.76545    | 0.89966 |
| 0.15 | 4.66404    | 1.09513 | 4.89805   | 1.15251 | 5.14429    | 1.21277 | 5.40341   | 1.27607 | 5.67603    | 1.34255 |
| 0.2  | 4.56181    | 1.44965 | 4.79058   | 1.52560 | 5.03153    | 1.60537 | 5.28496   | 1.68916 | 5.55162    | 1.77716 |
| 0.25 | 4.43145    | 1.79523 | 4.65378   | 1.88930 | 4.88776    | 1.98808 | 5.13394   | 2.09184 | 5.39298    | 2.20082 |
| 0.3  | 4.27377    | 2.12975 | 4.48820   | 2.24134 | 4.71384    | 2.35853 | 4.95127   | 2.48162 | 5.20109    | 2.61091 |
| 0.35 | 4.08975    | 2.45113 | 4.29494   | 2.57956 | 4.51087    | 2.71443 | 4.73808   | 2.85610 | 4.97713    | 3.00490 |
| 0.4  | 3.88050    | 2.75740 | 4.07520   | 2.90188 | 4.28008    | 3.05360 | 4.49567   | 3.21297 | 4.72249    | 3.38036 |
| 0.45 | 3.64734    | 3.04677 | 3.83034   | 3.20630 | 4.02290    | 3.37395 | 4.22554   | 3.55003 | 4.43873    | 3.73499 |
| 0.5  | 3.39168    | 3.31716 | 3.56185   | 3.49096 | 3.74093    | 3.67351 | 3.92935   | 3.86521 | 4.12761    | 4.06659 |
| 0.55 | 3.11512    | 3.56719 | 3.27141   | 3.75410 | 3.43588    | 3.95038 | 3.60894   | 4.15656 | 3.79104    | 4.37311 |
| 0.6  | 2.81935    | 3.79523 | 2.96081   | 3.99409 | 3.10966    | 4.20292 | 3.26629   | 4.42228 | 3.43109    | 4.65268 |
| 0.65 | 2.50620    | 3.99987 | 2.63194   | 4.20946 | 2.76426    | 4.42955 | 2.90349   | 4.66073 | 3.04999    | 4.90356 |
| 0.7  | 2.17760    | 4.17986 | 2.28685   | 4.39887 | 2.40182    | 4.62887 | 2.52280   | 4.87045 | 2.65009    | 5.12420 |
| 0.75 | 1.83557    | 4.33407 | 1.92766   | 4.56116 | 2.02457    | 4.79965 | 2.12655   | 5.05014 | 2.23385    | 5.31325 |
| 0.8  | 1.48222    | 4.46157 | 1.55659   | 4.69533 | 1.63485    | 4.94083 | 1.71719   | 5.19869 | 1.80383    | 5.46955 |
| 0.85 | 1.11974    | 4.56155 | 1.17592   | 4.80056 | 1.23504    | 5.05156 | 1.29724   | 5.31521 | 1.36269    | 5.59213 |
| 0.9  | 0.75035    | 4.63341 | 0.78799   | 4.87618 | 0.82761    | 5.13114 | 0.86930   | 5.39893 | 0.91316    | 5.68022 |
| 0.95 | 0.37633    | 4.67671 | 0.39522   | 4.92174 | 0.41509    | 5.17909 | 0.43599   | 5.44938 | 0.45799    | 5.73330 |
| 1.0  | 0.00       | 4.69117 | 0.00      | 4.93696 | 0.00       | 5.19510 | 0.00      | 5.46623 | 0.00       | 5.75103 |
| 1.05 | 0.37633    | 4.67671 | 0.39522   | 4.92174 | 0.41509    | 5.17909 | 0.43599   | 5.44938 | 0.45799    | 5.73330 |
| 1.1  | 0.75035    | 4.63341 | 0.78799   | 4.87618 | 0.82761    | 5.13114 | 0.86930   | 5.39893 | 0.91316    | 5.68022 |
| 1.15 | 1.11974    | 4.56155 | 1.17592   | 4.80056 | 1.23504    | 5.05156 | 1.29724   | 5.31521 | 1.36269    | 5.59213 |
| 1.2  | 1.48222    | 4.46157 | 1.55659   | 4.69533 | 1.63485    | 4.94083 | 1.71719   | 5.19869 | 1.80383    | 5.46955 |
| 1.25 | 1.83557    | 4.33407 | 1.92766   | 4.56116 | 2.02457    | 4.79965 | 2.12655   | 5.05014 | 2.23385    | 5.31325 |
| 1.3  | 2.17760    | 4.17986 | 2.28685   | 4.39887 | 2.40182    | 4.62887 | 2.52280   | 4.87045 | 2.65009    | 5.12420 |
| 1.35 | 2.50620    | 3.99987 | 2.63194   | 4.20946 | 2.76426    | 4.42955 | 2.90349   | 4.66073 | 3.04999    | 4.90356 |
| 1.4  | 2.81935    | 3.79523 | 2.96081   | 3.99409 | 3.10966    | 4.20292 | 3.26629   | 4.42228 | 3.43109    | 4.65268 |
| 1.45 | 3.11512    | 3.56719 | 3.27141   | 3.75410 | 3.43588    | 3.95038 | 3.60894   | 4.15656 | 3.79104    | 4.37311 |
| 1.5  | 3.39168    | 3.31716 | 3.56185   | 3.49096 | 3.74093    | 3.67351 | 3.92935   | 3.86521 | 4.12761    | 4.06659 |
| 1.55 | 3.64734    | 3.04677 | 3.83034   | 3.20630 | 4.02290    | 3.37395 | 4.22554   | 3.55003 | 4.43873    | 3.73499 |
| 1.6  | 3.88050    | 2.75740 | 4.07520   | 2.90188 | 4.28008    | 3.05360 | 4.49567   | 3.21297 | 4.72249    | 3.38036 |
| 1.65 | 4.08975    | 2.45113 | 4.29494   | 2.57956 | 4.51087    | 2.71443 | 4.73808   | 2.85610 | 4.97713    | 3.00490 |
| 1.7  | 4.27377    | 2.12975 | 4.48820   | 2.24134 | 4.71384    | 2.35853 | 4.95127   | 2.48162 | 5.20109    | 2.61091 |
| 1.75 | 4.43145    | 1.79523 | 4.65378   | 1.88930 | 4.88776    | 1.98808 | 5.13394   | 2.09184 | 5.39298    | 2.20082 |
| 1.8  | 4.56181    | 1.44965 | 4.79058   | 1.52560 | 5.03153    | 1.60537 | 5.28496   | 1.68916 | 5.55162    | 1.77716 |
| 1.85 | 4.66404    | 1.09513 | 4.89805   | 1.15251 | 5.14429    | 1.21277 | 5.40341   | 1.27607 | 5.67603    | 1.34255 |
| 1.9  | 4.73751    | 0.73386 | 4.97521   | 0.77231 | 5.22533    | 0.81269 | 5.48853   | 0.85511 | 5.76545    | 0.89966 |
| 1.95 | 4.78178    | 0.36807 | 5.02169   | 0.38735 | 5.27416    | 0.40760 | 5.53981   | 0.42888 | 5.81933    | 0.45122 |
| 2.0  | 4.79657    | 0.00    | 5.03722   | 0.00    | 5.29047    | 0.00    | 5.55695   | 0.00    | 5.83732    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(2.4 + i0.4) = 4.49567 + i3.21297$ .  
 $\cosh(2.4 + i1.5) = -3.92935 + i3.86521$

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.5$       | $x = 2.55$      | $x = 2.6$       | $x = 2.65$      | $x = 2.7$       |
|------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0    | 6.13229 0.00    | 6.44259 0.00    | 6.76901 0.00    | 7.11234 0.00    | 7.47347 0.00    |
| 0.05 | 6.11339 0.47469 | 6.42273 0.49935 | 6.74814 0.52526 | 7.09042 0.55249 | 7.45043 0.58109 |
| 0.1  | 6.05680 0.94646 | 6.36327 0.99563 | 6.68567 1.04729 | 7.02478 1.10156 | 7.38146 1.15859 |
| 0.15 | 5.96287 1.41239 | 6.26458 1.48577 | 6.58199 1.56285 | 6.91583 1.64385 | 7.26697 1.72896 |
| 0.2  | 5.83215 1.86961 | 6.12727 1.96674 | 6.43770 2.06878 | 6.76424 2.17600 | 7.10769 2.28866 |
| 0.25 | 5.66550 2.31531 | 5.95218 2.43559 | 6.25374 2.56196 | 6.57095 2.69474 | 6.90458 2.83425 |
| 0.3  | 5.46392 2.74674 | 5.74039 2.88943 | 6.03123 3.03934 | 6.33714 3.19686 | 6.65891 3.36237 |
| 0.35 | 5.22864 3.16122 | 5.49321 3.32545 | 5.77153 3.49799 | 6.06427 3.67927 | 6.37218 3.86976 |
| 0.4  | 4.96113 3.55622 | 5.21217 3.74097 | 5.47624 3.93506 | 5.75401 4.13900 | 6.04606 4.35329 |
| 0.45 | 4.66304 3.92929 | 4.89898 4.13342 | 5.14719 4.34788 | 5.40827 4.57321 | 5.68287 4.80998 |
| 0.5  | 4.33619 4.27814 | 4.55560 4.50039 | 4.78641 4.73389 | 5.02919 4.97923 | 5.28454 5.23702 |
| 0.55 | 3.98260 4.60062 | 4.18413 4.83961 | 4.39612 5.09071 | 4.61910 5.35454 | 4.85303 5.63177 |
| 0.6  | 3.60448 4.89472 | 3.78686 5.14900 | 3.97872 5.41615 | 4.18053 5.69685 | 4.39279 5.99179 |
| 0.65 | 3.20411 5.15805 | 3.36624 5.42664 | 3.53680 5.70819 | 3.71619 6.00403 | 3.90488 6.31488 |
| 0.7  | 2.78401 5.39077 | 2.92488 5.67082 | 3.07306 5.96505 | 3.22894 6.27419 | 3.39288 6.59903 |
| 0.75 | 2.34673 5.58966 | 2.46547 5.88004 | 2.59039 6.18512 | 2.72178 6.50567 | 2.85997 6.84249 |
| 0.8  | 1.89498 5.75408 | 1.99087 6.05301 | 2.09174 6.36706 | 2.19784 6.69705 | 2.30943 7.04377 |
| 0.85 | 1.43155 5.88304 | 1.50399 6.18866 | 1.58019 6.50976 | 1.66034 6.84713 | 1.74465 7.20163 |
| 0.9  | 0.95930 5.97572 | 1.00784 6.28615 | 1.05891 6.61231 | 1.11262 6.95500 | 1.16911 7.31508 |
| 0.95 | 0.48113 6.03155 | 0.50548 6.34489 | 0.53109 6.67409 | 0.55803 7.01998 | 0.58636 7.38343 |
| 1.0  | 0.00 6.05020    | 0.00 6.36451    | 0.00 6.69473    | 0.00 7.04169    | 0.00 7.40626    |
| 1.05 | 0.48113 6.03155 | 0.50548 6.34489 | 0.53109 6.67409 | 0.55803 7.01998 | 0.58636 7.38343 |
| 1.1  | 0.95930 5.97572 | 1.00784 6.28615 | 1.05891 6.61231 | 1.11262 6.95500 | 1.16911 7.31508 |
| 1.15 | 1.43155 5.88304 | 1.50399 6.18866 | 1.58019 6.50976 | 1.66034 6.84713 | 1.74465 7.20163 |
| 1.2  | 1.89498 5.75408 | 1.99087 6.05301 | 2.09174 6.36706 | 2.19784 6.69705 | 2.30943 7.04377 |
| 1.25 | 2.34673 5.58966 | 2.46547 5.88004 | 2.59039 6.18512 | 2.72178 6.50567 | 2.85997 6.84249 |
| 1.3  | 2.78401 5.39077 | 2.92488 5.67082 | 3.07306 5.96505 | 3.22894 6.27419 | 3.39288 6.59903 |
| 1.35 | 3.20411 5.15805 | 3.36624 5.42664 | 3.53680 5.70819 | 3.71619 6.00403 | 3.90488 6.31488 |
| 1.4  | 3.60448 4.89472 | 3.78586 5.14900 | 3.97872 5.41615 | 4.18053 5.69685 | 4.39279 5.99179 |
| 1.45 | 3.98260 4.60062 | 4.18413 4.83661 | 4.39612 5.09071 | 4.61910 5.35454 | 4.85303 5.63177 |
| 1.5  | 4.33619 4.27814 | 4.55560 4.50039 | 4.78641 4.73389 | 5.02919 4.97923 | 5.28454 5.23702 |
| 1.55 | 4.66304 3.92929 | 4.89898 4.13342 | 5.14719 4.34788 | 5.40827 4.57321 | 5.68287 4.80998 |
| 1.6  | 4.96113 3.55622 | 5.21217 3.74097 | 5.47624 3.93506 | 5.75401 4.13900 | 6.04606 4.35329 |
| 1.65 | 5.22864 3.16122 | 5.49321 3.32545 | 5.77153 3.49799 | 6.06427 3.67927 | 6.37218 3.86976 |
| 1.7  | 5.46392 2.74674 | 5.74039 2.88943 | 6.03123 3.03934 | 6.33714 3.19686 | 6.65891 3.36237 |
| 1.75 | 5.66550 2.31531 | 5.95218 2.43559 | 6.25374 2.56196 | 6.57095 2.69474 | 6.90458 2.83425 |
| 1.8  | 5.83215 1.86961 | 6.12727 1.96674 | 6.43770 2.06878 | 6.76424 2.17600 | 7.10769 2.28866 |
| 1.85 | 5.96287 1.41239 | 6.26458 1.48577 | 6.58199 1.56285 | 6.91583 1.64385 | 7.26697 1.72896 |
| 1.9  | 6.05680 0.94646 | 6.36327 0.99563 | 6.68567 1.04729 | 7.02478 1.10156 | 7.38146 1.15859 |
| 1.95 | 6.11339 0.47469 | 6.42273 0.49935 | 6.74814 0.52526 | 7.09042 0.55249 | 7.45043 0.58109 |
| 2.0  | 6.13229 0.00    | 6.44259 0.00    | 6.76901 0.00    | 7.11234 0.00    | 7.47347 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(2.7 + i 1.00) = 0 + i 7.40626$ .  
 $\cosh(2.6 + i 1.2) = -2.09174 + i 6.36706$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.75$ |         | $x = 2.80$ |         | $x = 2.85$ |         | $x = 2.90$ |         | $x = 2.95$ |         |
|------|------------|---------|------------|---------|------------|---------|------------|---------|------------|---------|
| 0    | 7.85328    | 0.00    | 8.25273    | 0.00    | 8.67281    | 0.00    | 9.11458    | 0.00    | 9.57915    | 0.00    |
| 0.05 | 7.82907    | 0.61115 | 8.22728    | 0.64273 | 8.64608    | 0.67592 | 9.08649    | 0.71081 | 9.54962    | 0.74747 |
| 0.1  | 7.75659    | 1.21852 | 8.15112    | 1.28150 | 8.56604    | 1.34768 | 9.00237    | 1.41723 | 9.46121    | 1.49032 |
| 0.15 | 7.63629    | 1.81839 | 8.02470    | 1.91237 | 8.43318    | 2.01112 | 8.86275    | 2.11491 | 9.31447    | 2.22399 |
| 0.2  | 7.46891    | 2.40704 | 7.84881    | 2.53144 | 8.24834    | 2.66217 | 8.66850    | 2.79956 | 9.11031    | 2.94395 |
| 0.25 | 7.25548    | 2.98086 | 7.62452    | 3.13491 | 8.01263    | 3.29681 | 8.42078    | 3.46694 | 8.84998    | 3.64575 |
| 0.3  | 6.99732    | 3.53629 | 7.35323    | 3.71905 | 7.72754    | 3.91112 | 8.12115    | 4.11295 | 8.53508    | 4.32508 |
| 0.35 | 6.69602    | 4.06993 | 7.03661    | 4.28026 | 7.39479    | 4.50131 | 7.77146    | 4.73361 | 8.16757    | 4.97774 |
| 0.4  | 6.35344    | 4.57847 | 6.67660    | 4.81509 | 7.01646    | 5.06375 | 7.37385    | 5.32508 | 7.74969    | 5.63048 |
| 0.45 | 5.97168    | 5.05878 | 6.27542    | 5.32022 | 6.59486    | 5.59498 | 6.93078    | 5.88371 | 7.28404    | 6.18717 |
| 0.5  | 5.55311    | 5.50790 | 5.83556    | 5.79256 | 6.13261    | 6.09170 | 6.44498    | 6.40608 | 6.77348    | 6.73647 |
| 0.55 | 5.10030    | 5.92307 | 5.35972    | 6.22918 | 5.63254    | 6.55088 | 5.91945    | 6.88894 | 6.22116    | 7.24424 |
| 0.6  | 4.61604    | 6.30172 | 4.85083    | 6.62740 | 5.09775    | 6.96966 | 5.35742    | 7.32934 | 5.63048    | 7.70735 |
| 0.65 | 4.10333    | 6.64151 | 4.31204    | 6.98476 | 4.53153    | 7.34547 | 4.76236    | 7.72455 | 5.00509    | 8.12294 |
| 0.7  | 3.56531    | 6.94036 | 3.74666    | 7.29905 | 3.93738    | 7.67599 | 4.13793    | 8.07213 | 4.34884    | 8.48845 |
| 0.75 | 3.00532    | 7.19642 | 3.15818    | 7.56834 | 3.31894    | 7.95919 | 3.48800    | 8.36994 | 3.66578    | 8.80162 |
| 0.8  | 2.42680    | 7.40811 | 2.55023    | 7.79098 | 2.68005    | 8.19332 | 2.81656    | 8.61616 | 2.96012    | 9.06053 |
| 0.85 | 1.83331    | 7.57413 | 1.92656    | 7.96557 | 2.02463    | 8.37694 | 2.12776    | 8.80924 | 2.23621    | 9.26358 |
| 0.9  | 1.22852    | 7.69345 | 1.29101    | 8.09106 | 1.35673    | 8.50891 | 1.42583    | 8.94802 | 1.49851    | 9.40952 |
| 0.95 | 0.61616    | 7.76534 | 0.64750    | 8.16666 | 0.68046    | 8.58841 | 0.71512    | 9.03163 | 0.75157    | 9.49744 |
| 1.0  | 0.00       | 7.78935 | 0.00       | 8.19102 | 0.00       | 8.61497 | 0.00       | 9.05956 | 0.00       | 9.52681 |
| 1.05 | 0.61616    | 7.76534 | 0.64750    | 8.16666 | 0.68046    | 8.58841 | 0.71512    | 9.03163 | 0.75157    | 9.49744 |
| 1.1  | 1.22852    | 7.69345 | 1.29101    | 8.09106 | 1.35673    | 8.50891 | 1.42583    | 8.94802 | 1.49851    | 9.40952 |
| 1.15 | 1.83331    | 7.57413 | 1.92656    | 7.96557 | 2.02463    | 8.37694 | 2.12776    | 8.80924 | 2.23621    | 9.26358 |
| 1.2  | 2.42680    | 7.40811 | 2.55023    | 7.79098 | 2.68005    | 8.19332 | 2.81656    | 8.61616 | 2.96012    | 9.06053 |
| 1.25 | 3.00532    | 7.19642 | 3.15818    | 7.56834 | 3.31894    | 7.95919 | 3.48800    | 8.36994 | 3.66578    | 8.80162 |
| 1.3  | 3.56531    | 6.94036 | 3.74666    | 7.29905 | 3.93738    | 7.67599 | 4.13793    | 8.07213 | 4.34884    | 8.48845 |
| 1.35 | 4.10333    | 6.64151 | 4.31204    | 6.98476 | 4.53153    | 7.34547 | 4.76236    | 7.72455 | 5.00509    | 8.12294 |
| 1.4  | 4.61604    | 6.30172 | 4.85083    | 6.62740 | 5.09775    | 6.96966 | 5.35742    | 7.32934 | 5.63048    | 7.70735 |
| 1.45 | 5.10030    | 5.92307 | 5.35972    | 6.22918 | 5.63254    | 6.55088 | 5.91945    | 6.88894 | 6.22116    | 7.24424 |
| 1.5  | 5.55311    | 5.50790 | 5.83556    | 5.79256 | 6.13261    | 6.09170 | 6.44498    | 6.40608 | 6.77348    | 6.73647 |
| 1.55 | 5.97168    | 5.05878 | 6.27542    | 5.32022 | 6.59486    | 5.59498 | 6.93078    | 5.88371 | 7.28404    | 6.18717 |
| 1.6  | 6.35344    | 4.57847 | 6.67660    | 4.81509 | 7.01646    | 5.06375 | 7.37385    | 5.32508 | 7.74969    | 5.63048 |
| 1.65 | 6.69602    | 4.06993 | 7.03661    | 4.28026 | 7.39479    | 4.50131 | 7.77146    | 4.73361 | 8.16757    | 4.97774 |
| 1.7  | 6.99732    | 3.53629 | 7.35323    | 3.71905 | 7.72754    | 3.91112 | 8.12115    | 4.11295 | 8.53508    | 4.32508 |
| 1.75 | 7.25548    | 2.98086 | 7.62452    | 3.13491 | 8.01263    | 3.29681 | 8.42078    | 3.46694 | 8.84998    | 3.64575 |
| 1.8  | 7.46891    | 2.40704 | 7.84881    | 2.53144 | 8.24834    | 2.66217 | 8.66850    | 2.79956 | 9.11031    | 2.94395 |
| 1.85 | 7.63629    | 1.81839 | 8.02470    | 1.91237 | 8.43318    | 2.01112 | 8.86275    | 2.11491 | 9.31447    | 2.22399 |
| 1.9  | 7.75659    | 1.21852 | 8.15112    | 1.28150 | 8.56604    | 1.34768 | 9.00237    | 1.41723 | 9.46121    | 1.49032 |
| 1.95 | 7.82907    | 0.61115 | 8.22728    | 0.64273 | 8.64608    | 0.67592 | 9.08649    | 0.71081 | 9.54962    | 0.74747 |
| 2.0  | 7.85328    | 0.00    | 8.25273    | 0.00    | 8.67281    | 0.00    | 9.11458    | 0.00    | 9.57915    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(2.95 + i0) = 9.57915 + i0$ .  
 $\cosh(2.8 + i1.2) = -2.55023 + i7.79098$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.0$ | $x = 3.05$ | $x = 3.10$ | $x = 3.15$ | $x = 3.20$ |          |          |          |          |          |
|------|-----------|------------|------------|------------|------------|----------|----------|----------|----------|----------|
| 0    | 10.06766  | 0.00       | 10.58135   | 0.00       | 11.12150   | 0.00     | 11.68946 | 0.00     | 12.28665 | 0.00     |
| 0.05 | 10.03660  | 0.78599    | 10.54870   | 0.82649    | 11.08720   | 0.86905  | 11.65340 | 0.91378  | 12.24880 | 0.96080  |
| 0.1  | 9.94371   | 1.56714    | 10.45110   | 1.64788    | 10.98460   | 1.73274  | 11.54550 | 1.82193  | 12.13540 | 1.91568  |
| 0.15 | 9.78949   | 2.33863    | 10.28900   | 2.45911    | 10.81420   | 2.58575  | 11.36650 | 2.71885  | 11.94720 | 2.85874  |
| 0.2  | 9.57492   | 3.09569    | 10.06350   | 3.25518    | 10.57720   | 3.42281  | 11.11730 | 3.59900  | 11.68530 | 3.78419  |
| 0.25 | 9.30131   | 3.83368    | 9.77589    | 4.03120    | 10.27490   | 4.23878  | 10.79970 | 4.45696  | 11.35140 | 4.68630  |
| 0.3  | 8.97035   | 4.54806    | 9.42805    | 4.78233    | 9.90933    | 5.02860  | 10.41540 | 5.28745  | 10.94750 | 5.55952  |
| 0.35 | 8.58409   | 5.23433    | 9.02209    | 5.50400    | 9.48264    | 5.78743  | 9.96690  | 6.08533  | 10.47610 | 6.39846  |
| 0.4  | 8.14491   | 5.88836    | 8.56049    | 6.19173    | 8.99749    | 6.51058  | 9.45697  | 6.84570  | 9.94011  | 7.19795  |
| 0.45 | 7.65551   | 6.50609    | 8.04612    | 6.84128    | 8.45686    | 7.19358  | 8.88873  | 7.56387  | 9.34284  | 7.95306  |
| 0.5  | 7.11891   | 7.08371    | 7.48215    | 7.44866    | 7.86409    | 7.83223  | 8.26570  | 8.23539  | 8.68797  | 8.65915  |
| 0.55 | 6.53842   | 7.61765    | 6.87204    | 8.01011    | 7.22284    | 8.42260  | 7.59170  | 8.85615  | 7.97954  | 9.31184  |
| 0.6  | 5.91762   | 8.10463    | 6.21956    | 8.52218    | 6.53705    | 8.96104  | 6.87089  | 9.42230  | 7.22191  | 9.90712  |
| 0.65 | 5.26034   | 8.54164    | 5.52874    | 8.98171    | 5.81097    | 9.44423  | 6.10773  | 9.93036  | 6.41976  | 10.44130 |
| 0.7  | 4.57062   | 8.92599    | 4.80383    | 9.38586    | 5.04906    | 9.86919  | 5.30690  | 10.37720 | 5.57802  | 10.91120 |
| 0.75 | 3.85273   | 9.25531    | 4.04931    | 9.73216    | 4.25602    | 10.23330 | 4.47336  | 10.76010 | 4.70190  | 11.31370 |
| 0.8  | 3.11108   | 9.52757    | 3.26982    | 10.01840   | 3.43673    | 10.53430 | 3.61224  | 11.07660 | 3.79678  | 11.64650 |
| 0.85 | 2.35025   | 9.74108    | 2.47017    | 10.24290   | 2.59626    | 10.77040 | 2.72885  | 11.32480 | 2.86826  | 11.90750 |
| 0.9  | 1.57493   | 9.89454    | 1.65529    | 10.40430   | 1.73979    | 10.94010 | 1.82863  | 11.50320 | 1.92205  | 12.09510 |
| 0.95 | 0.78990   | 9.98699    | 0.83020    | 10.50150   | 0.87258    | 11.04230 | 0.91714  | 11.61070 | 0.96400  | 12.20810 |
| 1.0  | 0.00      | 10.01787   | 0.00       | 10.53399   | 0.00       | 11.07645 | 0.00     | 11.64661 | 0.00     | 12.24588 |
| 1.05 | 0.78990   | 9.98699    | 0.83020    | 10.50150   | 0.87258    | 11.04230 | 0.91714  | 11.61070 | 0.96400  | 12.20810 |
| 1.1  | 1.57493   | 9.89454    | 1.65529    | 10.40430   | 1.73979    | 10.94010 | 1.82863  | 11.50320 | 1.92205  | 12.09510 |
| 1.15 | 2.35025   | 9.74108    | 2.47017    | 10.24290   | 2.59626    | 10.77040 | 2.72885  | 11.32480 | 2.86826  | 11.90750 |
| 1.2  | 3.11108   | 9.52757    | 3.26982    | 10.01840   | 3.43673    | 10.53430 | 3.61224  | 11.07660 | 3.79678  | 11.64650 |
| 1.25 | 3.85273   | 9.25531    | 4.04931    | 9.73216    | 4.25602    | 10.23330 | 4.47336  | 10.76010 | 4.70190  | 11.31370 |
| 1.3  | 4.57062   | 8.92599    | 4.80383    | 9.38586    | 5.04906    | 9.86919  | 5.30690  | 10.37720 | 5.57802  | 10.91120 |
| 1.35 | 5.26034   | 8.54164    | 5.52874    | 8.98171    | 5.81097    | 9.44423  | 6.10773  | 9.93036  | 6.41976  | 10.44130 |
| 1.4  | 5.91762   | 8.10463    | 6.21956    | 8.52218    | 6.53705    | 8.96104  | 6.87089  | 9.42230  | 7.22191  | 9.90712  |
| 1.45 | 6.53842   | 7.61765    | 6.87204    | 8.01011    | 7.22284    | 8.42260  | 7.59170  | 8.85615  | 7.97954  | 9.31184  |
| 1.5  | 7.11891   | 7.08371    | 7.48215    | 7.44866    | 7.86409    | 7.83223  | 8.26570  | 8.23539  | 8.68797  | 8.65915  |
| 1.55 | 7.65551   | 6.50609    | 8.04612    | 6.84128    | 8.45686    | 7.19358  | 8.88873  | 7.56387  | 9.34284  | 7.95306  |
| 1.6  | 8.14491   | 5.88836    | 8.56049    | 6.19173    | 8.99749    | 6.51058  | 9.45697  | 6.84570  | 9.94011  | 7.19795  |
| 1.65 | 8.58409   | 5.23433    | 9.02209    | 5.50400    | 9.48264    | 5.78743  | 9.96690  | 6.08533  | 10.47610 | 6.39846  |
| 1.7  | 8.97035   | 4.54806    | 9.42805    | 4.78233    | 9.90933    | 5.02860  | 10.41540 | 5.28745  | 10.94750 | 5.55952  |
| 1.75 | 9.30131   | 3.83368    | 9.77589    | 4.03120    | 10.27490   | 4.23878  | 10.79970 | 4.45696  | 11.35140 | 4.68630  |
| 1.8  | 9.57492   | 3.09569    | 10.06350   | 3.25518    | 10.57720   | 3.42281  | 11.11730 | 3.59900  | 11.68530 | 3.78419  |
| 1.85 | 9.78949   | 2.33863    | 10.28900   | 2.45911    | 10.81420   | 2.58575  | 11.36650 | 2.71885  | 11.94720 | 2.85874  |
| 1.9  | 9.94371   | 1.56714    | 10.45110   | 1.64788    | 10.98460   | 1.73274  | 11.54550 | 1.82193  | 12.13540 | 1.91568  |
| 1.95 | 10.03660  | 0.78599    | 10.54870   | 0.82649    | 11.08720   | 0.86905  | 11.65340 | 0.91378  | 12.24880 | 0.96080  |
| 2.0  | 10.06766  | 0.00       | 10.58135   | 0.00       | 11.12150   | 0.00     | 11.68946 | 0.00     | 12.28665 | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(3.10 + i 0.5) = 7.86409 + i 7.83223$ .  
 $\cosh(3.10 + i 1.55) = -8.45686 + i 7.19358$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.25$ |          | $x = 3.30$ |          | $x = 3.35$ |          | $x = 3.40$ |          | $x = 3.45$ |          |
|------|------------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| 0    | 12.91456   | 0.00     | 13.57476   | 0.00     | 14.26891   | 0.00     | 14.99874   | 0.00     | 15.76607   | 0.00     |
| 0.05 | 12.87474   | 1.01022  | 13.53290   | 1.06217  | 14.22498   | 1.11677  | 14.95250   | 1.17417  | 15.71746   | 1.23450  |
| 0.1  | 12.75555   | 2.01422  | 13.40764   | 2.11780  | 14.09323   | 2.22666  | 14.81410   | 2.34110  | 15.57196   | 2.46139  |
| 0.15 | 12.55773   | 3.00579  | 13.19970   | 3.16036  | 13.87465   | 3.32282  | 14.58432   | 3.49359  | 15.33045   | 3.67311  |
| 0.2  | 12.28247   | 3.97883  | 12.91035   | 4.18343  | 13.57054   | 4.39850  | 14.26405   | 4.62455  | 14.99442   | 4.86217  |
| 0.25 | 11.93150   | 4.92735  | 12.54145   | 5.18072  | 13.18275   | 5.44705  | 13.85702   | 5.72700  | 14.56595   | 6.02127  |
| 0.3  | 11.50695   | 5.84548  | 12.09520   | 6.14670  | 12.71369   | 6.46202  | 13.36397   | 6.79414  | 14.04764   | 7.14324  |
| 0.35 | 11.01147   | 6.72758  | 11.57440   | 7.07352  | 12.16623   | 7.43715  | 12.78852   | 7.81938  | 13.44278   | 8.22116  |
| 0.4  | 10.44810   | 7.56820  | 10.98222   | 7.95737  | 11.54379   | 8.36643  | 12.13423   | 8.79642  | 12.75501   | 9.24840  |
| 0.45 | 9.82031    | 8.36217  | 10.32233   | 8.79215  | 10.85016   | 9.24413  | 11.40512   | 9.71923  | 11.98861   | 10.21863 |
| 0.5  | 9.13197    | 9.10456  | 9.59881    | 9.57273  | 10.08964   | 10.06483 | 10.60570   | 10.58212 | 11.14830   | 11.12585 |
| 0.55 | 8.38733    | 9.79082  | 8.81610    | 10.29428 | 9.26691    | 10.82349 | 9.74090    | 11.37975 | 10.23924   | 11.96447 |
| 0.6  | 7.59099    | 10.41673 | 7.97905    | 10.95235 | 8.38705    | 11.51541 | 8.81604    | 12.10723 | 9.26706    | 12.72933 |
| 0.65 | 6.74784    | 10.97841 | 7.09279    | 11.54294 | 7.45549    | 12.13633 | 7.83682    | 12.76006 | 8.23775    | 13.41572 |
| 0.7  | 5.86309    | 11.47240 | 6.16381    | 12.06234 | 6.47795    | 12.68244 | 6.80928    | 13.33423 | 7.15765    | 14.01940 |
| 0.75 | 4.94218    | 11.89566 | 5.19485    | 12.50736 | 5.46075    | 13.15033 | 5.73977    | 13.82620 | 6.03341    | 14.53662 |
| 0.8  | 3.99082    | 12.24560 | 4.19483    | 12.87530 | 4.40933    | 13.53717 | 4.63487    | 14.23291 | 4.87198    | 14.96423 |
| 0.85 | 3.01484    | 12.52002 | 3.16897    | 13.16388 | 3.33101    | 13.84054 | 3.50139    | 14.55188 | 3.68053    | 15.29960 |
| 0.9  | 2.02028    | 12.71726 | 2.21356    | 13.37120 | 2.32115    | 14.05851 | 2.34632    | 14.78111 | 2.46636    | 15.54061 |
| 0.95 | 1.01326    | 12.83610 | 1.06507    | 13.49615 | 1.11953    | 14.18950 | 1.17679    | 14.91923 | 1.23699    | 15.68581 |
| 1.0  | 0.00       | 12.87578 | 0.00       | 13.53788 | 0.00       | 14.23382 | 0.00       | 14.96536 | 0.00       | 15.73432 |
| 1.05 | 1.01326    | 12.83610 | 1.06507    | 13.49615 | 1.11953    | 14.18950 | 1.17679    | 14.91923 | 1.23699    | 15.68581 |
| 1.1  | 2.02028    | 12.71726 | 2.12356    | 13.37120 | 2.23215    | 14.05851 | 2.34632    | 14.78111 | 2.46636    | 15.54061 |
| 1.15 | 3.01484    | 12.52002 | 3.16897    | 13.16388 | 3.33101    | 13.84054 | 3.50139    | 14.55188 | 3.68053    | 15.29960 |
| 1.2  | 3.99082    | 12.24560 | 4.19483    | 12.87530 | 4.40933    | 13.53717 | 4.63487    | 14.23291 | 4.87198    | 14.96423 |
| 1.25 | 4.94218    | 11.89566 | 5.19485    | 12.50736 | 5.46075    | 13.15033 | 5.73977    | 13.82620 | 6.03341    | 14.53662 |
| 1.3  | 5.86309    | 11.47240 | 6.16381    | 12.06234 | 6.47795    | 12.68244 | 6.80928    | 13.33423 | 7.15765    | 14.01940 |
| 1.35 | 6.74784    | 10.97841 | 7.09279    | 11.54294 | 7.45549    | 12.13633 | 7.83682    | 12.76006 | 8.23775    | 13.41572 |
| 1.4  | 7.59099    | 10.41673 | 7.97905    | 10.95235 | 8.38705    | 11.51541 | 8.81604    | 12.10723 | 9.26706    | 12.72933 |
| 1.45 | 8.38733    | 9.79082  | 8.81610    | 10.29428 | 9.26691    | 10.82349 | 9.74090    | 11.37975 | 10.23924   | 11.96447 |
| 1.5  | 9.13197    | 9.10456  | 9.59881    | 9.57273  | 10.08964   | 10.06483 | 10.60570   | 10.58212 | 11.14830   | 11.12585 |
| 1.55 | 9.82031    | 8.36217  | 10.32233   | 8.79215  | 10.85016   | 9.24413  | 11.40512   | 9.71923  | 11.98861   | 10.21863 |
| 1.6  | 10.44810   | 7.56820  | 10.98222   | 7.95737  | 11.54379   | 8.36643  | 12.13423   | 8.79642  | 12.75501   | 9.24840  |
| 1.65 | 11.01147   | 6.72758  | 11.57440   | 7.07352  | 12.16623   | 7.43715  | 12.78852   | 7.81938  | 13.44278   | 8.22116  |
| 1.7  | 11.50695   | 5.84548  | 12.09520   | 6.14670  | 12.71369   | 6.46202  | 13.36397   | 6.79414  | 14.04764   | 7.14324  |
| 1.75 | 11.93150   | 4.92735  | 12.54145   | 5.18072  | 13.18275   | 5.44705  | 13.85702   | 5.72700  | 14.56595   | 6.02127  |
| 1.8  | 12.28247   | 3.97883  | 12.91035   | 4.18343  | 13.57054   | 4.39850  | 14.26465   | 4.62455  | 14.99442   | 4.86217  |
| 1.85 | 12.55773   | 3.00579  | 13.19970   | 3.16036  | 13.87465   | 3.32282  | 14.58432   | 3.49359  | 15.33045   | 3.67311  |
| 1.9  | 12.75555   | 2.01422  | 13.40764   | 2.11780  | 14.09323   | 2.22666  | 14.81410   | 2.34110  | 15.57196   | 2.46139  |
| 1.95 | 12.87474   | 1.01022  | 13.53290   | 1.06217  | 14.22498   | 1.11677  | 14.95250   | 1.17417  | 15.71746   | 1.23450  |
| 2.0  | 12.91456   | 0.00     | 13.57476   | 0.00     | 14.26891   | 0.00     | 14.99874   | 0.00     | 15.76607   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(3.45 + i0.05) = 15.71746 + i1.23450$ .  
 $\cosh(3.25 + i1.95) = -12.87474 + i1.01022$ .



TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.50$ |          | $x = 3.55$ |          | $x = 3.60$ |          | $x = 3.65$ |          | $x = 3.70$ |          |
|------|------------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| 0    | 16.57282   | 0.00     | 17.42102   | 0.00     | 18.31278   | 0.00     | 19.25033   | 0.00     | 20.23601   | 0.00     |
| 0.05 | 16.52173   | 1.29792  | 17.36731   | 1.36458  | 18.25632   | 1.43466  | 19.19100   | 1.50832  | 20.17362   | 1.58576  |
| 0.1  | 16.36878   | 2.58783  | 17.20653   | 2.72075  | 18.08732   | 2.86047  | 19.01331   | 3.00735  | 19.98688   | 3.16174  |
| 0.15 | 16.11491   | 3.86180  | 16.93970   | 4.06015  | 17.80680   | 4.26865  | 18.71843   | 4.48783  | 19.67690   | 4.71823  |
| 0.2  | 15.76170   | 5.11195  | 16.56840   | 5.37451  | 17.41650   | 5.65052  | 18.30814   | 5.94065  | 19.24560   | 6.24563  |
| 0.25 | 15.31130   | 6.33059  | 16.09492   | 6.65574  | 16.91880   | 6.99754  | 17.78497   | 7.35683  | 18.69565   | 7.73453  |
| 0.3  | 14.76650   | 7.51020  | 15.52225   | 7.89594  | 16.31680   | 8.30143  | 17.15216   | 8.72766  | 18.03040   | 9.17573  |
| 0.35 | 14.13065   | 8.64360  | 14.85386   | 9.08745  | 15.61421   | 9.55412  | 16.41360   | 10.04469 | 17.25404   | 10.56037 |
| 0.4  | 13.40770   | 9.72361  | 14.09390   | 10.22293 | 14.81535   | 10.74789 | 15.57383   | 11.29972 | 16.37127   | 11.87990 |
| 0.45 | 12.60210   | 10.74357 | 13.24705   | 11.29540 | 13.92515   | 11.87545 | 14.63806   | 12.48520 | 15.38760   | 13.12620 |
| 0.5  | 11.71820   | 11.69740 | 12.31852   | 12.29820 | 12.94910   | 12.92978 | 13.61203   | 13.59365 | 14.30902   | 14.29155 |
| 0.55 | 10.76320   | 12.57910 | 11.31405   | 13.22520 | 11.89320   | 13.90436 | 12.50208   | 14.61830 | 13.14223   | 15.36878 |
| 0.6  | 9.74126    | 13.38330 | 10.23982   | 14.07066 | 10.76400   | 14.79326 | 11.31505   | 15.55281 | 11.89444   | 16.35127 |
| 0.65 | 8.65928    | 14.10490 | 9.10246    | 14.82933 | 9.56840    | 15.59090 | 10.05827   | 16.39143 | 10.57330   | 17.23295 |
| 0.7  | 7.52391    | 14.73960 | 7.90898    | 15.49665 | 8.31383    | 16.29246 | 8.73946    | 17.12900 | 9.18696    | 18.00840 |
| 0.75 | 6.34215    | 15.28340 | 6.66674    | 16.06840 | 7.00800    | 16.89356 | 7.36678    | 17.76096 | 7.74399    | 18.67280 |
| 0.8  | 5.12129    | 15.73300 | 5.38339    | 16.54105 | 5.65896    | 17.39050 | 5.94868    | 18.28342 | 6.25327    | 19.22208 |
| 0.85 | 3.86885    | 16.08555 | 4.06686    | 16.91175 | 4.27503    | 17.78022 | 4.49390    | 18.69316 | 4.72401    | 19.65290 |
| 0.9  | 2.59256    | 16.33896 | 2.72525    | 17.17817 | 2.86475    | 18.06033 | 3.01142    | 18.98765 | 3.16561    | 19.96246 |
| 0.95 | 1.30029    | 16.49163 | 1.36684    | 17.33870 | 1.43680    | 18.22900 | 1.51036    | 19.16506 | 1.58770    | 20.14900 |
| 1.0  | 0.00       | 16.54263 | 0.00       | 17.39230 | 0.00       | 18.28546 | 0.00       | 19.22434 | 0.00       | 20.21129 |
| 1.05 | 1.30029    | 16.49163 | 1.36684    | 17.33870 | 1.43680    | 18.22900 | 1.51036    | 19.16506 | 1.58770    | 20.14900 |
| 1.1  | 2.59256    | 16.33896 | 2.72525    | 17.17817 | 2.86475    | 18.06033 | 3.01142    | 18.98765 | 3.16561    | 19.96246 |
| 1.15 | 3.86885    | 16.08555 | 4.06686    | 16.91175 | 4.27503    | 17.78022 | 4.49390    | 18.69316 | 4.72401    | 19.65290 |
| 1.2  | 5.12129    | 15.73300 | 5.38339    | 16.54105 | 5.65896    | 17.39050 | 5.94868    | 18.28342 | 6.25327    | 19.22208 |
| 1.25 | 6.34215    | 15.28340 | 6.66674    | 16.06840 | 7.00800    | 16.89356 | 7.36678    | 17.76096 | 7.74399    | 18.67280 |
| 1.3  | 7.52391    | 14.73960 | 7.90898    | 15.49665 | 8.31383    | 16.29246 | 8.73946    | 17.12900 | 9.18696    | 18.00840 |
| 1.35 | 8.65928    | 14.10490 | 9.10246    | 14.82933 | 9.56840    | 15.59090 | 10.05827   | 16.39143 | 10.57330   | 17.23295 |
| 1.4  | 9.74126    | 13.38330 | 10.23982   | 14.07066 | 10.76400   | 14.79326 | 11.31505   | 15.55281 | 11.89444   | 16.35127 |
| 1.45 | 10.76320   | 12.57910 | 11.31405   | 13.22520 | 11.89320   | 13.90436 | 12.50208   | 14.61830 | 13.14223   | 15.36878 |
| 1.5  | 11.71820   | 11.69740 | 12.31852   | 12.29820 | 12.94910   | 12.92978 | 13.61203   | 13.59365 | 14.30902   | 14.29155 |
| 1.55 | 12.60210   | 10.74357 | 13.24705   | 11.29540 | 13.92515   | 11.87545 | 14.63806   | 12.48520 | 15.38760   | 13.12620 |
| 1.6  | 13.40770   | 9.72361  | 14.09390   | 10.22293 | 14.81535   | 10.74789 | 15.57383   | 11.29972 | 16.37127   | 11.87990 |
| 1.65 | 14.13065   | 8.64360  | 14.85386   | 9.08745  | 15.61421   | 9.55412  | 16.41360   | 10.04469 | 17.25404   | 10.56037 |
| 1.7  | 14.76650   | 7.51020  | 15.52225   | 7.89594  | 16.31680   | 8.30143  | 17.15216   | 8.72766  | 18.03040   | 9.17573  |
| 1.75 | 15.31130   | 6.33059  | 16.09492   | 6.65574  | 16.91880   | 6.99754  | 17.78497   | 7.35683  | 18.69565   | 7.73453  |
| 1.8  | 15.76170   | 5.11195  | 16.56840   | 5.37451  | 17.41650   | 5.65052  | 18.30814   | 5.94065  | 19.24560   | 6.24563  |
| 1.85 | 16.11491   | 3.86180  | 16.93970   | 4.06015  | 17.80680   | 4.26865  | 18.71843   | 4.48783  | 19.67690   | 4.71823  |
| 1.9  | 16.36878   | 2.58783  | 17.20653   | 2.72075  | 18.08732   | 2.86047  | 19.01331   | 3.00735  | 19.98688   | 3.16174  |
| 1.95 | 16.52173   | 1.29792  | 17.36731   | 1.36458  | 18.25632   | 1.43466  | 19.19100   | 1.50832  | 20.17362   | 1.58576  |
| 2.0  | 16.57282   | 0.00     | 17.42102   | 0.00     | 18.31278   | 0.00     | 19.25033   | 0.00     | 20.23601   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(3.50 + i 0.70) = 7.52391 + i 14.73960$ .  
 $\cosh(3.60 + i 1.55) = -13.92515 + i 11.87545$ .

TABLE VIII. HYPERBOLIC COSINES.  $\cosh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.75$ |          | $x = 3.80$ |          | $x = 3.85$ |          | $x = 3.90$ |          | $x = 3.95$ |          |
|------|------------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| 0    | 21.27230   | 0.00     | 22.36178   | 0.00     | 23.50717   | 0.00     | 24.71135   | 0.00     | 25.97731   | 0.00     |
| 0.05 | 21.20670   | 1.66716  | 22.29283   | 1.75273  | 23.43470   | 1.84268  | 24.63516   | 1.93724  | 25.89724   | 2.03665  |
| 0.1  | 21.01038   | 3.32404  | 22.08646   | 3.49465  | 23.21775   | 3.67400  | 24.40710   | 3.86254  | 25.65749   | 4.06074  |
| 0.15 | 20.68452   | 4.96043  | 21.74391   | 5.21503  | 22.85766   | 5.48267  | 24.02856   | 5.76402  | 25.25957   | 6.05979  |
| 0.2  | 20.23113   | 6.56624  | 21.26731   | 6.90325  | 22.35664   | 7.25754  | 23.50188   | 7.62997  | 24.70590   | 8.02149  |
| 0.25 | 19.65301   | 8.13156  | 20.65958   | 8.54892  | 21.71778   | 8.98766  | 22.83030   | 9.44887  | 23.99991   | 9.93373  |
| 0.3  | 18.95373   | 9.64674  | 19.92448   | 10.14188 | 20.94503   | 10.66233 | 22.01797   | 11.20952 | 23.14597   | 11.78474 |
| 0.35 | 18.13760   | 11.10246 | 19.06655   | 11.67230 | 20.04315   | 12.27134 | 21.06988   | 12.90106 | 22.14931   | 13.56305 |
| 0.4  | 17.20963   | 12.48971 | 18.09105   | 13.13076 | 19.01770   | 13.80465 | 19.99190   | 14.51307 | 21.01610   | 15.25776 |
| 0.45 | 16.17556   | 13.79998 | 17.00402   | 14.50828 | 17.87500   | 15.25286 | 18.79065   | 16.03558 | 19.75331   | 16.85842 |
| 0.5  | 15.04177   | 15.02516 | 15.81216   | 15.79634 | 16.62208   | 16.60702 | 17.47355   | 17.45924 | 18.36873   | 18.35512 |
| 0.55 | 13.81524   | 16.15770 | 14.52281   | 16.98701 | 15.26668   | 17.85880 | 16.04874   | 18.77526 | 16.87098   | 19.73867 |
| 0.6  | 12.50353   | 17.19062 | 13.14392   | 18.07296 | 13.81716   | 19.00048 | 14.52497   | 19.97556 | 15.26910   | 21.00052 |
| 0.65 | 11.11473   | 18.11756 | 11.68400   | 19.04746 | 12.28246   | 20.02501 | 12.91164   | 21.05272 | 13.57315   | 22.13290 |
| 0.7  | 9.65741    | 18.93280 | 10.15203   | 19.90455 | 10.67203   | 20.92608 | 11.21871   | 21.99993 | 11.79366   | 23.12881 |
| 0.75 | 8.14055    | 19.63131 | 8.55748    | 20.63891 | 8.99580    | 21.69813 | 9.45662    | 22.81160 | 9.94109    | 23.98212 |
| 0.8  | 6.57350    | 20.20879 | 6.91017    | 21.24603 | 7.26411    | 22.33640 | 7.63623    | 23.48262 | 8.02744    | 24.68760 |
| 0.85 | 4.96592    | 20.66167 | 5.22025    | 21.72216 | 5.48764    | 22.83696 | 5.76875    | 24.00888 | 6.06429    | 25.24084 |
| 0.9  | 3.32772    | 20.98716 | 3.49815    | 22.06437 | 3.67733    | 23.19673 | 3.86571    | 24.38710 | 4.06375    | 25.63849 |
| 0.95 | 1.66900    | 21.18327 | 1.75448    | 22.27052 | 1.84435    | 23.41348 | 1.93883    | 24.61500 | 2.03816    | 25.87805 |
| 1.0  | 0.00       | 21.24878 | 0.00       | 22.33941 | 0.00       | 23.48589 | 0.00       | 24.69110 | 0.00       | 25.95806 |
| 1.05 | 1.66900    | 21.18327 | 1.75448    | 22.27052 | 1.84435    | 23.41348 | 1.93883    | 24.61500 | 2.03816    | 25.87805 |
| 1.1  | 3.32772    | 20.98716 | 3.49815    | 22.06437 | 3.67733    | 23.19673 | 3.86571    | 24.38710 | 4.06375    | 25.63849 |
| 1.15 | 4.96592    | 20.66167 | 5.22025    | 21.72216 | 5.48764    | 22.83696 | 5.76875    | 24.00888 | 6.06429    | 25.24084 |
| 1.2  | 6.57350    | 20.20879 | 6.91017    | 21.24603 | 7.26411    | 22.33640 | 7.63623    | 23.48262 | 8.02744    | 24.68760 |
| 1.25 | 8.14055    | 19.63131 | 8.55748    | 20.63891 | 8.99580    | 21.69813 | 9.45662    | 22.81160 | 9.94109    | 23.98212 |
| 1.3  | 9.65741    | 18.93280 | 10.15203   | 19.90455 | 10.67203   | 20.92608 | 11.21871   | 21.99993 | 11.79366   | 23.12881 |
| 1.35 | 11.11473   | 18.11756 | 11.68400   | 19.04746 | 12.28246   | 20.02501 | 12.91164   | 21.05272 | 13.57315   | 22.13290 |
| 1.4  | 12.50353   | 17.19062 | 13.14392   | 18.07296 | 13.81716   | 19.00048 | 14.52497   | 19.97556 | 15.26910   | 21.00052 |
| 1.45 | 13.81524   | 16.15770 | 14.52281   | 16.98701 | 15.26668   | 17.85880 | 16.04874   | 18.77526 | 16.87098   | 19.73867 |
| 1.5  | 15.04177   | 15.02516 | 15.81216   | 15.79634 | 16.62208   | 16.60702 | 17.47355   | 17.45924 | 18.36873   | 18.35512 |
| 1.55 | 16.17556   | 13.79998 | 17.00402   | 14.50828 | 17.87500   | 15.25286 | 18.79065   | 16.03558 | 19.75331   | 16.85842 |
| 1.6  | 17.20963   | 12.48971 | 18.09105   | 13.13076 | 19.01770   | 13.80465 | 19.99190   | 14.51307 | 21.01610   | 15.25776 |
| 1.65 | 18.13760   | 11.10246 | 19.06655   | 11.67230 | 20.04315   | 12.27134 | 21.06988   | 12.90106 | 22.14931   | 13.56305 |
| 1.7  | 18.95373   | 9.64674  | 19.92448   | 10.14188 | 20.94503   | 10.66233 | 22.01797   | 11.20952 | 23.14597   | 11.78474 |
| 1.75 | 19.65301   | 8.13156  | 20.65958   | 8.54892  | 21.71778   | 8.98766  | 22.83030   | 9.44887  | 23.99991   | 9.93373  |
| 1.8  | 20.23113   | 6.56624  | 21.26731   | 6.90325  | 22.35664   | 7.25754  | 23.50188   | 7.62997  | 24.70590   | 8.02149  |
| 1.85 | 20.68452   | 4.96043  | 21.74391   | 5.21503  | 22.85766   | 5.48267  | 24.02856   | 5.76402  | 25.25957   | 6.05979  |
| 1.9  | 21.01038   | 3.32404  | 22.08646   | 3.49465  | 23.21775   | 3.67400  | 24.40710   | 3.86254  | 25.65749   | 4.06074  |
| 1.95 | 21.20670   | 1.66716  | 22.29283   | 1.75273  | 23.43470   | 1.84268  | 24.63516   | 1.93724  | 25.89724   | 2.03665  |
| 2.0  | 21.27230   | 0.00     | 22.36178   | 0.00     | 23.50717   | 0.00     | 24.71135   | 0.00     | 25.97731   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\cosh(3.90 + i 0.25) = 22.83030 + i 9.44887$ .  
 $\cosh(3.75 + i 1.25) = -8.14055 + i 19.63131$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$

| $q$  | $x = 0$ |          | $x = 0.05$ |         | $x = 0.1$ |         | $x = 0.15$ |          | $x = 0.2$ |         |
|------|---------|----------|------------|---------|-----------|---------|------------|----------|-----------|---------|
| 0    | 0.00    | 0.00     | 0.04996    | 0.00    | 0.09967   | 0.00    | 0.14889    | 0.00     | 0.19738   | 0.00    |
| 0.05 | 0.00    | 0.07870  | 0.05027    | 0.07850 | 0.10028   | 0.07792 | 0.14979    | 0.07695  | 0.19855   | 0.07562 |
| 0.1  | 0.00    | 0.15838  | 0.05121    | 0.15798 | 0.10214   | 0.15677 | 0.15254    | 0.15479  | 0.20213   | 0.15207 |
| 0.15 | 0.00    | 0.24008  | 0.05283    | 0.23944 | 0.10535   | 0.23755 | 0.15727    | 0.23446* | 0.20828   | 0.23021 |
| 0.2  | 0.00    | 0.32492  | 0.05522    | 0.32402 | 0.11008   | 0.32136 | 0.16422    | 0.31698  | 0.21732   | 0.31098 |
| 0.25 | 0.00    | 0.41421  | 0.05850    | 0.41300 | 0.11657   | 0.40940 | 0.17377    | 0.40350  | 0.22970   | 0.39543 |
| 0.3  | 0.00    | 0.50953  | 0.06289    | 0.50792 | 0.12522   | 0.50317 | 0.18647    | 0.49538  | 0.24613   | 0.48477 |
| 0.35 | 0.00    | 0.61280  | 0.06865    | 0.61070 | 0.13659   | 0.60446 | 0.20310    | 0.59427  | 0.26758   | 0.58044 |
| 0.4  | 0.00    | 0.72654  | 0.07623    | 0.72378 | 0.15149   | 0.71557 | 0.22484    | 0.70222  | 0.29549   | 0.68417 |
| 0.45 | 0.00    | 0.85408  | 0.08624    | 0.85040 | 0.17113   | 0.83951 | 0.25339    | 0.82186  | 0.33192   | 0.79813 |
| 0.5  | 0.00    | 1.0000   | 0.09967    | 0.99503 | 0.19738   | 0.98033 | 0.29131    | 0.95663  | 0.37995   | 0.92501 |
| 0.55 | 0.00    | 1.17085  | 0.11804    | 1.16395 | 0.23313   | 1.14365 | 0.34258    | 1.11113  | 0.44423   | 1.06819 |
| 0.6  | 0.00    | 1.37638  | 0.14392    | 1.36649 | 0.28315   | 1.33754 | 0.41357    | 1.29164  | 0.53203   | 1.23185 |
| 0.65 | 0.00    | 1.63185  | 0.18179    | 1.61702 | 0.35567   | 1.57401 | 0.51496    | 1.50674  | 0.65502   | 1.42088 |
| 0.7  | 0.00    | 1.96261  | 0.24007    | 1.93900 | 0.46575   | 1.87150 | 0.66202    | 1.75880  | 0.83268   | 1.64005 |
| 0.75 | 0.00    | 2.41421  | 0.33624    | 2.37365 | 0.64333   | 2.25941 | 0.90034    | 2.09061  | 1.09837   | 1.89083 |
| 0.8  | 0.00    | 3.07768  | 0.51109    | 2.99911 | 0.95397   | 2.78504 | 1.28858    | 2.48723  | 1.50982   | 2.16055 |
| 0.85 | 0.00    | 4.16530  | 0.87867    | 3.98246 | 1.56000   | 3.51765 | 1.97316    | 2.94167  | 2.16111   | 2.38860 |
| 0.9  | 0.00    | 6.31375  | 1.85674    | 5.72808 | 2.91746   | 4.47780 | 3.22989    | 3.27758  | 3.15925   | 2.37676 |
| 0.95 | 0.00    | 12.70620 | 5.79801    | 9.05499 | 6.21808   | 4.83133 | 5.28217    | 2.71349  | 4.39854   | 1.67517 |
| 1.0  | 0.00    | $\infty$ | 20.01667   | 0.00    | 10.03331  | 0.00    | 6.71659    | 0.00     | 5.06649   | 0.00    |
| 1.05 | 0.00    | 12.70620 | 5.79801    | 9.05499 | 6.21808   | 4.83133 | 5.28217    | 2.71349  | 4.39854   | 1.67517 |
| 1.1  | 0.00    | 6.31375  | 1.85674    | 5.72808 | 2.91746   | 4.47780 | 3.22989    | 3.27758  | 3.15925   | 2.37676 |
| 1.15 | 0.00    | 4.16530  | 0.87867    | 3.98246 | 1.56000   | 3.51765 | 1.97316    | 2.94167  | 2.16111   | 2.38860 |
| 1.2  | 0.00    | 3.07768  | 0.51109    | 2.99911 | 0.95397   | 2.78504 | 1.28858    | 2.48723  | 1.50982   | 2.16055 |
| 1.25 | 0.00    | 2.41421  | 0.33624    | 2.37365 | 0.64333   | 2.25941 | 0.90034    | 2.09061  | 1.09837   | 1.89083 |
| 1.3  | 0.00    | 1.96261  | 0.24007    | 1.93900 | 0.46575   | 1.87150 | 0.66202    | 1.75880  | 0.83268   | 1.64005 |
| 1.35 | 0.00    | 1.63185  | 0.18179    | 1.61702 | 0.35567   | 1.57401 | 0.51496    | 1.50674  | 0.65502   | 1.42088 |
| 1.4  | 0.00    | 1.37638  | 0.14392    | 1.36649 | 0.28315   | 1.33754 | 0.41357    | 1.29164  | 0.53203   | 1.23185 |
| 1.45 | 0.00    | 1.17085  | 0.11804    | 1.16395 | 0.23313   | 1.14365 | 0.34258    | 1.11113  | 0.44423   | 1.06819 |
| 1.5  | 0.00    | 1.0000   | 0.09967    | 0.99503 | 0.19738   | 0.98033 | 0.29131    | 0.95663  | 0.37995   | 0.92501 |
| 1.55 | 0.00    | 0.85408  | 0.08624    | 0.85040 | 0.17113   | 0.83951 | 0.25339    | 0.82186  | 0.33192   | 0.79813 |
| 1.6  | 0.00    | 0.72654  | 0.07623    | 0.72378 | 0.15149   | 0.71557 | 0.22484    | 0.70222  | 0.29549   | 0.68417 |
| 1.65 | 0.00    | 0.61280  | 0.06865    | 0.61070 | 0.13659   | 0.60446 | 0.20310    | 0.59427  | 0.26758   | 0.58044 |
| 1.7  | 0.00    | 0.50953  | 0.06289    | 0.50792 | 0.12522   | 0.50317 | 0.18647    | 0.49538  | 0.24613   | 0.48477 |
| 1.75 | 0.00    | 0.41421  | 0.05850    | 0.41300 | 0.11657   | 0.40940 | 0.17377    | 0.40350  | 0.22970   | 0.39543 |
| 1.8  | 0.00    | 0.32492  | 0.05522    | 0.32402 | 0.11008   | 0.32136 | 0.16422    | 0.31698  | 0.21732   | 0.31098 |
| 1.85 | 0.00    | 0.24008  | 0.05283    | 0.23944 | 0.10535   | 0.23755 | 0.15727    | 0.23446  | 0.20828   | 0.23021 |
| 1.9  | 0.00    | 0.15838  | 0.05121    | 0.15798 | 0.10214   | 0.15677 | 0.15254    | 0.15479  | 0.20213   | 0.15207 |
| 1.95 | 0.00    | 0.07870  | 0.05027    | 0.07850 | 0.10028   | 0.07792 | 0.14979    | 0.07695  | 0.19855   | 0.07562 |
| 2.0  | 0.00    | 0.00     | 0.04996    | 0.00    | 0.09967   | 0.00    | 0.14889    | 0.00     | 0.19738   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(0 + i \underline{0.95}) = 0 + i \underline{12.70620}$ .  
 $\tanh(0 + i \underline{1.45}) = 0 - i \underline{1.17085}$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 0.25$ |         | $x = 0.3$ |         | $x = 0.35$ |         | $x = 0.4$ |         | $x = 0.45$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 0.24492    | 0.00    | 0.29131   | 0.00    | 0.33638    | 0.00    | 0.37995   | 0.00    | 0.42190    | 0.00    |
| 0.05 | 0.24635    | 0.07395 | 0.29296   | 0.07199 | 0.33322    | 0.06975 | 0.38196   | 0.06728 | 0.42405    | 0.06462 |
| 0.1  | 0.25069    | 0.14866 | 0.29799   | 0.14464 | 0.34384    | 0.14007 | 0.38808   | 0.13503 | 0.43056    | 0.12961 |
| 0.15 | 0.25814    | 0.22490 | 0.30660   | 0.21864 | 0.35346    | 0.21153 | 0.39853   | 0.20373 | 0.44169    | 0.19534 |
| 0.2  | 0.26907    | 0.30351 | 0.31921   | 0.29471 | 0.36750    | 0.28475 | 0.41376   | 0.27384 | 0.45784    | 0.26216 |
| 0.25 | 0.28402    | 0.38540 | 0.33640   | 0.37362 | 0.38658    | 0.36035 | 0.43438   | 0.34585 | 0.47964    | 0.33039 |
| 0.3  | 0.30377    | 0.47162 | 0.35903   | 0.45623 | 0.41161    | 0.43898 | 0.46130   | 0.42022 | 0.50796    | 0.40033 |
| 0.35 | 0.32947    | 0.56335 | 0.38833   | 0.54348 | 0.44383    | 0.52131 | 0.49575   | 0.49737 | 0.54397    | 0.47216 |
| 0.4  | 0.36272    | 0.66200 | 0.42600   | 0.63638 | 0.48497    | 0.60802 | 0.53941   | 0.57764 | 0.58924    | 0.54593 |
| 0.45 | 0.40582    | 0.76919 | 0.47444   | 0.73604 | 0.53739    | 0.69969 | 0.59450   | 0.66116 | 0.64580    | 0.62138 |
| 0.5  | 0.46212    | 0.88682 | 0.53705   | 0.84355 | 0.60437    | 0.79671 | 0.66404   | 0.74770 | 0.71630    | 0.69779 |
| 0.55 | 0.53655    | 1.01699 | 0.61869   | 0.95983 | 0.69041    | 0.89893 | 0.75200   | 0.83632 | 0.80407    | 0.77365 |
| 0.6  | 0.63656    | 1.16180 | 0.72640   | 1.08513 | 0.80176    | 1.00519 | 0.86357   | 0.92478 | 0.91321    | 0.84608 |
| 0.65 | 0.77356    | 1.32269 | 0.87037   | 1.21810 | 0.94684    | 1.11212 | 1.00528   | 1.00856 | 1.04844    | 0.91003 |
| 0.7  | 0.96528    | 1.49862 | 1.06521   | 1.35360 | 1.13666    | 1.21222 | 1.18469   | 1.07919 | 1.21438    | 0.95708 |
| 0.75 | 1.23914    | 1.68151 | 1.33091   | 1.47820 | 1.38412    | 1.29020 | 1.40806   | 1.12181 | 1.41397    | 0.97400 |
| 0.8  | 1.63553    | 1.84484 | 1.69121   | 1.56140 | 1.70028    | 1.31745 | 1.68068   | 1.11235 | 1.64487    | 0.94186 |
| 0.85 | 2.20223    | 1.91863 | 2.16210   | 1.54177 | 2.08309    | 1.24667 | 1.98936   | 1.01695 | 1.89366    | 0.83750 |
| 0.9  | 2.95122    | 1.75011 | 2.71602   | 1.31829 | 2.49443    | 1.01613 | 2.29855   | 0.79978 | 2.12957    | 0.64107 |
| 0.95 | 3.72316    | 1.11789 | 3.21905   | 0.79096 | 2.83603    | 0.58484 | 2.53928   | 0.44728 | 2.30471    | 0.35122 |
| 1.0  | 4.08299    | 0.00    | 3.43274   | 0.00    | 2.97287    | 0.00    | 2.63193   | 0.00    | 2.37024    | 0.00    |
| 1.05 | 3.72316    | 1.11789 | 3.21905   | 0.79096 | 2.83603    | 0.58484 | 2.53928   | 0.44728 | 2.30471    | 0.35122 |
| 1.1  | 2.95122    | 1.75011 | 2.71602   | 1.31829 | 2.49443    | 1.01613 | 2.29855   | 0.79978 | 2.12957    | 0.64107 |
| 1.15 | 2.20223    | 1.91863 | 2.16210   | 1.54177 | 2.08309    | 1.24667 | 1.98936   | 1.01695 | 1.89366    | 0.83750 |
| 1.2  | 1.63553    | 1.84484 | 1.69121   | 1.56140 | 1.70028    | 1.31745 | 1.68068   | 1.11235 | 1.64487    | 0.94186 |
| 1.25 | 1.23914    | 1.68151 | 1.33091   | 1.47820 | 1.38412    | 1.29020 | 1.40806   | 1.12181 | 1.41397    | 0.97400 |
| 1.3  | 0.96528    | 1.49862 | 1.06521   | 1.35360 | 1.13666    | 1.21222 | 1.18469   | 1.07919 | 1.21438    | 0.95708 |
| 1.35 | 0.77356    | 1.32269 | 0.87037   | 1.21810 | 0.94684    | 1.11212 | 1.00528   | 1.00856 | 1.04844    | 0.91003 |
| 1.4  | 0.63656    | 1.16180 | 0.72640   | 1.08513 | 0.80176    | 1.00519 | 0.86357   | 0.92478 | 0.91321    | 0.84608 |
| 1.45 | 0.53655    | 1.01699 | 0.61869   | 0.95983 | 0.69041    | 0.89893 | 0.75200   | 0.83632 | 0.80407    | 0.77365 |
| 1.5  | 0.46212    | 0.88682 | 0.53705   | 0.84355 | 0.60437    | 0.79671 | 0.66404   | 0.74770 | 0.71630    | 0.69779 |
| 1.55 | 0.40582    | 0.76919 | 0.47444   | 0.73604 | 0.53739    | 0.69969 | 0.59450   | 0.66116 | 0.64580    | 0.62138 |
| 1.6  | 0.36272    | 0.66200 | 0.42600   | 0.63638 | 0.48497    | 0.60802 | 0.53941   | 0.57764 | 0.58924    | 0.54593 |
| 1.65 | 0.32947    | 0.56335 | 0.38833   | 0.54348 | 0.44383    | 0.52131 | 0.49575   | 0.49737 | 0.54397    | 0.47216 |
| 1.7  | 0.30377    | 0.47162 | 0.35903   | 0.45623 | 0.41161    | 0.43898 | 0.46130   | 0.42022 | 0.50796    | 0.40033 |
| 1.75 | 0.28402    | 0.38540 | 0.33640   | 0.37362 | 0.38658    | 0.36035 | 0.43438   | 0.34585 | 0.47964    | 0.33039 |
| 1.8  | 0.26907    | 0.30351 | 0.31921   | 0.29471 | 0.36750    | 0.28475 | 0.41376   | 0.27384 | 0.45784    | 0.26216 |
| 1.85 | 0.25814    | 0.22490 | 0.30660   | 0.21864 | 0.35346    | 0.21153 | 0.39853   | 0.20373 | 0.44169    | 0.19534 |
| 1.9  | 0.25069    | 0.14866 | 0.29799   | 0.14464 | 0.34384    | 0.14007 | 0.38808   | 0.13503 | 0.43056    | 0.12961 |
| 1.95 | 0.24635    | 0.07395 | 0.29296   | 0.07199 | 0.33822    | 0.06975 | 0.38196   | 0.06728 | 0.42405    | 0.06462 |
| 2.0  | 0.24492    | 0.00    | 0.29131   | 0.00    | 0.33638    | 0.00    | 0.37995   | 0.00    | 0.42190    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(0.4 + i 0.4) = 0.53941 + i 0.57764$ .  
 $\tanh(0.45 + i 1.75) = 0.47964 - i 0.33039$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 0.5$       | $x = 0.55$      | $x = 0.6$       | $x = 0.65$      | $x = 0.7$       |
|------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0    | 0.46211 0.00    | 0.50052 0.00    | 0.53704 0.00    | 0.57167 0.00    | 0.60437 0.00    |
| 0.05 | 0.46436 0.06181 | 0.50284 0.05889 | 0.53941 0.05590 | 0.57405 0.05288 | 0.60674 0.04984 |
| 0.1  | 0.47119 0.12300 | 0.50987 0.11796 | 0.54657 0.11189 | 0.58125 0.10576 | 0.61390 0.09962 |
| 0.15 | 0.48281 0.18651 | 0.52183 0.17737 | 0.55872 0.16804 | 0.59344 0.15863 | 0.62602 0.14925 |
| 0.2  | 0.49964 0.24990 | 0.53910 0.23725 | 0.57620 0.22437 | 0.61094 0.21144 | 0.64336 0.19858 |
| 0.25 | 0.52227 0.31424 | 0.56223 0.29765 | 0.59953 0.28085 | 0.63419 0.26404 | 0.66631 0.24741 |
| 0.3  | 0.55151 0.37967 | 0.59196 0.35856 | 0.62935 0.33731 | 0.66377 0.31618 | 0.69534 0.29540 |
| 0.35 | 0.58846 0.44616 | 0.62928 0.41979 | 0.66653 0.39344 | 0.70039 0.36748 | 0.73105 0.34205 |
| 0.4  | 0.63452 0.51350 | 0.67541 0.48093 | 0.71212 0.44869 | 0.74493 0.41714 | 0.77471 0.38662 |
| 0.45 | 0.69149 0.58116 | 0.73188 0.54121 | 0.76736 0.50211 | 0.79836 0.46428 | 0.82533 0.42806 |
| 0.5  | 0.76159 0.64805 | 0.80050 0.59933 | 0.83365 0.55229 | 0.86173 0.50738 | 0.88535 0.46492 |
| 0.55 | 0.84752 0.71229 | 0.88332 0.65320 | 0.91249 0.59707 | 0.93607 0.54434 | 0.95480 0.49522 |
| 0.6  | 0.95230 0.77067 | 0.98247 0.69957 | 1.00521 0.63346 | 1.02195 0.57227 | 1.03389 0.51635 |
| 0.65 | 1.07907 0.81812 | 1.09973 0.73362 | 1.11262 0.65676 | 1.11962 0.58738 | 1.12222 0.52508 |
| 0.7  | 1.23020 0.84688 | 1.23587 0.74858 | 1.23436 0.66157 | 1.22793 0.58492 | 1.21827 0.51757 |
| 0.75 | 1.40579 0.84585 | 1.38926 0.73549 | 1.36782 0.64076 | 1.34386 0.55951 | 1.31896 0.48976 |
| 0.8  | 1.60095 0.80073 | 1.55398 0.68387 | 1.50695 0.58681 | 1.46173 0.50588 | 1.41909 0.43803 |
| 0.85 | 1.80225 0.69623 | 1.71785 0.58390 | 1.64135 0.49366 | 1.57271 0.42040 | 1.51148 0.36034 |
| 0.9  | 1.98505 0.52197 | 1.86183 0.43071 | 1.75601 0.35949 | 1.66532 0.30300 | 1.58713 0.25755 |
| 0.95 | 2.11599 0.28167 | 1.96180 0.22977 | 1.83417 0.19009 | 1.72736 0.15910 | 1.63711 0.13449 |
| 1.0  | 2.16395 0.00    | 1.99792 0.00    | 1.86202 0.00    | 1.74926 0.00    | 1.65462 0.00    |
| 1.05 | 2.11599 0.28167 | 1.96180 0.22977 | 1.83417 0.19009 | 1.72736 0.15910 | 1.63711 0.13449 |
| 1.1  | 1.98505 0.52197 | 1.86183 0.43071 | 1.75601 0.35949 | 1.66532 0.30300 | 1.58713 0.25755 |
| 1.15 | 1.80225 0.69623 | 1.71785 0.58390 | 1.64135 0.49366 | 1.57271 0.42040 | 1.51148 0.36034 |
| 1.2  | 1.60095 0.80073 | 1.55398 0.68387 | 1.50695 0.58681 | 1.46173 0.50588 | 1.41909 0.43803 |
| 1.25 | 1.40579 0.84585 | 1.38926 0.73549 | 1.36782 0.64076 | 1.34386 0.55951 | 1.31896 0.48976 |
| 1.3  | 1.23020 0.84688 | 1.23587 0.74858 | 1.23436 0.66157 | 1.22793 0.58492 | 1.21827 0.51757 |
| 1.35 | 1.07907 0.81812 | 1.09973 0.73362 | 1.11262 0.65676 | 1.11962 0.58738 | 1.12222 0.52508 |
| 1.4  | 0.95230 0.77067 | 0.98247 0.69957 | 1.00521 0.63346 | 1.02195 0.57227 | 1.03389 0.51635 |
| 1.45 | 0.84752 0.71229 | 0.88332 0.65320 | 0.91249 0.59707 | 0.93607 0.54434 | 0.95480 0.49522 |
| 1.5  | 0.76159 0.64805 | 0.80050 0.59933 | 0.83365 0.55229 | 0.86173 0.50738 | 0.88535 0.46492 |
| 1.55 | 0.69149 0.58116 | 0.73188 0.54121 | 0.76736 0.50211 | 0.79836 0.46428 | 0.82533 0.42806 |
| 1.6  | 0.63452 0.51350 | 0.67541 0.48093 | 0.71212 0.44869 | 0.74493 0.41714 | 0.77471 0.38662 |
| 1.65 | 0.58846 0.44616 | 0.62928 0.41979 | 0.66653 0.39344 | 0.70039 0.36748 | 0.73105 0.34205 |
| 1.7  | 0.55151 0.37967 | 0.59196 0.35856 | 0.62935 0.33731 | 0.66377 0.31618 | 0.69534 0.29540 |
| 1.75 | 0.52227 0.31424 | 0.56223 0.29765 | 0.59953 0.28085 | 0.63419 0.26404 | 0.66631 0.25741 |
| 1.8  | 0.49964 0.24990 | 0.53910 0.23725 | 0.57620 0.22437 | 0.61094 0.21144 | 0.64336 0.19858 |
| 1.85 | 0.48281 0.18651 | 0.52183 0.17737 | 0.55872 0.16804 | 0.59344 0.15863 | 0.62602 0.14925 |
| 1.9  | 0.47119 0.12300 | 0.50987 0.11796 | 0.54657 0.11189 | 0.58125 0.10576 | 0.61390 0.09962 |
| 1.95 | 0.46436 0.06181 | 0.50284 0.05889 | 0.53941 0.05590 | 0.57405 0.05288 | 0.60674 0.04984 |
| 2.0  | 0.46211 0.00    | 0.50052 0.00    | 0.53704 0.00    | 0.57167 0.00    | 0.60437 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(0.6 + i0.6) = 1.00521 + i0.63346$ .  
 $\tanh(0.6 + i1.5) = 0.83365 - i0.55229$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 0.75$ |         | $x = 0.8$ |         | $x = 0.85$ |         | $x = 0.9$ |         | $x = 0.95$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 0.63515    | 0.00    | 0.66403   | 0.00    | 0.69107    | 0.00    | 0.71629   | 0.00    | 0.73978    | 0.00    |
| 0.05 | 0.63749    | 0.04684 | 0.66633   | 0.04388 | 0.69330    | 0.04099 | 0.71845   | 0.03820 | 0.74185    | 0.03551 |
| 0.1  | 0.64456    | 0.09354 | 0.67325   | 0.08758 | 0.70002    | 0.08176 | 0.72494   | 0.07614 | 0.74807    | 0.07073 |
| 0.15 | 0.65649    | 0.13997 | 0.68490   | 0.13089 | 0.71132    | 0.12206 | 0.73582   | 0.11354 | 0.75850    | 0.10537 |
| 0.2  | 0.67352    | 0.18592 | 0.70148   | 0.17357 | 0.72736    | 0.16160 | 0.75123   | 0.15008 | 0.77321    | 0.13906 |
| 0.25 | 0.69595    | 0.23112 | 0.72325   | 0.21528 | 0.74832    | 0.20001 | 0.77130   | 0.18537 | 0.79231    | 0.17143 |
| 0.3  | 0.72420    | 0.27516 | 0.75051   | 0.25559 | 0.77446    | 0.23683 | 0.79620   | 0.21893 | 0.81592    | 0.20198 |
| 0.35 | 0.75872    | 0.31749 | 0.78364   | 0.29392 | 0.80603    | 0.27146 | 0.82611   | 0.25018 | 0.84411    | 0.23013 |
| 0.4  | 0.80005    | 0.35735 | 0.82300   | 0.32949 | 0.84328    | 0.30314 | 0.86117   | 0.27837 | 0.87695    | 0.25520 |
| 0.45 | 0.84871    | 0.39368 | 0.86893   | 0.36128 | 0.88638    | 0.33091 | 0.90143   | 0.30261 | 0.91438    | 0.27634 |
| 0.5  | 0.90515    | 0.42510 | 0.92167   | 0.38798 | 0.93541    | 0.35357 | 0.94681   | 0.32181 | 0.95624    | 0.29259 |
| 0.55 | 0.96963    | 0.44978 | 0.98122   | 0.40796 | 0.99018    | 0.36966 | 0.99700   | 0.33469 | 1.00211    | 0.30285 |
| 0.6  | 1.04203    | 0.46543 | 1.04722   | 0.41926 | 1.05015    | 0.37751 | 1.05136   | 0.33985 | 1.05129    | 0.30593 |
| 0.65 | 1.12161    | 0.46934 | 1.11872   | 0.42057 | 1.11427    | 0.37527 | 1.10885   | 0.33580 | 1.10271    | 0.30064 |
| 0.7  | 1.20665    | 0.45847 | 1.19395   | 0.40661 | 1.18081    | 0.36108 | 1.16765   | 0.32108 | 1.15485    | 0.28588 |
| 0.75 | 1.29416    | 0.42977 | 1.27012   | 0.37806 | 1.24723    | 0.33335 | 1.22572   | 0.29458 | 1.20569    | 0.26087 |
| 0.8  | 1.37961    | 0.38084 | 1.34331   | 0.33238 | 1.31017    | 0.29108 | 1.28006   | 0.25573 | 1.25279    | 0.22532 |
| 0.85 | 1.45701    | 0.31065 | 1.40862   | 0.26920 | 1.36562    | 0.23434 | 1.32742   | 0.20483 | 1.29344    | 0.17968 |
| 0.9  | 1.51945    | 0.22051 | 1.46062   | 0.19000 | 1.40931    | 0.16461 | 1.36438   | 0.14330 | 1.32493    | 0.12528 |
| 0.95 | 1.56023    | 0.11463 | 1.49428   | 0.09840 | 1.43735    | 0.08499 | 1.38796   | 0.07380 | 1.34490    | 0.06438 |
| 1.0  | 1.57443    | 0.00    | 1.50594   | 0.00    | 1.44703    | 0.00    | 1.39606   | 0.00    | 1.35175    | 0.00    |
| 1.05 | 1.56023    | 0.11463 | 1.49428   | 0.09840 | 1.43735    | 0.08499 | 1.38796   | 0.07380 | 1.34490    | 0.06438 |
| 1.1  | 1.51945    | 0.22051 | 1.46062   | 0.19000 | 1.40931    | 0.16461 | 1.36438   | 0.14330 | 1.32493    | 0.12528 |
| 1.15 | 1.45701    | 0.31065 | 1.40862   | 0.26920 | 1.36562    | 0.23434 | 1.32742   | 0.20483 | 1.29344    | 0.17968 |
| 1.2  | 1.37961    | 0.38084 | 1.34331   | 0.33238 | 1.31017    | 0.29108 | 1.28006   | 0.25573 | 1.25279    | 0.22532 |
| 1.25 | 1.29416    | 0.42977 | 1.27012   | 0.37806 | 1.24723    | 0.33335 | 1.22572   | 0.29458 | 1.20569    | 0.26087 |
| 1.3  | 1.20665    | 0.45847 | 1.19395   | 0.40661 | 1.18081    | 0.36108 | 1.16765   | 0.32108 | 1.15485    | 0.28588 |
| 1.35 | 1.12161    | 0.46934 | 1.11872   | 0.42057 | 1.11427    | 0.37527 | 1.10885   | 0.33580 | 1.10271    | 0.30064 |
| 1.4  | 1.04203    | 0.46543 | 1.04722   | 0.41926 | 1.05015    | 0.37751 | 1.05136   | 0.33985 | 1.05129    | 0.30593 |
| 1.45 | 0.96963    | 0.44978 | 0.98122   | 0.40796 | 0.99018    | 0.36966 | 0.99700   | 0.33469 | 1.00211    | 0.30285 |
| 1.5  | 0.90515    | 0.42510 | 0.92167   | 0.38798 | 0.93541    | 0.35357 | 0.94681   | 0.32181 | 0.95624    | 0.29259 |
| 1.55 | 0.84871    | 0.39368 | 0.86893   | 0.36128 | 0.88638    | 0.33091 | 0.90143   | 0.30261 | 0.91438    | 0.27634 |
| 1.6  | 0.80005    | 0.35735 | 0.82300   | 0.32949 | 0.84328    | 0.30314 | 0.86117   | 0.27837 | 0.87695    | 0.25520 |
| 1.65 | 0.75872    | 0.31749 | 0.78364   | 0.29392 | 0.80603    | 0.27146 | 0.82611   | 0.25018 | 0.84411    | 0.23013 |
| 1.7  | 0.72420    | 0.27516 | 0.75051   | 0.25559 | 0.77446    | 0.23683 | 0.79620   | 0.21893 | 0.81592    | 0.20198 |
| 1.75 | 0.69595    | 0.23112 | 0.72325   | 0.21528 | 0.74832    | 0.20001 | 0.77130   | 0.18537 | 0.79231    | 0.17143 |
| 1.8  | 0.67352    | 0.18592 | 0.70148   | 0.17357 | 0.72736    | 0.16160 | 0.75123   | 0.15008 | 0.77321    | 0.13906 |
| 1.85 | 0.65649    | 0.13997 | 0.68490   | 0.13089 | 0.71132    | 0.12206 | 0.73582   | 0.11354 | 0.75850    | 0.10537 |
| 1.9  | 0.64456    | 0.09354 | 0.67325   | 0.08758 | 0.70002    | 0.08176 | 0.72494   | 0.07614 | 0.74807    | 0.07073 |
| 1.95 | 0.63749    | 0.04684 | 0.66633   | 0.04388 | 0.69330    | 0.04099 | 0.71845   | 0.03820 | 0.74185    | 0.03551 |
| 2.0  | 0.63515    | 0.00    | 0.66403   | 0.00    | 0.69107    | 0.00    | 0.71629   | 0.00    | 0.73978    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(0.95 + i0) = 0.73978 + i0$ .

$\tanh(0.9 + i1.9) = 0.72494 - i0.07614$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.0$ |         | $x = 1.05$ |         | $x = 1.1$ |         | $x = 1.15$ |         | $x = 1.2$ |         |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 0.76159   | 0.00    | 0.78181    | 0.00    | 0.80050   | 0.00    | 0.81775    | 0.00    | 0.83365   | 0.00    |
| 0.05 | 0.76357   | 0.03293 | 0.78368    | 0.03048 | 0.80227   | 0.02816 | 0.81943    | 0.02597 | 0.83522   | 0.02390 |
| 0.1  | 0.76950   | 0.06556 | 0.78932    | 0.06065 | 0.80760   | 0.05599 | 0.82444    | 0.05160 | 0.83992   | 0.04748 |
| 0.15 | 0.77943   | 0.09757 | 0.79873    | 0.09016 | 0.81648   | 0.08317 | 0.83279    | 0.07658 | 0.84775   | 0.07041 |
| 0.2  | 0.79341   | 0.12858 | 0.81195    | 0.11867 | 0.82893   | 0.10932 | 0.84447    | 0.10054 | 0.85867   | 0.09233 |
| 0.25 | 0.81151   | 0.15821 | 0.82901    | 0.14575 | 0.84495   | 0.13405 | 0.85945    | 0.12310 | 0.87264   | 0.11288 |
| 0.3  | 0.83377   | 0.18598 | 0.84991    | 0.17096 | 0.86450   | 0.15692 | 0.87768    | 0.14383 | 0.88958   | 0.13166 |
| 0.35 | 0.86022   | 0.21133 | 0.87464    | 0.19377 | 0.88753   | 0.17742 | 0.89097    | 0.16226 | 0.90938   | 0.14823 |
| 0.4  | 0.89086   | 0.23361 | 0.90311    | 0.21356 | 0.91392   | 0.19501 | 0.92345    | 0.17789 | 0.93186   | 0.16213 |
| 0.45 | 0.92554   | 0.25205 | 0.93515    | 0.22966 | 0.94344   | 0.20906 | 0.95058    | 0.19017 | 0.95674   | 0.17287 |
| 0.5  | 0.96403   | 0.26580 | 0.97045    | 0.24130 | 0.97574   | 0.21892 | 0.98010    | 0.19852 | 0.98368   | 0.17996 |
| 0.55 | 1.00585   | 0.27392 | 1.00852    | 0.24767 | 1.01034   | 0.22389 | 1.01151    | 0.20236 | 1.01217   | 0.18288 |
| 0.6  | 1.05030   | 0.27542 | 1.04864    | 0.24798 | 1.04654   | 0.22331 | 1.04415    | 0.20115 | 1.04160   | 0.18123 |
| 0.65 | 1.09632   | 0.26933 | 1.08984    | 0.24144 | 1.08342   | 0.21658 | 1.07718    | 0.19441 | 1.07119   | 0.17461 |
| 0.7  | 1.14516   | 0.25486 | 1.13084    | 0.22747 | 1.11984   | 0.20326 | 1.10957    | 0.18182 | 1.10003   | 0.16281 |
| 0.75 | 1.18715   | 0.23145 | 1.17009    | 0.20572 | 1.15445   | 0.18315 | 1.14015    | 0.16330 | 1.12709   | 0.14580 |
| 0.8  | 1.22812   | 0.19094 | 1.20585    | 0.17623 | 1.18575   | 0.15637 | 1.16763    | 0.13902 | 1.15129   | 0.12380 |
| 0.85 | 1.26319   | 0.15812 | 1.23624    | 0.13955 | 1.21219   | 0.12347 | 1.19346    | 0.10950 | 1.17152   | 0.09730 |
| 0.9  | 1.29017   | 0.10993 | 1.25949    | 0.09677 | 1.23231   | 0.08544 | 1.20821    | 0.07563 | 1.18680   | 0.06709 |
| 0.95 | 1.30721   | 0.05638 | 1.27410    | 0.04956 | 1.24492   | 0.04369 | 1.21914    | 0.03863 | 1.19631   | 0.03424 |
| 1.0  | 1.31304   | 0.00    | 1.27908    | 0.00    | 1.24922   | 0.00    | 1.22286    | 0.00    | 1.19954   | 0.00    |
| 1.05 | 1.30721   | 0.05638 | 1.27410    | 0.04956 | 1.24492   | 0.04369 | 1.21914    | 0.03863 | 1.19631   | 0.03424 |
| 1.1  | 1.29017   | 0.10993 | 1.25949    | 0.09677 | 1.23231   | 0.08544 | 1.20821    | 0.07563 | 1.18680   | 0.06709 |
| 1.15 | 1.26319   | 0.15812 | 1.23624    | 0.13955 | 1.21219   | 0.12347 | 1.19346    | 0.10950 | 1.17152   | 0.09730 |
| 1.2  | 1.22812   | 0.19094 | 1.20585    | 0.17623 | 1.18575   | 0.15637 | 1.16763    | 0.13902 | 1.15129   | 0.12380 |
| 1.25 | 1.18715   | 0.23145 | 1.17009    | 0.20572 | 1.15445   | 0.18315 | 1.14015    | 0.16330 | 1.12709   | 0.14580 |
| 1.3  | 1.14516   | 0.25486 | 1.13084    | 0.22747 | 1.11984   | 0.20326 | 1.10957    | 0.18182 | 1.10003   | 0.16281 |
| 1.35 | 1.09632   | 0.26933 | 1.08984    | 0.24144 | 1.08342   | 0.21658 | 1.07718    | 0.19441 | 1.07119   | 0.17461 |
| 1.4  | 1.05030   | 0.27542 | 1.04864    | 0.24798 | 1.04654   | 0.22331 | 1.04415    | 0.20115 | 1.04160   | 0.18123 |
| 1.45 | 1.00585   | 0.27392 | 1.00852    | 0.24767 | 1.01034   | 0.22389 | 1.01151    | 0.20236 | 1.01217   | 0.18288 |
| 1.5  | 0.96403   | 0.26580 | 0.97045    | 0.24130 | 0.97574   | 0.21892 | 0.98010    | 0.19852 | 0.98368   | 0.17996 |
| 1.55 | 0.92554   | 0.25205 | 0.93515    | 0.22966 | 0.94344   | 0.20906 | 0.95058    | 0.19017 | 0.95674   | 0.17287 |
| 1.6  | 0.89086   | 0.23361 | 0.90311    | 0.21356 | 0.91392   | 0.19501 | 0.92345    | 0.17789 | 0.93186   | 0.16213 |
| 1.65 | 0.86022   | 0.21133 | 0.87464    | 0.19377 | 0.88753   | 0.17742 | 0.89097    | 0.16226 | 0.90938   | 0.14823 |
| 1.7  | 0.83377   | 0.18598 | 0.84991    | 0.17096 | 0.86450   | 0.15692 | 0.87768    | 0.14383 | 0.88958   | 0.13166 |
| 1.75 | 0.81151   | 0.15821 | 0.82901    | 0.14575 | 0.84495   | 0.13405 | 0.85945    | 0.12310 | 0.87264   | 0.11288 |
| 1.8  | 0.79341   | 0.12858 | 0.81195    | 0.11867 | 0.82893   | 0.10932 | 0.84447    | 0.10054 | 0.85867   | 0.09233 |
| 1.85 | 0.77943   | 0.09757 | 0.79873    | 0.09016 | 0.81648   | 0.08317 | 0.83279    | 0.07658 | 0.84775   | 0.07041 |
| 1.9  | 0.76950   | 0.06556 | 0.78932    | 0.06065 | 0.80760   | 0.05599 | 0.82444    | 0.05160 | 0.83992   | 0.04748 |
| 1.95 | 0.76357   | 0.03293 | 0.78368    | 0.03048 | 0.80227   | 0.02816 | 0.81943    | 0.02597 | 0.83522   | 0.02390 |
| 2.0  | 0.76159   | 0.00    | 0.78181    | 0.00    | 0.80050   | 0.00    | 0.81775    | 0.00    | 0.83365   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(1.2 + i\,0.75) = 1.12709 + i\,0.14580$ .  
 $\tanh(1.2 + i\,1.25) = 1.12709 - i\,0.14580$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.25$ | $x = 1.3$ | $x = 1.35$ | $x = 1.4$ | $x = 1.45$ |
|------|------------|-----------|------------|-----------|------------|
| 0    | 0.84828    | 0.86172   | 0.87405    | 0.88535   | 0.89569    |
| 0.05 | 0.84975    | 0.86309   | 0.87533    | 0.88653   | 0.89678    |
| 0.1  | 0.85144    | 0.86719   | 0.87913    | 0.89006   | 0.90005    |
| 0.15 | 0.86145    | 0.87398   | 0.88544    | 0.89591   | 0.90545    |
| 0.2  | 0.87162    | 0.88468   | 0.89421    | 0.90401   | 0.91293    |
| 0.25 | 0.88461    | 0.10339   | 0.89548    | 0.90535   | 0.92240    |
| 0.3  | 0.90032    | 0.12039   | 0.90791    | 0.91875   | 0.93375    |
| 0.35 | 0.91861    | 0.13528   | 0.92686    | 0.93425   | 0.94680    |
| 0.4  | 0.93928    | 0.14765   | 0.94585    | 0.95166   | 0.96137    |
| 0.45 | 0.96207    | 0.15706   | 0.96669    | 0.97069   | 0.97719    |
| 0.5  | 0.98661    | 0.16307   | 0.98903    | 0.99101   | 0.99396    |
| 0.55 | 1.01244    | 0.16528   | 1.01243    | 1.01219   | 1.01181    |
| 0.6  | 1.03897    | 0.16332   | 1.03634    | 1.03375   | 1.03125    |
| 0.65 | 1.06550    | 0.15691   | 1.06013    | 1.05510   | 1.05042    |
| 0.7  | 1.09121    | 0.14591   | 1.08308    | 1.07560   | 1.06875    |
| 0.75 | 1.11521    | 0.13034   | 1.10439    | 1.09457   | 1.08565    |
| 0.8  | 1.13656    | 0.11042   | 1.12328    | 1.11131   | 1.10052    |
| 0.85 | 1.15434    | 0.08662   | 1.13895    | 1.12512   | 1.11277    |
| 0.9  | 1.16772    | 0.05964   | 1.15070    | 1.13551   | 1.12192    |
| 0.95 | 1.17603    | 0.03041   | 1.15799    | 1.14192   | 1.12758    |
| 1.0  | 1.17885    | 0.00      | 1.16047    | 1.14410   | 1.12950    |
| 1.05 | 1.17603    | 0.03041   | 1.15799    | 1.14192   | 1.12758    |
| 1.1  | 1.16772    | 0.05964   | 1.15070    | 1.13551   | 1.12192    |
| 1.15 | 1.15434    | 0.08662   | 1.13895    | 1.12512   | 1.11277    |
| 1.2  | 1.13656    | 0.11042   | 1.12328    | 1.11131   | 1.10052    |
| 1.25 | 1.11521    | 0.13034   | 1.10439    | 1.09457   | 1.08565    |
| 1.3  | 1.09121    | 0.14591   | 1.08308    | 1.07560   | 1.06875    |
| 1.35 | 1.06550    | 0.15691   | 1.06013    | 1.05510   | 1.05042    |
| 1.4  | 1.03897    | 0.16332   | 1.03634    | 1.03375   | 1.03125    |
| 1.45 | 1.01244    | 0.16528   | 1.01243    | 1.01219   | 1.01181    |
| 1.5  | 0.98661    | 0.16307   | 0.98903    | 0.99101   | 0.99263    |
| 1.55 | 0.96207    | 0.15706   | 0.96669    | 0.97069   | 0.97417    |
| 1.6  | 0.93928    | 0.14765   | 0.94585    | 0.95166   | 0.95680    |
| 1.65 | 0.91861    | 0.13528   | 0.92686    | 0.93425   | 0.94087    |
| 1.7  | 0.90032    | 0.12039   | 0.90791    | 0.91875   | 0.92663    |
| 1.75 | 0.88461    | 0.10339   | 0.89548    | 0.90535   | 0.91429    |
| 1.8  | 0.87162    | 0.08468   | 0.88461    | 0.89421   | 0.90401    |
| 1.85 | 0.86145    | 0.06464   | 0.87398    | 0.88544   | 0.89591    |
| 1.9  | 0.85414    | 0.04363   | 0.86719    | 0.87913   | 0.89006    |
| 1.95 | 0.84975    | 0.02197   | 0.86309    | 0.87533   | 0.88653    |
| 2.0  | 0.84828    | 0.00      | 0.86172    | 0.87405   | 0.88535    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(1.4 + i0.8) = 1.10052 + i0.07896$ .  
 $\tanh(1.3 + i1.3) = 1.08308 - i0.13088$ .



TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.5$       | $x = 1.55$      | $x = 1.6$       | $x = 1.65$      | $x = 1.7$       |
|------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0    | 0.90515 0.00    | 0.91379 0.00    | 0.92167 0.00    | 0.92886 0.00    | 0.93541 0.00    |
| 0.05 | 0.90616 0.01415 | 0.91471 0.01292 | 0.92253 0.01178 | 0.92964 0.01074 | 0.93613 0.00979 |
| 0.1  | 0.90917 0.02804 | 0.91749 0.02560 | 0.92508 0.02334 | 0.93199 0.02127 | 0.93828 0.01937 |
| 0.15 | 0.91415 0.04143 | 0.92208 0.03779 | 0.92929 0.03445 | 0.93586 0.03138 | 0.94183 0.02857 |
| 0.2  | 0.92104 0.05404 | 0.92842 0.04927 | 0.93511 0.04488 | 0.94119 0.04086 | 0.94671 0.03718 |
| 0.25 | 0.92975 0.06563 | 0.93641 0.05978 | 0.94245 0.05442 | 0.94791 0.04951 | 0.95285 0.04502 |
| 0.3  | 0.94017 0.07593 | 0.94595 0.06909 | 0.95118 0.06284 | 0.95589 0.05712 | 0.96015 0.05190 |
| 0.35 | 0.95212 0.08468 | 0.95689 0.07697 | 0.96117 0.06993 | 0.96501 0.06351 | 0.96846 0.05766 |
| 0.4  | 0.96542 0.09165 | 0.96903 0.08320 | 0.97223 0.07551 | 0.97509 0.06850 | 0.97763 0.06213 |
| 0.45 | 0.97983 0.09660 | 0.98214 0.08758 | 0.98415 0.07938 | 0.98592 0.07193 | 0.98748 0.06517 |
| 0.5  | 0.99506 0.09933 | 0.99595 0.08992 | 0.99668 0.08139 | 0.99728 0.07367 | 0.99777 0.06667 |
| 0.55 | 1.01076 0.09965 | 1.01016 0.09008 | 1.00954 0.08142 | 1.00891 0.07361 | 1.00829 0.06655 |
| 0.6  | 1.02657 0.09746 | 1.02441 0.08796 | 1.02240 0.07940 | 1.02052 0.07169 | 1.01877 0.06474 |
| 0.65 | 1.04204 0.09268 | 1.03834 0.08353 | 1.03492 0.07530 | 1.03179 0.06791 | 1.02892 0.06126 |
| 0.7  | 1.05675 0.08534 | 1.05152 0.07680 | 1.04676 0.06915 | 1.04242 0.06230 | 1.03847 0.05614 |
| 0.75 | 1.07022 0.07554 | 1.06357 0.06790 | 1.05755 0.06107 | 1.05209 0.05495 | 1.04714 0.04948 |
| 0.8  | 1.08200 0.06349 | 1.07408 0.05700 | 1.06693 0.05121 | 1.06049 0.04604 | 1.05466 0.04142 |
| 0.85 | 1.09167 0.04947 | 1.08269 0.04438 | 1.07461 0.03984 | 1.06734 0.03579 | 1.06079 0.03218 |
| 0.9  | 1.09886 0.03390 | 1.08909 0.03038 | 1.08030 0.02726 | 1.07241 0.02448 | 1.06533 0.02200 |
| 0.95 | 1.10329 0.01723 | 1.09302 0.01544 | 1.08380 0.01385 | 1.07554 0.01243 | 1.06811 0.01117 |
| 1.0  | 1.10479 0.00    | 1.09436 0.00    | 1.08500 0.00    | 1.07659 0.00    | 1.06906 0.00    |
| 1.05 | 1.10329 0.01723 | 1.09302 0.01544 | 1.08380 0.01385 | 1.07554 0.01243 | 1.06811 0.01117 |
| 1.1  | 1.09886 0.03390 | 1.08909 0.03038 | 1.08030 0.02726 | 1.07241 0.02448 | 1.06533 0.02200 |
| 1.15 | 1.09167 0.04947 | 1.08269 0.04438 | 1.07461 0.03984 | 1.06734 0.03579 | 1.06079 0.03218 |
| 1.2  | 1.08200 0.06349 | 1.07408 0.05700 | 1.06693 0.05121 | 1.06049 0.04604 | 1.05466 0.04142 |
| 1.25 | 1.07022 0.07554 | 1.06357 0.06790 | 1.05755 0.06107 | 1.05209 0.05495 | 1.04714 0.04948 |
| 1.3  | 1.05675 0.08534 | 1.05152 0.07680 | 1.04676 0.06915 | 1.04242 0.06230 | 1.03847 0.05614 |
| 1.35 | 1.04204 0.09268 | 1.03834 0.08353 | 1.03492 0.07530 | 1.03179 0.06791 | 1.02892 0.06126 |
| 1.4  | 1.02657 0.09746 | 1.02441 0.08796 | 1.02240 0.07940 | 1.02052 0.07169 | 1.01877 0.06474 |
| 1.45 | 1.01076 0.09965 | 1.01016 0.09008 | 1.00954 0.08142 | 1.00891 0.07361 | 1.00829 0.06655 |
| 1.5  | 0.99506 0.09933 | 0.99595 0.08992 | 0.99668 0.08139 | 0.99728 0.07367 | 0.99777 0.06667 |
| 1.55 | 0.97983 0.09660 | 0.98214 0.08758 | 0.98415 0.07938 | 0.98592 0.07193 | 0.98748 0.06517 |
| 1.6  | 0.96542 0.09165 | 0.96903 0.08320 | 0.97223 0.07551 | 0.97509 0.06850 | 0.97763 0.06213 |
| 1.65 | 0.95212 0.08468 | 0.95689 0.07697 | 0.96117 0.06993 | 0.96501 0.06351 | 0.96846 0.05766 |
| 1.7  | 0.94017 0.07593 | 0.94595 0.06909 | 0.95118 0.06284 | 0.95589 0.05712 | 0.96015 0.05190 |
| 1.75 | 0.92975 0.06563 | 0.93641 0.05978 | 0.94245 0.05442 | 0.94791 0.04951 | 0.95285 0.04502 |
| 1.8  | 0.92104 0.05404 | 0.92842 0.04927 | 0.93511 0.04488 | 0.94119 0.04086 | 0.94671 0.03718 |
| 1.85 | 0.91415 0.04143 | 0.92208 0.03779 | 0.92929 0.03445 | 0.93586 0.03138 | 0.94183 0.02857 |
| 1.9  | 0.90917 0.02804 | 0.91749 0.02560 | 0.92508 0.02334 | 0.93199 0.02127 | 0.93828 0.01937 |
| 1.95 | 0.90616 0.01415 | 0.91471 0.01292 | 0.92253 0.01178 | 0.92964 0.01074 | 0.93613 0.00979 |
| 2.0  | 0.90515 0.00    | 0.91379 0.00    | 0.92167 0.00    | 0.92886 0.00    | 0.93541 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(1.7 + i \underline{0.7}) = 1.03847 + i 0.05614$ .  
 $\tanh(1.6 + i \underline{1.6}) = 0.97223 - i 0.07551$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 1.75$ |         | $x = 1.8$ |         | $x = 1.85$ |         | $x = 1.9$ |         | $x = 1.95$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 0.94138    | 0.00    | 0.94681   | 0.00    | 0.95175    | 0.00    | 0.95624   | 0.00    | 0.96032    | 0.00    |
| 0.05 | 0.94204    | 0.00891 | 0.94741   | 0.00811 | 0.95230    | 0.00737 | 0.95674   | 0.00670 | 0.96078    | 0.00609 |
| 0.1  | 0.94400    | 0.01763 | 0.94921   | 0.01604 | 0.95394    | 0.01459 | 0.95825   | 0.01326 | 0.96215    | 0.01204 |
| 0.15 | 0.94725    | 0.02600 | 0.95218   | 0.02364 | 0.95666    | 0.02149 | 0.96072   | 0.01952 | 0.96441    | 0.01773 |
| 0.2  | 0.95172    | 0.03382 | 0.95626   | 0.03074 | 0.96038    | 0.02793 | 0.96412   | 0.02537 | 0.96750    | 0.02303 |
| 0.25 | 0.95733    | 0.04092 | 0.96139   | 0.03718 | 0.96506    | 0.03376 | 0.96838   | 0.03065 | 0.97138    | 0.02782 |
| 0.3  | 0.96399    | 0.04714 | 0.96746   | 0.04280 | 0.97059    | 0.03885 | 0.97342   | 0.03525 | 0.97597    | 0.03198 |
| 0.35 | 0.97156    | 0.05233 | 0.97435   | 0.04748 | 0.97686    | 0.04307 | 0.97912   | 0.03905 | 0.98116    | 0.03541 |
| 0.4  | 0.97991    | 0.05634 | 0.98194   | 0.05107 | 0.98376    | 0.04629 | 0.98538   | 0.04195 | 0.98684    | 0.03801 |
| 0.45 | 0.98884    | 0.05904 | 0.99005   | 0.05348 | 0.99111    | 0.04843 | 0.99206   | 0.04386 | 0.99289    | 0.03972 |
| 0.5  | 0.99818    | 0.06034 | 0.99851   | 0.05461 | 0.99878    | 0.04942 | 0.99900   | 0.04472 | 0.99918    | 0.04047 |
| 0.55 | 1.00769    | 0.06017 | 1.00711   | 0.05440 | 1.00656    | 0.04919 | 1.00604   | 0.04448 | 1.00555    | 0.04022 |
| 0.6  | 1.01714    | 0.05848 | 1.01565   | 0.05283 | 1.01427    | 0.04773 | 1.01300   | 0.04313 | 1.01184    | 0.03897 |
| 0.65 | 1.02629    | 0.05528 | 1.02389   | 0.04989 | 1.02170    | 0.04504 | 1.01970   | 0.04067 | 1.01788    | 0.03673 |
| 0.7  | 1.03488    | 0.05061 | 1.03162   | 0.04564 | 1.02866    | 0.04118 | 1.02596   | 0.03716 | 1.02352    | 0.03354 |
| 0.75 | 1.04266    | 0.04457 | 1.03861   | 0.04016 | 1.03494    | 0.03621 | 1.03162   | 0.03265 | 1.02862    | 0.02946 |
| 0.8  | 1.04940    | 0.03729 | 1.04466   | 0.03358 | 1.04037    | 0.03026 | 1.03650   | 0.02727 | 1.03300    | 0.02459 |
| 0.85 | 1.05489    | 0.02895 | 1.04958   | 0.02606 | 1.04478    | 0.02347 | 1.04046   | 0.02115 | 1.03656    | 0.01906 |
| 0.9  | 1.05895    | 0.01978 | 1.05320   | 0.01780 | 1.04804    | 0.01602 | 1.04337   | 0.01443 | 1.03918    | 0.01301 |
| 0.95 | 1.06143    | 0.01004 | 1.05543   | 0.00903 | 1.05003    | 0.00813 | 1.04516   | 0.00732 | 1.04078    | 0.00659 |
| 1.0  | 1.06228    | 0.00    | 1.05619   | 0.00    | 1.05070    | 0.00    | 1.04576   | 0.00    | 1.04131    | 0.00    |
| 1.05 | 1.06143    | 0.01004 | 1.05543   | 0.00903 | 1.05003    | 0.00813 | 1.04516   | 0.00732 | 1.04078    | 0.00659 |
| 1.1  | 1.05895    | 0.01978 | 1.05320   | 0.01780 | 1.04804    | 0.01602 | 1.04337   | 0.01443 | 1.03918    | 0.01301 |
| 1.15 | 1.05489    | 0.02895 | 1.04958   | 0.02606 | 1.04478    | 0.02347 | 1.04046   | 0.02115 | 1.03656    | 0.01906 |
| 1.2  | 1.04940    | 0.03729 | 1.04466   | 0.03358 | 1.04037    | 0.03026 | 1.03650   | 0.02727 | 1.03300    | 0.02459 |
| 1.25 | 1.04266    | 0.04457 | 1.03861   | 0.04016 | 1.03494    | 0.03621 | 1.03162   | 0.03265 | 1.02862    | 0.02946 |
| 1.3  | 1.03488    | 0.05061 | 1.03162   | 0.04564 | 1.02866    | 0.04118 | 1.02596   | 0.03716 | 1.02352    | 0.03354 |
| 1.35 | 1.02629    | 0.05528 | 1.02389   | 0.04989 | 1.02170    | 0.04504 | 1.01970   | 0.04067 | 1.01788    | 0.03673 |
| 1.4  | 1.01714    | 0.05848 | 1.01565   | 0.05283 | 1.01427    | 0.04773 | 1.01300   | 0.04313 | 1.01184    | 0.03897 |
| 1.45 | 1.00769    | 0.06017 | 1.00711   | 0.05440 | 1.00656    | 0.04919 | 1.00604   | 0.04448 | 1.00555    | 0.04022 |
| 1.5  | 0.99818    | 0.06034 | 0.99851   | 0.05461 | 0.99878    | 0.04942 | 0.99900   | 0.04472 | 0.99918    | 0.04047 |
| 1.55 | 0.98884    | 0.05904 | 0.99005   | 0.05348 | 0.99111    | 0.04843 | 0.99206   | 0.04386 | 0.99289    | 0.03972 |
| 1.6  | 0.97991    | 0.05634 | 0.98194   | 0.05107 | 0.98376    | 0.04629 | 0.98538   | 0.04195 | 0.98684    | 0.03801 |
| 1.65 | 0.97156    | 0.05233 | 0.97435   | 0.04748 | 0.97686    | 0.04307 | 0.97912   | 0.03905 | 0.98116    | 0.03541 |
| 1.7  | 0.96399    | 0.04714 | 0.96746   | 0.04280 | 0.97059    | 0.03885 | 0.97342   | 0.03525 | 0.97597    | 0.03198 |
| 1.75 | 0.95733    | 0.04092 | 0.96139   | 0.03718 | 0.96506    | 0.03376 | 0.96838   | 0.03065 | 0.97138    | 0.02782 |
| 1.8  | 0.95172    | 0.03382 | 0.95626   | 0.03074 | 0.96038    | 0.02793 | 0.96412   | 0.02537 | 0.96750    | 0.02303 |
| 1.85 | 0.94725    | 0.02600 | 0.95218   | 0.02364 | 0.95666    | 0.02149 | 0.96072   | 0.01952 | 0.96441    | 0.01773 |
| 1.9  | 0.94400    | 0.01763 | 0.94921   | 0.01604 | 0.95394    | 0.01459 | 0.95825   | 0.01326 | 0.96215    | 0.01204 |
| 1.95 | 0.94204    | 0.00891 | 0.94741   | 0.00811 | 0.95230    | 0.00737 | 0.95674   | 0.00670 | 0.96078    | 0.00609 |
| 2.0  | 0.94138    | 0.00    | 0.94681   | 0.00    | 0.95175    | 0.00    | 0.95624   | 0.00    | 0.96032    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(1.85 + i \underline{0.85}) = 1.04478 + i \underline{0.02347}$ .  
 $\tanh(1.95 + i \underline{1.25}) = 1.02862 - i \underline{0.02946}$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.0$ |         | $x = 2.05$ |         | $x = 2.1$ |         | $x = 2.15$ |         | $x = 2.2$ |         |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 0.96403   | 0.00    | 0.96740    | 0.00    | 0.97045   | 0.00    | 0.97323    | 0.00    | 0.97574   | 0.00    |
| 0.05 | 0.96445   | 0.00553 | 0.96778    | 0.00502 | 0.97080   | 0.00456 | 0.97354    | 0.00413 | 0.97603   | 0.00375 |
| 0.1  | 0.96570   | 0.01094 | 0.96892    | 0.00993 | 0.97184   | 0.00901 | 0.97449    | 0.00817 | 0.97689   | 0.00741 |
| 0.15 | 0.96775   | 0.01610 | 0.97079    | 0.01461 | 0.97354   | 0.01326 | 0.97604    | 0.01203 | 0.97830   | 0.01091 |
| 0.2  | 0.97058   | 0.02090 | 0.97336    | 0.01897 | 0.97588   | 0.01721 | 0.97816    | 0.01561 | 0.98023   | 0.01415 |
| 0.25 | 0.97411   | 0.02524 | 0.97657    | 0.02289 | 0.97880   | 0.02076 | 0.98081    | 0.01882 | 0.98264   | 0.01706 |
| 0.3  | 0.97827   | 0.02900 | 0.98036    | 0.02630 | 0.98224   | 0.02384 | 0.98394    | 0.02161 | 0.98548   | 0.01958 |
| 0.35 | 0.98299   | 0.03209 | 0.98464    | 0.02909 | 0.98613   | 0.02636 | 0.98747    | 0.02388 | 0.98868   | 0.02163 |
| 0.4  | 0.98815   | 0.03444 | 0.98932    | 0.03120 | 0.99037   | 0.02826 | 0.99132    | 0.02559 | 0.99217   | 0.02317 |
| 0.45 | 0.99364   | 0.03596 | 0.99430    | 0.03256 | 0.99489   | 0.02948 | 0.99541    | 0.02669 | 0.99587   | 0.02416 |
| 0.5  | 0.99933   | 0.03662 | 0.99945    | 0.03314 | 0.99955   | 0.02998 | 0.99963    | 0.02713 | 0.99970   | 0.02455 |
| 0.55 | 1.00509   | 0.03638 | 1.00466    | 0.03290 | 1.00426   | 0.02976 | 1.00389    | 0.02691 | 1.00355   | 0.02434 |
| 0.6  | 1.01077   | 0.03523 | 1.00979    | 0.03184 | 1.00890   | 0.02878 | 1.00808    | 0.02602 | 1.00734   | 0.02353 |
| 0.65 | 1.01623   | 0.03318 | 1.01471    | 0.02998 | 1.01334   | 0.02709 | 1.01210    | 0.02448 | 1.01097   | 0.02212 |
| 0.7  | 1.02131   | 0.03028 | 1.01930    | 0.02734 | 1.01748   | 0.02469 | 1.01583    | 0.02231 | 1.01434   | 0.02015 |
| 0.75 | 1.02589   | 0.02658 | 1.02343    | 0.02399 | 1.02120   | 0.02166 | 1.01919    | 0.01956 | 1.01736   | 0.01767 |
| 0.8  | 1.02984   | 0.02218 | 1.02608    | 0.02001 | 1.02440   | 0.01806 | 1.02207    | 0.01631 | 1.01996   | 0.01472 |
| 0.85 | 1.03304   | 0.01719 | 1.02986    | 0.01550 | 1.02699   | 0.01399 | 1.02440    | 0.01262 | 1.02206   | 0.01140 |
| 0.9  | 1.03539   | 0.01172 | 1.03197    | 0.01057 | 1.02889   | 0.00954 | 1.02611    | 0.00861 | 1.02360   | 0.00777 |
| 0.95 | 1.03683   | 0.00594 | 1.03327    | 0.00536 | 1.03005   | 0.00483 | 1.02715    | 0.00436 | 1.02454   | 0.00394 |
| 1.0  | 1.03731   | 0.00    | 1.03370    | 0.00    | 1.03045   | 0.00    | 1.02751    | 0.00    | 1.02486   | 0.00    |
| 1.05 | 1.03683   | 0.00594 | 1.03327    | 0.00536 | 1.03005   | 0.00483 | 1.02715    | 0.00436 | 1.02454   | 0.00394 |
| 1.1  | 1.03539   | 0.01172 | 1.03197    | 0.01057 | 1.02889   | 0.00954 | 1.02611    | 0.00861 | 1.02360   | 0.00777 |
| 1.15 | 1.03304   | 0.01719 | 1.02986    | 0.01550 | 1.02699   | 0.01399 | 1.02440    | 0.01262 | 1.02206   | 0.01140 |
| 1.2  | 1.02984   | 0.02218 | 1.02608    | 0.02001 | 1.02440   | 0.01806 | 1.02207    | 0.01631 | 1.01996   | 0.01472 |
| 1.25 | 1.02589   | 0.02658 | 1.02343    | 0.02399 | 1.02120   | 0.02166 | 1.01919    | 0.01956 | 1.01736   | 0.01767 |
| 1.3  | 1.02131   | 0.03028 | 1.01930    | 0.02734 | 1.01748   | 0.02469 | 1.01583    | 0.02231 | 1.01434   | 0.02015 |
| 1.35 | 1.01623   | 0.03318 | 1.01471    | 0.02998 | 1.01334   | 0.02709 | 1.01210    | 0.02448 | 1.01097   | 0.02212 |
| 1.4  | 1.01077   | 0.03523 | 1.00979    | 0.03184 | 1.00890   | 0.02878 | 1.00808    | 0.02602 | 1.00734   | 0.02353 |
| 1.45 | 1.00509   | 0.03638 | 1.00466    | 0.03290 | 1.00426   | 0.02976 | 1.00389    | 0.02691 | 1.00355   | 0.02434 |
| 1.5  | 0.99933   | 0.03662 | 0.99945    | 0.03314 | 0.99955   | 0.02998 | 0.99963    | 0.02713 | 0.99970   | 0.02455 |
| 1.55 | 0.99364   | 0.03596 | 0.99430    | 0.03256 | 0.99489   | 0.02948 | 0.99541    | 0.02669 | 0.99587   | 0.02416 |
| 1.6  | 0.98815   | 0.03444 | 0.98932    | 0.03120 | 0.99037   | 0.02826 | 0.99132    | 0.02559 | 0.99217   | 0.02317 |
| 1.65 | 0.98299   | 0.03209 | 0.98464    | 0.02909 | 0.98613   | 0.02636 | 0.98747    | 0.02388 | 0.98868   | 0.02163 |
| 1.7  | 0.97827   | 0.02900 | 0.98036    | 0.02630 | 0.98224   | 0.02384 | 0.98394    | 0.02161 | 0.98548   | 0.01958 |
| 1.75 | 0.97411   | 0.02524 | 0.97657    | 0.02289 | 0.97880   | 0.02076 | 0.98081    | 0.01882 | 0.98264   | 0.01706 |
| 1.8  | 0.97058   | 0.02090 | 0.97336    | 0.01897 | 0.97588   | 0.01721 | 0.97816    | 0.01561 | 0.98023   | 0.01415 |
| 1.85 | 0.96775   | 0.01610 | 0.97079    | 0.01461 | 0.97354   | 0.01326 | 0.97604    | 0.01203 | 0.97830   | 0.01091 |
| 1.9  | 0.96570   | 0.01094 | 0.96892    | 0.00993 | 0.97184   | 0.00901 | 0.97449    | 0.00817 | 0.97689   | 0.00741 |
| 1.95 | 0.96445   | 0.00553 | 0.96778    | 0.00502 | 0.97080   | 0.00456 | 0.97354    | 0.00413 | 0.97603   | 0.00375 |
| 2.0  | 0.96403   | 0.00    | 0.96740    | 0.00    | 0.97045   | 0.00    | 0.97323    | 0.00    | 0.97574   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(2.2 + i0) = 0.97574 + i0$ .  
 $\tanh(2.15 + i1.15) = 1.02440 - i0.01262$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.25$ |         | $x = 2.3$ |         | $x = 2.35$ |         | $x = 2.4$ |         | $x = 2.45$ |         |
|------|------------|---------|-----------|---------|------------|---------|-----------|---------|------------|---------|
| 0    | 0.97803    | 0.00    | 0.98010   | 0.00    | 0.98197    | 0.00    | 0.98367   | 0.00    | 0.98522    | 0.00    |
| 0.05 | 0.97829    | 0.00340 | 0.98034   | 0.00308 | 0.98219    | 0.00280 | 0.98387   | 0.00253 | 0.98540    | 0.00230 |
| 0.1  | 0.97907    | 0.00672 | 0.98105   | 0.00610 | 0.98283    | 0.00553 | 0.98446   | 0.00501 | 0.98592    | 0.00454 |
| 0.15 | 0.98035    | 0.00989 | 0.98221   | 0.00897 | 0.98389    | 0.00813 | 0.98541   | 0.00736 | 0.98680    | 0.00667 |
| 0.2  | 0.98210    | 0.01283 | 0.98380   | 0.01163 | 0.98534    | 0.01054 | 0.98673   | 0.00955 | 0.98799    | 0.00865 |
| 0.25 | 0.98429    | 0.01547 | 0.98579   | 0.01401 | 0.98714    | 0.01270 | 0.98836   | 0.01150 | 0.98947    | 0.01042 |
| 0.3  | 0.98687    | 0.01774 | 0.98813   | 0.01607 | 0.98926    | 0.01456 | 0.99029   | 0.01319 | 0.99121    | 0.01194 |
| 0.35 | 0.98977    | 0.01960 | 0.99076   | 0.01775 | 0.99165    | 0.01607 | 0.99245   | 0.01456 | 0.99317    | 0.01318 |
| 0.4  | 0.99294    | 0.02098 | 0.99363   | 0.01900 | 0.99425    | 0.01720 | 0.99481   | 0.01557 | 0.99531    | 0.01410 |
| 0.45 | 0.99629    | 0.02187 | 0.99667   | 0.01979 | 0.99700    | 0.01791 | 0.99730   | 0.01621 | 0.99757    | 0.01468 |
| 0.5  | 0.99975    | 0.02222 | 0.99980   | 0.02010 | 0.99984    | 0.01819 | 0.99986   | 0.01646 | 0.99989    | 0.01489 |
| 0.55 | 1.00324    | 0.02202 | 1.00295   | 0.01992 | 1.00269    | 0.01802 | 1.00245   | 0.01630 | 1.00222    | 0.01474 |
| 0.6  | 1.00666    | 0.02127 | 1.00604   | 0.01924 | 1.00549    | 0.01740 | 1.00498   | 0.01573 | 1.00451    | 0.01423 |
| 0.65 | 1.00994    | 0.02000 | 1.00900   | 0.01808 | 1.00816    | 0.01634 | 1.00739   | 0.01478 | 1.00693    | 0.01336 |
| 0.7  | 1.01293    | 0.01821 | 1.01175   | 0.01646 | 1.01064    | 0.01487 | 1.00963   | 0.01345 | 1.00872    | 0.01216 |
| 0.75 | 1.01571    | 0.01596 | 1.01421   | 0.01442 | 1.01286    | 0.01303 | 1.01164   | 0.01178 | 1.01053    | 0.01064 |
| 0.8  | 1.01805    | 0.01330 | 1.01632   | 0.01201 | 1.01477    | 0.01085 | 1.01336   | 0.00981 | 1.01208    | 0.00886 |
| 0.85 | 1.01994    | 0.01029 | 1.01803   | 0.00929 | 1.01631    | 0.00839 | 1.01475   | 0.00758 | 1.01333    | 0.00685 |
| 0.9  | 1.02133    | 0.00701 | 1.01928   | 0.00633 | 1.01744    | 0.00572 | 1.01576   | 0.00517 | 1.01425    | 0.00467 |
| 0.95 | 1.02218    | 0.00355 | 1.02006   | 0.00322 | 1.01812    | 0.00290 | 1.01639   | 0.00262 | 1.01482    | 0.00237 |
| 1.0  | 1.02247    | 0.00    | 1.02031   | 0.00    | 1.01836    | 0.00    | 1.01659   | 0.00    | 1.01500    | 0.00    |
| 1.05 | 1.02218    | 0.00355 | 1.02006   | 0.00322 | 1.01812    | 0.00290 | 1.01639   | 0.00262 | 1.01482    | 0.00237 |
| 1.1  | 1.02133    | 0.00701 | 1.01928   | 0.00633 | 1.01744    | 0.00572 | 1.01576   | 0.00517 | 1.01425    | 0.00467 |
| 1.15 | 1.01994    | 0.01029 | 1.01803   | 0.00929 | 1.01631    | 0.00839 | 1.01475   | 0.00758 | 1.01333    | 0.00685 |
| 1.2  | 1.01805    | 0.01330 | 1.01632   | 0.01201 | 1.01477    | 0.01085 | 1.01336   | 0.00981 | 1.01208    | 0.00886 |
| 1.25 | 1.01571    | 0.01596 | 1.01421   | 0.01442 | 1.01286    | 0.01303 | 1.01164   | 0.01178 | 1.01053    | 0.01064 |
| 1.3  | 1.01298    | 0.01821 | 1.01175   | 0.01646 | 1.01064    | 0.01487 | 1.00963   | 0.01345 | 1.00872    | 0.01216 |
| 1.35 | 1.00994    | 0.02000 | 1.00900   | 0.01808 | 1.00816    | 0.01634 | 1.00739   | 0.01478 | 1.00693    | 0.01336 |
| 1.4  | 1.00666    | 0.02127 | 1.00604   | 0.01924 | 1.00549    | 0.01740 | 1.00498   | 0.01573 | 1.00451    | 0.01423 |
| 1.45 | 1.00324    | 0.02202 | 1.00295   | 0.01992 | 1.00269    | 0.01802 | 1.00245   | 0.01630 | 1.00222    | 0.01474 |
| 1.5  | 0.99975    | 0.02222 | 0.99980   | 0.02010 | 0.99984    | 0.01819 | 0.99986   | 0.01646 | 0.99989    | 0.01489 |
| 1.55 | 0.99629    | 0.02187 | 0.99667   | 0.01979 | 0.99700    | 0.01791 | 0.99730   | 0.01621 | 0.99757    | 0.01468 |
| 1.6  | 0.99294    | 0.02098 | 0.99363   | 0.01900 | 0.99425    | 0.01720 | 0.99481   | 0.01557 | 0.99531    | 0.01410 |
| 1.65 | 0.98977    | 0.01960 | 0.99076   | 0.01775 | 0.99165    | 0.01607 | 0.99245   | 0.01456 | 0.99317    | 0.01318 |
| 1.7  | 0.98687    | 0.01774 | 0.98813   | 0.01607 | 0.98926    | 0.01456 | 0.99029   | 0.01319 | 0.99121    | 0.01194 |
| 1.75 | 0.98429    | 0.01547 | 0.98579   | 0.01401 | 0.98714    | 0.01270 | 0.98836   | 0.01150 | 0.98947    | 0.01042 |
| 1.8  | 0.98210    | 0.01283 | 0.98380   | 0.01163 | 0.98534    | 0.01054 | 0.98673   | 0.00955 | 0.98799    | 0.00865 |
| 1.85 | 0.98035    | 0.00989 | 0.98221   | 0.00897 | 0.98389    | 0.00813 | 0.98541   | 0.00736 | 0.98680    | 0.00667 |
| 1.9  | 0.97907    | 0.00672 | 0.98105   | 0.00610 | 0.98283    | 0.00553 | 0.98446   | 0.00501 | 0.98592    | 0.00454 |
| 1.95 | 0.97829    | 0.00340 | 0.98034   | 0.00308 | 0.98219    | 0.00280 | 0.98387   | 0.00253 | 0.98540    | 0.00230 |
| 2.0  | 0.97803    | 0.00    | 0.98010   | 0.00    | 0.98197    | 0.00    | 0.98367   | 0.00    | 0.98522    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(2.25 + i \underline{0.25}) = 0.98429 + i \underline{0.01547}$ .  
 $\tanh(2.45 + i \underline{1.45}) = 1.00222 - i \underline{0.01474}$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.5$ |         | $x = 2.55$ |         | $x = 2.6$ |         | $x = 2.65$ |         | $x = 2.7$ |         |
|------|-----------|---------|------------|---------|-----------|---------|------------|---------|-----------|---------|
| 0    | 0.8661    | 0.00    | 0.98788    | 0.00    | 0.98903   | 0.00    | 0.99007    | 0.00    | 0.99101   | 0.00    |
| 0.05 | 0.98678   | 0.00208 | 0.98803    | 0.00189 | 0.98916   | 0.00171 | 0.99019    | 0.00155 | 0.99112   | 0.00140 |
| 0.1  | 0.98726   | 0.00411 | 0.98846    | 0.00373 | 0.98956   | 0.00337 | 0.99055    | 0.00306 | 0.99144   | 0.00277 |
| 0.15 | 0.98805   | 0.00605 | 0.98918    | 0.00548 | 0.99021   | 0.00496 | 0.99113    | 0.00449 | 0.99198   | 0.00407 |
| 0.2  | 0.98913   | 0.00784 | 0.99016    | 0.00710 | 0.99109   | 0.00643 | 0.99194    | 0.00582 | 0.99270   | 0.00527 |
| 0.25 | 0.99047   | 0.00944 | 0.99138    | 0.00855 | 0.99220   | 0.00774 | 0.99294    | 0.00701 | 0.99361   | 0.00635 |
| 0.3  | 0.99205   | 0.01082 | 0.99281    | 0.00979 | 0.99350   | 0.00887 | 0.99412    | 0.00803 | 0.99468   | 0.00727 |
| 0.35 | 0.99383   | 0.01193 | 0.99442    | 0.01080 | 0.99495   | 0.00978 | 0.99544    | 0.00886 | 0.99588   | 0.00802 |
| 0.4  | 0.99576   | 0.01276 | 0.99617    | 0.01155 | 0.99654   | 0.01046 | 0.99688    | 0.00947 | 0.99718   | 0.00857 |
| 0.45 | 0.99781   | 0.01328 | 0.99802    | 0.01202 | 0.99822   | 0.01088 | 0.99839    | 0.00985 | 0.99855   | 0.00891 |
| 0.5  | 0.99991   | 0.01348 | 0.99993    | 0.01219 | 0.99994   | 0.01103 | 0.99995    | 0.00998 | 0.99996   | 0.00903 |
| 0.55 | 1.00202   | 0.01334 | 1.00184    | 0.01207 | 1.00167   | 0.01092 | 1.00151    | 0.00988 | 1.00137   | 0.00893 |
| 0.6  | 1.00409   | 0.01287 | 1.00371    | 0.01164 | 1.00336   | 0.01053 | 1.00304    | 0.00952 | 1.00276   | 0.00862 |
| 0.65 | 1.00606   | 0.01208 | 1.00549    | 0.01093 | 1.00498   | 0.00988 | 1.00450    | 0.00894 | 1.00408   | 0.00808 |
| 0.7  | 1.00789   | 0.01099 | 1.00714    | 0.00994 | 1.00647   | 0.00898 | 1.00585    | 0.00812 | 1.00530   | 0.00735 |
| 0.75 | 1.00953   | 0.00962 | 1.00862    | 0.00870 | 1.00780   | 0.00786 | 1.00706    | 0.00711 | 1.00639   | 0.00643 |
| 0.8  | 1.01093   | 0.00801 | 1.00989    | 0.00726 | 1.00895   | 0.00654 | 1.00809    | 0.00592 | 1.00732   | 0.00535 |
| 0.85 | 1.01206   | 0.00619 | 1.01091    | 0.00560 | 1.00987   | 0.00506 | 1.00893    | 0.00457 | 1.00807   | 0.00413 |
| 0.9  | 1.01289   | 0.00422 | 1.01166    | 0.00381 | 1.01054   | 0.00345 | 1.00954    | 0.00312 | 1.00862   | 0.00282 |
| 0.95 | 1.01340   | 0.00214 | 1.01211    | 0.00193 | 1.01096   | 0.00175 | 1.00991    | 0.00158 | 1.00896   | 0.00143 |
| 1.0  | 1.01357   | 0.00    | 1.01227    | 0.00    | 1.01110   | 0.00    | 1.01003    | 0.00    | 1.00907   | 0.00    |
| 1.05 | 1.01340   | 0.00214 | 1.01211    | 0.00193 | 1.01096   | 0.00175 | 1.00991    | 0.00158 | 1.00896   | 0.00143 |
| 1.1  | 1.01289   | 0.00422 | 1.01166    | 0.00381 | 1.01054   | 0.00345 | 1.00954    | 0.00312 | 1.00862   | 0.00282 |
| 1.15 | 1.01206   | 0.00619 | 1.01091    | 0.00560 | 1.00987   | 0.00506 | 1.00893    | 0.00457 | 1.00807   | 0.00413 |
| 1.2  | 1.01093   | 0.00801 | 1.00989    | 0.00726 | 1.00895   | 0.00654 | 1.00809    | 0.00592 | 1.00732   | 0.00535 |
| 1.25 | 1.00953   | 0.00962 | 1.00862    | 0.00870 | 1.00780   | 0.00786 | 1.00706    | 0.00711 | 1.00639   | 0.00643 |
| 1.3  | 1.00789   | 0.01099 | 1.00714    | 0.00994 | 1.00647   | 0.00898 | 1.00585    | 0.00812 | 1.00530   | 0.00735 |
| 1.35 | 1.00606   | 0.01208 | 1.00549    | 0.01093 | 1.00498   | 0.00988 | 1.00450    | 0.00894 | 1.00408   | 0.00808 |
| 1.4  | 1.00409   | 0.01287 | 1.00371    | 0.01164 | 1.00336   | 0.01053 | 1.00304    | 0.00952 | 1.00276   | 0.00862 |
| 1.45 | 1.00202   | 0.01334 | 1.00184    | 0.01207 | 1.00167   | 0.01092 | 1.00151    | 0.00988 | 1.00137   | 0.00893 |
| 1.5  | 0.99991   | 0.01348 | 0.99993    | 0.01219 | 0.99994   | 0.01103 | 0.99995    | 0.00998 | 0.99996   | 0.00903 |
| 1.55 | 0.99781   | 0.01328 | 0.99802    | 0.01202 | 0.99822   | 0.01088 | 0.99839    | 0.00985 | 0.99855   | 0.00891 |
| 1.6  | 0.99576   | 0.01276 | 0.99617    | 0.01155 | 0.99654   | 0.01046 | 0.99688    | 0.00947 | 0.99718   | 0.00857 |
| 1.65 | 0.99383   | 0.01193 | 0.99442    | 0.01080 | 0.99495   | 0.00978 | 0.99544    | 0.00886 | 0.99588   | 0.00802 |
| 1.7  | 0.99205   | 0.01082 | 0.99281    | 0.00979 | 0.99350   | 0.00887 | 0.99412    | 0.00803 | 0.99468   | 0.00727 |
| 1.75 | 0.99047   | 0.00944 | 0.99138    | 0.00855 | 0.99220   | 0.00774 | 0.99294    | 0.00701 | 0.99361   | 0.00635 |
| 1.8  | 0.98913   | 0.00784 | 0.99016    | 0.00710 | 0.99109   | 0.00643 | 0.99194    | 0.00582 | 0.99270   | 0.00527 |
| 1.85 | 0.98805   | 0.00605 | 0.98918    | 0.00548 | 0.99021   | 0.00496 | 0.99113    | 0.00449 | 0.99198   | 0.00407 |
| 1.9  | 0.98726   | 0.00411 | 0.98846    | 0.00373 | 0.98956   | 0.00337 | 0.99055    | 0.00306 | 0.99144   | 0.00277 |
| 1.95 | 0.98678   | 0.00208 | 0.98803    | 0.00189 | 0.98916   | 0.00171 | 0.99019    | 0.00155 | 0.99112   | 0.00140 |
| 2.0  | 0.98661   | 0.00    | 0.98788    | 0.00    | 0.98903   | 0.00    | 0.99007    | 0.00    | 0.99101   | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(2.60 + i \underline{0.35}) = 0.99495 + i \underline{0.00978}$ .  
 $\tanh(2.70 + i \underline{1.35}) = 1.00408 - i \underline{0.00808}$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 2.75$ |         | $x = 2.80$ |         | $x = 2.85$ |         | $x = 2.90$ |         | $x = 2.95$ |         |
|------|------------|---------|------------|---------|------------|---------|------------|---------|------------|---------|
| 0    | 0.99186    | 0.00    | 0.99263    | 0.00    | 0.99333    | 0.00    | 0.99396    | 0.00    | 0.99454    | 0.00    |
| 0.05 | 0.99196    | 0.00127 | 0.99272    | 0.00115 | 0.99341    | 0.00104 | 0.99404    | 0.00094 | 0.99460    | 0.00085 |
| 0.1  | 0.99225    | 0.00251 | 0.99299    | 0.00227 | 0.99365    | 0.00206 | 0.99426    | 0.00186 | 0.99480    | 0.00168 |
| 0.15 | 0.99274    | 0.00368 | 0.99343    | 0.00334 | 0.99405    | 0.00302 | 0.99462    | 0.00273 | 0.99513    | 0.00248 |
| 0.2  | 0.99340    | 0.00477 | 0.99403    | 0.00432 | 0.99459    | 0.00391 | 0.99511    | 0.00354 | 0.99557    | 0.00321 |
| 0.25 | 0.99422    | 0.00575 | 0.99477    | 0.00519 | 0.99527    | 0.00471 | 0.99572    | 0.00426 | 0.99613    | 0.00386 |
| 0.3  | 0.99519    | 0.00658 | 0.99564    | 0.00596 | 0.99606    | 0.00539 | 0.99644    | 0.00488 | 0.99678    | 0.00442 |
| 0.35 | 0.99627    | 0.00726 | 0.99663    | 0.00657 | 0.99695    | 0.00594 | 0.99724    | 0.00538 | 0.99750    | 0.00487 |
| 0.4  | 0.99745    | 0.00775 | 0.99769    | 0.00702 | 0.99791    | 0.00635 | 0.99811    | 0.00575 | 0.99830    | 0.00520 |
| 0.45 | 0.99869    | 0.00806 | 0.99882    | 0.00730 | 0.99893    | 0.00660 | 0.99904    | 0.00598 | 0.99913    | 0.00541 |
| 0.5  | 0.99997    | 0.00817 | 0.99997    | 0.00740 | 0.99998    | 0.00669 | 0.99998    | 0.00606 | 0.99999    | 0.00548 |
| 0.55 | 1.00125    | 0.00808 | 1.00113    | 0.00731 | 1.00103    | 0.00662 | 1.00093    | 0.00599 | 1.00084    | 0.00542 |
| 0.6  | 1.00250    | 0.00779 | 1.00226    | 0.00705 | 1.00205    | 0.00638 | 1.00186    | 0.00577 | 1.00168    | 0.00522 |
| 0.65 | 1.00369    | 0.00731 | 1.00334    | 0.00661 | 1.00303    | 0.00598 | 1.00274    | 0.00541 | 1.00248    | 0.00489 |
| 0.7  | 1.00479    | 0.00664 | 1.00434    | 0.00601 | 1.00393    | 0.00544 | 1.00355    | 0.00480 | 1.00322    | 0.00445 |
| 0.75 | 1.00578    | 0.00581 | 1.00523    | 0.00525 | 1.00473    | 0.00475 | 1.00428    | 0.00430 | 1.00387    | 0.00389 |
| 0.8  | 1.00662    | 0.00484 | 1.00599    | 0.00437 | 1.00542    | 0.00396 | 1.00490    | 0.00358 | 1.00444    | 0.00324 |
| 0.85 | 1.00730    | 0.00374 | 1.00661    | 0.00338 | 1.00598    | 0.00306 | 1.00540    | 0.00276 | 1.00489    | 0.00250 |
| 0.9  | 1.00780    | 0.00255 | 1.00706    | 0.00230 | 1.00638    | 0.00208 | 1.00577    | 0.00188 | 1.00522    | 0.00170 |
| 0.95 | 1.00810    | 0.00129 | 1.00733    | 0.00117 | 1.00663    | 0.00105 | 1.00600    | 0.00095 | 1.00542    | 0.00086 |
| 1.0  | 1.00821    | 0.00    | 1.00742    | 0.00    | 1.00671    | 0.00    | 1.00607    | 0.00    | 1.00549    | 0.00    |
| 1.05 | 1.00810    | 0.00129 | 1.00733    | 0.00117 | 1.00663    | 0.00105 | 1.00600    | 0.00095 | 1.00542    | 0.00086 |
| 1.1  | 1.00780    | 0.00255 | 1.00706    | 0.00230 | 1.00638    | 0.00208 | 1.00577    | 0.00188 | 1.00522    | 0.00170 |
| 1.15 | 1.00730    | 0.00374 | 1.00661    | 0.00338 | 1.00598    | 0.00306 | 1.00540    | 0.00276 | 1.00489    | 0.00250 |
| 1.2  | 1.00662    | 0.00484 | 1.00599    | 0.00437 | 1.00542    | 0.00396 | 1.00490    | 0.00358 | 1.00444    | 0.00324 |
| 1.25 | 1.00578    | 0.00581 | 1.00523    | 0.00525 | 1.00473    | 0.00475 | 1.00428    | 0.00430 | 1.00387    | 0.00389 |
| 1.3  | 1.00479    | 0.00664 | 1.00434    | 0.00601 | 1.00393    | 0.00544 | 1.00355    | 0.00480 | 1.00322    | 0.00445 |
| 1.35 | 1.00369    | 0.00731 | 1.00334    | 0.00661 | 1.00303    | 0.00598 | 1.00274    | 0.00541 | 1.00248    | 0.00489 |
| 1.4  | 1.00250    | 0.00779 | 1.00226    | 0.00705 | 1.00205    | 0.00638 | 1.00186    | 0.00577 | 1.00168    | 0.00522 |
| 1.45 | 1.00125    | 0.00808 | 1.00113    | 0.00731 | 1.00103    | 0.00662 | 1.00093    | 0.00599 | 1.00084    | 0.00542 |
| 1.5  | 0.99997    | 0.00817 | 0.99997    | 0.00740 | 0.99998    | 0.00669 | 0.99998    | 0.00606 | 0.99999    | 0.00548 |
| 1.55 | 0.99869    | 0.00806 | 0.99882    | 0.00730 | 0.99893    | 0.00660 | 0.99904    | 0.00598 | 0.99913    | 0.00541 |
| 1.6  | 0.99745    | 0.00775 | 0.99769    | 0.00702 | 0.99791    | 0.00635 | 0.99811    | 0.00575 | 0.99830    | 0.00520 |
| 1.65 | 0.99627    | 0.00726 | 0.99663    | 0.00657 | 0.99695    | 0.00594 | 0.99724    | 0.00538 | 0.99750    | 0.00487 |
| 1.7  | 0.99519    | 0.00658 | 0.99564    | 0.00596 | 0.99606    | 0.00539 | 0.99644    | 0.00488 | 0.99678    | 0.00442 |
| 1.75 | 0.99422    | 0.00575 | 0.99477    | 0.00519 | 0.99527    | 0.00471 | 0.99572    | 0.00426 | 0.99613    | 0.00386 |
| 1.8  | 0.99340    | 0.00477 | 0.99403    | 0.00432 | 0.99459    | 0.00391 | 0.99511    | 0.00354 | 0.99557    | 0.00321 |
| 1.85 | 0.99274    | 0.00368 | 0.99343    | 0.00334 | 0.99405    | 0.00302 | 0.99462    | 0.00273 | 0.99513    | 0.00248 |
| 1.9  | 0.99225    | 0.00251 | 0.99299    | 0.00227 | 0.99365    | 0.00206 | 0.99426    | 0.00186 | 0.99480    | 0.00168 |
| 1.95 | 0.99196    | 0.00127 | 0.99272    | 0.00115 | 0.99341    | 0.00104 | 0.99404    | 0.00094 | 0.99460    | 0.00085 |
| 2.0  | 0.99186    | 0.00    | 0.99263    | 0.00    | 0.99333    | 0.00    | 0.99396    | 0.00    | 0.99454    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(2.9 + i\mathbf{0.9}) = 1.00577 + i\mathbf{0.00188}$ .

$\tanh(2.95 + i\mathbf{1.95}) = 0.99460 - i\mathbf{0.00085}$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.0$ |         | $x = 3.05$ |         | $x = 3.10$ |         | $x = 3.15$ |         | $x = 3.20$ |         |
|------|-----------|---------|------------|---------|------------|---------|------------|---------|------------|---------|
| 0    | 0.99505   | 0.00    | 0.99552    | 0.00    | 0.99595    | 0.00    | 0.99633    | 0.00    | 0.99668    | 0.00    |
| 0.05 | 0.99512   | 0.00077 | 0.99558    | 0.00070 | 0.99600    | 0.00063 | 0.99638    | 0.00057 | 0.99672    | 0.00052 |
| 0.1  | 0.99530   | 0.00153 | 0.99574    | 0.00138 | 0.99615    | 0.00125 | 0.99651    | 0.00113 | 0.99684    | 0.00102 |
| 0.15 | 0.99559   | 0.00224 | 0.99601    | 0.00203 | 0.99639    | 0.00184 | 0.99673    | 0.00166 | 0.99704    | 0.00150 |
| 0.2  | 0.99599   | 0.00290 | 0.99637    | 0.00263 | 0.99672    | 0.00238 | 0.99703    | 0.00215 | 0.99731    | 0.00195 |
| 0.25 | 0.99649   | 0.00349 | 0.99683    | 0.00316 | 0.99713    | 0.00286 | 0.99740    | 0.00259 | 0.99765    | 0.00234 |
| 0.3  | 0.99708   | 0.00400 | 0.99736    | 0.00362 | 0.99761    | 0.00328 | 0.99784    | 0.00297 | 0.99805    | 0.00268 |
| 0.35 | 0.99774   | 0.00441 | 0.99796    | 0.00399 | 0.99815    | 0.00361 | 0.99833    | 0.00327 | 0.99849    | 0.00296 |
| 0.4  | 0.99846   | 0.00471 | 0.99861    | 0.00426 | 0.99874    | 0.00386 | 0.99886    | 0.00349 | 0.99897    | 0.00316 |
| 0.45 | 0.99921   | 0.00489 | 0.99929    | 0.00443 | 0.99936    | 0.00401 | 0.99942    | 0.00363 | 0.99947    | 0.00328 |
| 0.5  | 0.99999   | 0.00496 | 0.99999    | 0.00449 | 0.99999    | 0.00406 | 0.99999    | 0.00367 | 0.99999    | 0.00332 |
| 0.55 | 1.00076   | 0.00490 | 1.00069    | 0.00443 | 1.00063    | 0.00401 | 1.00057    | 0.00363 | 1.00051    | 0.00328 |
| 0.6  | 1.00152   | 0.00472 | 1.00138    | 0.00427 | 1.00125    | 0.00387 | 1.00113    | 0.00350 | 1.00102    | 0.00316 |
| 0.65 | 1.00224   | 0.00443 | 1.00203    | 0.00401 | 1.00184    | 0.00362 | 1.00166    | 0.00328 | 1.00151    | 0.00297 |
| 0.7  | 1.00291   | 0.00402 | 1.00263    | 0.00364 | 1.00238    | 0.00329 | 1.00216    | 0.00298 | 1.00195    | 0.00269 |
| 0.75 | 1.00351   | 0.00352 | 1.00317    | 0.00318 | 1.00287    | 0.00288 | 1.00260    | 0.00260 | 1.00235    | 0.00236 |
| 0.8  | 1.00401   | 0.00293 | 1.00363    | 0.00265 | 1.00329    | 0.00239 | 1.00297    | 0.00217 | 1.00269    | 0.00196 |
| 0.85 | 1.00443   | 0.00226 | 1.00400    | 0.00205 | 1.00362    | 0.00185 | 1.00328    | 0.00167 | 1.00297    | 0.00151 |
| 0.9  | 1.00473   | 0.00154 | 1.00426    | 0.00139 | 1.00387    | 0.00126 | 1.00350    | 0.00114 | 1.00317    | 0.00103 |
| 0.95 | 1.00491   | 0.00078 | 1.00444    | 0.00071 | 1.00402    | 0.00064 | 1.00363    | 0.00058 | 1.00329    | 0.00052 |
| 1.0  | 1.00497   | 0.00    | 1.00450    | 0.00    | 1.00407    | 0.00    | 1.00368    | 0.00    | 1.00333    | 0.00    |
| 1.05 | 1.00491   | 0.00078 | 1.00444    | 0.00071 | 1.00402    | 0.00064 | 1.00363    | 0.00058 | 1.00329    | 0.00052 |
| 1.1  | 1.00473   | 0.00154 | 1.00426    | 0.00139 | 1.00387    | 0.00126 | 1.00350    | 0.00114 | 1.00317    | 0.00103 |
| 1.15 | 1.00443   | 0.00226 | 1.00400    | 0.00205 | 1.00362    | 0.00185 | 1.00328    | 0.00167 | 1.00297    | 0.00151 |
| 1.2  | 1.00401   | 0.00293 | 1.00363    | 0.00265 | 1.00329    | 0.00239 | 1.00297    | 0.00217 | 1.00269    | 0.00196 |
| 1.25 | 1.00351   | 0.00352 | 1.00317    | 0.00318 | 1.00287    | 0.00288 | 1.00260    | 0.00260 | 1.00235    | 0.00236 |
| 1.3  | 1.00291   | 0.00402 | 1.00263    | 0.00364 | 1.00238    | 0.00329 | 1.00216    | 0.00298 | 1.00195    | 0.00269 |
| 1.35 | 1.00224   | 0.00443 | 1.00203    | 0.00401 | 1.00184    | 0.00362 | 1.00166    | 0.00328 | 1.00151    | 0.00297 |
| 1.4  | 1.00152   | 0.00472 | 1.00138    | 0.00427 | 1.00125    | 0.00387 | 1.00113    | 0.00350 | 1.00102    | 0.00316 |
| 1.45 | 1.00076   | 0.00490 | 1.00069    | 0.00443 | 1.00063    | 0.00401 | 1.00057    | 0.00363 | 1.00051    | 0.00328 |
| 1.5  | 0.99999   | 0.00496 | 0.99999    | 0.00449 | 0.99999    | 0.00406 | 0.99999    | 0.00367 | 0.99999    | 0.00332 |
| 1.55 | 0.99921   | 0.00489 | 0.99929    | 0.00443 | 0.99936    | 0.00401 | 0.99942    | 0.00363 | 0.99947    | 0.00328 |
| 1.6  | 0.99846   | 0.00471 | 0.99861    | 0.00426 | 0.99874    | 0.00386 | 0.99886    | 0.00349 | 0.99897    | 0.00316 |
| 1.65 | 0.99774   | 0.00441 | 0.99796    | 0.00399 | 0.99815    | 0.00361 | 0.99833    | 0.00327 | 0.99849    | 0.00296 |
| 1.7  | 0.99708   | 0.00400 | 0.99736    | 0.00362 | 0.99761    | 0.00328 | 0.99784    | 0.00297 | 0.99805    | 0.00268 |
| 1.75 | 0.99649   | 0.00349 | 0.99683    | 0.00316 | 0.99713    | 0.00286 | 0.99740    | 0.00259 | 0.99765    | 0.00234 |
| 1.8  | 0.99599   | 0.00290 | 0.99637    | 0.00263 | 0.99672    | 0.00238 | 0.99703    | 0.00215 | 0.99731    | 0.00195 |
| 1.85 | 0.99559   | 0.00224 | 0.99601    | 0.00203 | 0.99639    | 0.00184 | 0.99673    | 0.00166 | 0.99704    | 0.00150 |
| 1.9  | 0.99530   | 0.00153 | 0.99574    | 0.00138 | 0.99615    | 0.00125 | 0.99651    | 0.00113 | 0.99684    | 0.00102 |
| 1.95 | 0.99512   | 0.00077 | 0.99558    | 0.00070 | 0.99600    | 0.00063 | 0.99638    | 0.00057 | 0.99672    | 0.00052 |
| 2.0  | 0.99505   | 0.00    | 0.99552    | 0.00    | 0.99595    | 0.00    | 0.99633    | 0.00    | 0.99668    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(3.0 + i \underline{1.00}) = 1.00497 + i 0.$   
 $\tanh(3.0 + i \underline{1.50}) = 0.99999 - i 0.00496.$

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.25$ |         | $x = 3.30$ |         | $x = 3.35$ |         | $x = 3.40$ |         | $x = 3.45$ |         |
|------|------------|---------|------------|---------|------------|---------|------------|---------|------------|---------|
| 0    | 0.99700    | 0.00    | 0.99728    | 0.00    | 0.99754    | 0.00    | 0.99777    | 0.00    | 0.99799    | 0.00    |
| 0.05 | 0.99704    | 0.00047 | 0.99732    | 0.00042 | 0.99757    | 0.00038 | 0.99780    | 0.00035 | 0.99801    | 0.00031 |
| 0.1  | 0.99715    | 0.00093 | 0.99742    | 0.00084 | 0.99766    | 0.00076 | 0.99788    | 0.00069 | 0.99809    | 0.00062 |
| 0.15 | 0.99732    | 0.00136 | 0.99758    | 0.00123 | 0.99781    | 0.00112 | 0.99802    | 0.00101 | 0.99821    | 0.00091 |
| 0.2  | 0.99757    | 0.00176 | 0.99780    | 0.00160 | 0.99801    | 0.00144 | 0.99820    | 0.00131 | 0.99837    | 0.00118 |
| 0.25 | 0.99787    | 0.00212 | 0.99808    | 0.00192 | 0.99826    | 0.00174 | 0.99843    | 0.00157 | 0.99857    | 0.00142 |
| 0.3  | 0.98823    | 0.00243 | 0.99840    | 0.00220 | 0.99855    | 0.00199 | 0.99869    | 0.00180 | 0.99881    | 0.00163 |
| 0.35 | 0.99863    | 0.00268 | 0.99876    | 0.00242 | 0.99888    | 0.00219 | 0.99899    | 0.00198 | 0.99908    | 0.00179 |
| 0.4  | 0.99907    | 0.00286 | 0.99916    | 0.00259 | 0.99924    | 0.00234 | 0.99931    | 0.00212 | 0.99938    | 0.00192 |
| 0.45 | 0.99953    | 0.00297 | 0.99957    | 0.00269 | 0.99961    | 0.00243 | 0.99965    | 0.00220 | 0.99968    | 0.00200 |
| 0.5  | 1.00000    | 0.00301 | 1.00000    | 0.00272 | 1.00000    | 0.00246 | 1.00000    | 0.00223 | 1.00000    | 0.00202 |
| 0.55 | 1.00047    | 0.00297 | 1.00042    | 0.00269 | 1.00038    | 0.00243 | 1.00035    | 0.00220 | 1.00031    | 0.00200 |
| 0.6  | 1.00091    | 0.00286 | 1.00084    | 0.00259 | 1.00076    | 0.00234 | 1.00069    | 0.00212 | 1.00062    | 0.00192 |
| 0.65 | 1.00136    | 0.00268 | 1.00123    | 0.00243 | 1.00112    | 0.00220 | 1.00101    | 0.00199 | 1.00091    | 0.00180 |
| 0.7  | 1.00177    | 0.00244 | 1.00160    | 0.00220 | 1.00145    | 0.00199 | 1.00131    | 0.00180 | 1.00118    | 0.00163 |
| 0.75 | 1.00213    | 0.00213 | 1.00192    | 0.00193 | 1.00174    | 0.00174 | 1.00158    | 0.00158 | 1.00143    | 0.00143 |
| 0.8  | 1.00244    | 0.00177 | 1.00222    | 0.00160 | 1.00199    | 0.00145 | 1.00180    | 0.00131 | 1.00163    | 0.00119 |
| 0.85 | 1.00268    | 0.00137 | 1.00242    | 0.00124 | 1.00220    | 0.00112 | 1.00199    | 0.00103 | 1.00180    | 0.00092 |
| 0.9  | 1.00286    | 0.00093 | 1.00260    | 0.00084 | 1.00234    | 0.00076 | 1.00212    | 0.00069 | 1.00192    | 0.00062 |
| 0.95 | 1.00298    | 0.00047 | 1.00269    | 0.00043 | 1.00243    | 0.00039 | 1.00220    | 0.00035 | 1.00200    | 0.00032 |
| 1.0  | 1.00301    | 0.00    | 1.00273    | 0.00    | 1.00246    | 0.00    | 1.00223    | 0.00    | 1.00202    | 0.00    |
| 1.05 | 1.00298    | 0.00047 | 1.00269    | 0.00043 | 1.00243    | 0.00039 | 1.00220    | 0.00035 | 1.00200    | 0.00032 |
| 1.1  | 1.00286    | 0.00093 | 1.00260    | 0.00084 | 1.00234    | 0.00076 | 1.00212    | 0.00069 | 1.00192    | 0.00062 |
| 1.15 | 1.00268    | 0.00137 | 1.00242    | 0.00124 | 1.00220    | 0.00112 | 1.00199    | 0.00103 | 1.00180    | 0.00092 |
| 1.2  | 1.00244    | 0.00177 | 1.00222    | 0.00160 | 1.00199    | 0.00145 | 1.00180    | 0.00131 | 1.00163    | 0.00119 |
| 1.25 | 1.00213    | 0.00213 | 1.00192    | 0.00193 | 1.00174    | 0.00174 | 1.00158    | 0.00158 | 1.00143    | 0.00143 |
| 1.3  | 1.00177    | 0.00244 | 1.00160    | 0.00220 | 1.00145    | 0.00199 | 1.00131    | 0.00180 | 1.00118    | 0.00163 |
| 1.35 | 1.00136    | 0.00268 | 1.00123    | 0.00243 | 1.00112    | 0.00220 | 1.00101    | 0.00199 | 1.00091    | 0.00180 |
| 1.4  | 1.00091    | 0.00286 | 1.00084    | 0.00259 | 1.00076    | 0.00234 | 1.00069    | 0.00212 | 1.00062    | 0.00192 |
| 1.45 | 1.00047    | 0.00297 | 1.00042    | 0.00269 | 1.00038    | 0.00243 | 1.00035    | 0.00220 | 1.00031    | 0.00200 |
| 1.5  | 1.00000    | 0.00301 | 1.00000    | 0.00272 | 1.00000    | 0.00246 | 1.00000    | 0.00223 | 1.00000    | 0.00202 |
| 1.55 | 0.99953    | 0.00297 | 0.99957    | 0.00269 | 0.99961    | 0.00243 | 0.99965    | 0.00220 | 0.99968    | 0.00200 |
| 1.6  | 0.99907    | 0.00286 | 0.99916    | 0.00259 | 0.99924    | 0.00234 | 0.99931    | 0.00212 | 0.99938    | 0.00192 |
| 1.65 | 0.99863    | 0.00268 | 0.99876    | 0.00242 | 0.99888    | 0.00219 | 0.99899    | 0.00198 | 0.99908    | 0.00179 |
| 1.7  | 0.99823    | 0.00243 | 0.99840    | 0.00220 | 0.99855    | 0.00199 | 0.99869    | 0.00180 | 0.99881    | 0.00163 |
| 1.75 | 0.99787    | 0.00212 | 0.99808    | 0.00192 | 0.99826    | 0.00174 | 0.99843    | 0.00157 | 0.99857    | 0.00142 |
| 1.8  | 0.99757    | 0.00176 | 0.99780    | 0.00160 | 0.99801    | 0.00144 | 0.99820    | 0.00131 | 0.99837    | 0.00118 |
| 1.85 | 0.99732    | 0.00136 | 0.99758    | 0.00123 | 0.99781    | 0.00112 | 0.99802    | 0.00101 | 0.99821    | 0.00091 |
| 1.9  | 0.99715    | 0.00093 | 0.99742    | 0.00084 | 0.99766    | 0.00076 | 0.99788    | 0.00069 | 0.99809    | 0.00062 |
| 1.95 | 0.99704    | 0.00047 | 0.99732    | 0.00042 | 0.99757    | 0.00038 | 0.99780    | 0.00035 | 0.99801    | 0.00031 |
| 2.0  | 0.99700    | 0.00    | 0.99728    | 0.00    | 0.99754    | 0.00    | 0.99777    | 0.00    | 0.99799    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(3.25 + i 0.75) = 1.00213 + i 0.00213$ .  
 $\tanh(3.30 + i 1.50) = 1.00000 - i 0.00272$ .



TABLE IX HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.50$ |         | $x = 3.55$ |         | $x = 3.60$ |         | $x = 3.65$ |         | $x = 3.70$ |         |
|------|------------|---------|------------|---------|------------|---------|------------|---------|------------|---------|
| 0    | 0.99818    | 0.00    | 0.99835    | 0.00    | 0.99851    | 0.00    | 0.99865    | 0.00    | 0.99878    | 0.00    |
| 0.05 | 0.99820    | 0.00028 | 0.99837    | 0.00026 | 0.99853    | 0.00023 | 0.99867    | 0.00021 | 0.99879    | 0.00019 |
| 0.1  | 0.99827    | 0.00056 | 0.99843    | 0.00051 | 0.99858    | 0.00046 | 0.99872    | 0.00042 | 0.99884    | 0.00038 |
| 0.15 | 0.99837    | 0.00083 | 0.99853    | 0.00075 | 0.99867    | 0.00068 | 0.99880    | 0.00061 | 0.99891    | 0.00055 |
| 0.2  | 0.99853    | 0.00107 | 0.99867    | 0.00097 | 0.99879    | 0.00088 | 0.99891    | 0.00079 | 0.99901    | 0.00072 |
| 0.25 | 0.99871    | 0.00129 | 0.99883    | 0.00117 | 0.99894    | 0.00105 | 0.99904    | 0.00095 | 0.99914    | 0.00086 |
| 0.3  | 0.99893    | 0.00147 | 0.99903    | 0.00133 | 0.99912    | 0.00121 | 0.99921    | 0.00109 | 0.99928    | 0.00099 |
| 0.35 | 0.99917    | 0.00162 | 0.99925    | 0.00147 | 0.99932    | 0.00133 | 0.99939    | 0.00120 | 0.99944    | 0.00109 |
| 0.4  | 0.99944    | 0.00173 | 0.99949    | 0.00157 | 0.99954    | 0.00142 | 0.99958    | 0.00128 | 0.99962    | 0.00116 |
| 0.45 | 0.99971    | 0.00180 | 0.99974    | 0.00163 | 0.99977    | 0.00147 | 0.99979    | 0.00133 | 0.99981    | 0.00121 |
| 0.5  | 1.00000    | 0.00182 | 1.00000    | 0.00165 | 1.00000    | 0.00149 | 1.00000    | 0.00135 | 1.00000    | 0.00122 |
| 0.55 | 1.00028    | 0.00180 | 1.00026    | 0.00163 | 1.00025    | 0.00148 | 1.00021    | 0.00133 | 1.00019    | 0.00121 |
| 0.6  | 1.00056    | 0.00174 | 1.00051    | 0.00157 | 1.00048    | 0.00142 | 1.00042    | 0.00129 | 1.00038    | 0.00116 |
| 0.65 | 1.00083    | 0.00163 | 1.00075    | 0.00147 | 1.00068    | 0.00133 | 1.00061    | 0.00120 | 1.00055    | 0.00109 |
| 0.7  | 1.00107    | 0.00148 | 1.00097    | 0.00134 | 1.00088    | 0.00121 | 1.00079    | 0.00109 | 1.00072    | 0.00099 |
| 0.75 | 1.00129    | 0.00129 | 1.00117    | 0.00117 | 1.00106    | 0.00106 | 1.00096    | 0.00096 | 1.00086    | 0.00086 |
| 0.8  | 1.00148    | 0.00107 | 1.00133    | 0.00097 | 1.00121    | 0.00088 | 1.00108    | 0.00080 | 1.00099    | 0.00072 |
| 0.85 | 1.00163    | 0.00083 | 1.00147    | 0.00075 | 1.00133    | 0.00068 | 1.00120    | 0.00061 | 1.00109    | 0.00056 |
| 0.9  | 1.00174    | 0.00056 | 1.00157    | 0.00051 | 1.00142    | 0.00046 | 1.00129    | 0.00042 | 1.00116    | 0.00038 |
| 0.95 | 1.00180    | 0.00029 | 1.00163    | 0.00026 | 1.00148    | 0.00023 | 1.00134    | 0.00021 | 1.00121    | 0.00019 |
| 1.0  | 1.00183    | 0.00    | 1.00165    | 0.00    | 1.00149    | 0.00    | 1.00135    | 0.00    | 1.00122    | 0.00    |
| 1.05 | 1.00180    | 0.00029 | 1.00163    | 0.00026 | 1.00148    | 0.00023 | 1.00134    | 0.00021 | 1.00121    | 0.00019 |
| 1.1  | 1.00174    | 0.00056 | 1.00157    | 0.00051 | 1.00142    | 0.00046 | 1.00129    | 0.00042 | 1.00116    | 0.00038 |
| 1.15 | 1.00163    | 0.00083 | 1.00147    | 0.00075 | 1.00133    | 0.00068 | 1.00120    | 0.00061 | 1.00109    | 0.00056 |
| 1.2  | 1.00148    | 0.00107 | 1.00133    | 0.00097 | 1.00121    | 0.00088 | 1.00108    | 0.00080 | 1.00099    | 0.00072 |
| 1.25 | 1.00129    | 0.00129 | 1.00117    | 0.00117 | 1.00106    | 0.00106 | 1.00096    | 0.00096 | 1.00086    | 0.00086 |
| 1.3  | 1.00107    | 0.00148 | 1.00097    | 0.00134 | 1.00088    | 0.00121 | 1.00079    | 0.00109 | 1.00072    | 0.00099 |
| 1.35 | 1.00083    | 0.00163 | 1.00075    | 0.00147 | 1.00068    | 0.00133 | 1.00061    | 0.00120 | 1.00055    | 0.00109 |
| 1.4  | 1.00056    | 0.00174 | 1.00051    | 0.00157 | 1.00048    | 0.00142 | 1.00042    | 0.00129 | 1.00038    | 0.00116 |
| 1.45 | 1.00028    | 0.00180 | 1.00026    | 0.00163 | 1.00025    | 0.00148 | 1.00021    | 0.00133 | 1.00019    | 0.00121 |
| 1.5  | 1.00000    | 0.00182 | 1.00000    | 0.00165 | 1.00000    | 0.00149 | 1.00000    | 0.00135 | 1.00000    | 0.00122 |
| 1.55 | 0.99971    | 0.00180 | 0.99974    | 0.00163 | 0.99977    | 0.00147 | 0.99979    | 0.00133 | 0.99981    | 0.00121 |
| 1.6  | 0.99944    | 0.00173 | 0.99949    | 0.00157 | 0.99954    | 0.00142 | 0.99958    | 0.00128 | 0.99962    | 0.00116 |
| 1.65 | 0.99917    | 0.00162 | 0.99925    | 0.00147 | 0.99932    | 0.00133 | 0.99939    | 0.00120 | 0.99944    | 0.00109 |
| 1.7  | 0.99893    | 0.00147 | 0.99903    | 0.00133 | 0.99912    | 0.00121 | 0.99921    | 0.00109 | 0.99928    | 0.00099 |
| 1.75 | 0.99871    | 0.00129 | 0.99883    | 0.00117 | 0.99894    | 0.00105 | 0.99904    | 0.00095 | 0.99914    | 0.00086 |
| 1.8  | 0.99853    | 0.00107 | 0.99867    | 0.00097 | 0.99879    | 0.00088 | 0.99891    | 0.00079 | 0.99901    | 0.00072 |
| 1.85 | 0.99837    | 0.00083 | 0.99853    | 0.00075 | 0.99867    | 0.00068 | 0.99880    | 0.00061 | 0.99891    | 0.00055 |
| 1.9  | 0.99827    | 0.00056 | 0.99843    | 0.00051 | 0.99858    | 0.00046 | 0.99872    | 0.00042 | 0.99884    | 0.00038 |
| 1.95 | 0.99820    | 0.00028 | 0.99837    | 0.00026 | 0.99853    | 0.00023 | 0.99867    | 0.00021 | 0.99879    | 0.00019 |
| 2.0  | 0.99818    | 0.00    | 0.99835    | 0.00    | 0.99851    | 0.00    | 0.99865    | 0.00    | 0.99878    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(3.60 + i \underline{0.80}) = 1.00121 + i \underline{0.00088}$ .  
 $\tanh(3.70 + i \underline{1.70}) = 0.99928 - i \underline{0.00099}$ .

TABLE IX. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = u + iv$ . CONTINUED

| $q$  | $x = 3.75$ |         | $x = 3.80$ |         | $x = 3.85$ |         | $x = 3.90$ |         | $x = 3.95$ |         |
|------|------------|---------|------------|---------|------------|---------|------------|---------|------------|---------|
| 0    | 0.99889    | 0.00    | 0.99900    | 0.00    | 0.99909    | 0.00    | 0.99918    | 0.00    | 0.99926    | 0.00    |
| 0.05 | 0.99891    | 0.00017 | 0.99901    | 0.00016 | 0.99911    | 0.00014 | 0.99919    | 0.00013 | 0.99927    | 0.00012 |
| 0.1  | 0.99895    | 0.00034 | 0.99905    | 0.00031 | 0.99914    | 0.00028 | 0.99922    | 0.00025 | 0.99930    | 0.00023 |
| 0.15 | 0.99901    | 0.00050 | 0.99911    | 0.00045 | 0.99919    | 0.00041 | 0.99927    | 0.00037 | 0.99934    | 0.00034 |
| 0.2  | 0.99911    | 0.00065 | 0.99919    | 0.00059 | 0.99927    | 0.00053 | 0.99934    | 0.00048 | 0.99940    | 0.00044 |
| 0.25 | 0.99922    | 0.00078 | 0.99929    | 0.00071 | 0.99936    | 0.00064 | 0.99942    | 0.00058 | 0.99948    | 0.00052 |
| 0.3  | 0.99935    | 0.00089 | 0.99941    | 0.00081 | 0.99947    | 0.00073 | 0.99952    | 0.00066 | 0.99956    | 0.00060 |
| 0.35 | 0.99950    | 0.00099 | 0.99955    | 0.00089 | 0.99959    | 0.00081 | 0.99963    | 0.00073 | 0.99966    | 0.00066 |
| 0.4  | 0.99966    | 0.00105 | 0.99969    | 0.00095 | 0.99972    | 0.00086 | 0.99975    | 0.00078 | 0.99977    | 0.00071 |
| 0.45 | 0.99983    | 0.00109 | 0.99984    | 0.00099 | 0.99986    | 0.00089 | 0.99987    | 0.00081 | 0.99988    | 0.00073 |
| 0.5  | 1.00000    | 0.00111 | 1.00000    | 0.00100 | 1.00000    | 0.00091 | 1.00000    | 0.00082 | 1.00000    | 0.00074 |
| 0.55 | 1.00017    | 0.00109 | 1.00016    | 0.00099 | 1.00014    | 0.00089 | 1.00013    | 0.00081 | 1.00012    | 0.00073 |
| 0.6  | 1.00034    | 0.00105 | 1.00031    | 0.00095 | 1.00028    | 0.00086 | 1.00025    | 0.00078 | 1.00023    | 0.00071 |
| 0.65 | 1.00050    | 0.00099 | 1.00045    | 0.00089 | 1.00041    | 0.00081 | 1.00037    | 0.00073 | 1.00034    | 0.00066 |
| 0.7  | 1.00065    | 0.00090 | 1.00059    | 0.00081 | 1.00053    | 0.00073 | 1.00048    | 0.00066 | 1.00044    | 0.00060 |
| 0.75 | 1.00078    | 0.00078 | 1.00071    | 0.00071 | 1.00064    | 0.00064 | 1.00058    | 0.00058 | 1.00052    | 0.00052 |
| 0.8  | 1.00089    | 0.00065 | 1.00081    | 0.00059 | 1.00073    | 0.00053 | 1.00066    | 0.00048 | 1.00060    | 0.00044 |
| 0.85 | 1.00099    | 0.00050 | 1.00089    | 0.00045 | 1.00081    | 0.00041 | 1.00073    | 0.00037 | 1.00066    | 0.00034 |
| 0.9  | 1.00105    | 0.00034 | 1.00096    | 0.00031 | 1.00086    | 0.00028 | 1.00078    | 0.00025 | 1.00071    | 0.00023 |
| 0.95 | 1.00109    | 0.00017 | 1.00099    | 0.00016 | 1.00089    | 0.00014 | 1.00081    | 0.00013 | 1.00073    | 0.00012 |
| 1.0  | 1.00111    | 0.00    | 1.00100    | 0.00    | 1.00090    | 0.00    | 1.00082    | 0.00    | 1.00074    | 0.00    |
| 1.05 | 1.00109    | 0.00017 | 1.00099    | 0.00016 | 1.00089    | 0.00014 | 1.00081    | 0.00013 | 1.00073    | 0.00012 |
| 1.1  | 1.00105    | 0.00034 | 1.00096    | 0.00031 | 1.00086    | 0.00028 | 1.00078    | 0.00025 | 1.00071    | 0.00023 |
| 1.15 | 1.00099    | 0.00050 | 1.00089    | 0.00045 | 1.00081    | 0.00041 | 1.00073    | 0.00037 | 1.00066    | 0.00034 |
| 1.2  | 1.00089    | 0.00065 | 1.00081    | 0.00059 | 1.00073    | 0.00053 | 1.00066    | 0.00048 | 1.00060    | 0.00044 |
| 1.25 | 1.00078    | 0.00078 | 1.00071    | 0.00071 | 1.00064    | 0.00064 | 1.00058    | 0.00058 | 1.00052    | 0.00052 |
| 1.3  | 1.00065    | 0.00090 | 1.00059    | 0.00081 | 1.00053    | 0.00073 | 1.00048    | 0.00066 | 1.00044    | 0.00060 |
| 1.35 | 1.00050    | 0.00099 | 1.00045    | 0.00089 | 1.00041    | 0.00081 | 1.00037    | 0.00073 | 1.00034    | 0.00066 |
| 1.4  | 1.00034    | 0.00105 | 1.00031    | 0.00095 | 1.00028    | 0.00086 | 1.00025    | 0.00078 | 1.00023    | 0.00071 |
| 1.45 | 1.00017    | 0.00109 | 1.00016    | 0.00099 | 1.00014    | 0.00089 | 1.00013    | 0.00081 | 1.00012    | 0.00073 |
| 1.5  | 1.00000    | 0.00111 | 1.00000    | 0.00100 | 1.00000    | 0.00091 | 1.00000    | 0.00082 | 1.00000    | 0.00074 |
| 1.55 | 0.99983    | 0.00109 | 0.99984    | 0.00099 | 0.99986    | 0.00089 | 0.99987    | 0.00081 | 0.99988    | 0.00073 |
| 1.6  | 0.99966    | 0.00105 | 0.99969    | 0.00095 | 0.99972    | 0.00086 | 0.99975    | 0.00078 | 0.99977    | 0.00071 |
| 1.65 | 0.99950    | 0.00099 | 0.99955    | 0.00089 | 0.99959    | 0.00081 | 0.99963    | 0.00073 | 0.99966    | 0.00066 |
| 1.7  | 0.99935    | 0.00089 | 0.99941    | 0.00081 | 0.99947    | 0.00073 | 0.99952    | 0.00066 | 0.99956    | 0.00060 |
| 1.75 | 0.99922    | 0.00078 | 0.99929    | 0.00071 | 0.99936    | 0.00064 | 0.99942    | 0.00058 | 0.99948    | 0.00052 |
| 1.8  | 0.99911    | 0.00065 | 0.99919    | 0.00059 | 0.99927    | 0.00053 | 0.99934    | 0.00048 | 0.99940    | 0.00044 |
| 1.85 | 0.99901    | 0.00050 | 0.99911    | 0.00045 | 0.99919    | 0.00041 | 0.99927    | 0.00037 | 0.99934    | 0.00034 |
| 1.9  | 0.99895    | 0.00034 | 0.99905    | 0.00031 | 0.99914    | 0.00028 | 0.99922    | 0.00025 | 0.99930    | 0.00023 |
| 1.95 | 0.99891    | 0.00017 | 0.99901    | 0.00016 | 0.99911    | 0.00014 | 0.99919    | 0.00013 | 0.99927    | 0.00012 |
| 2.0  | 0.99889    | 0.00    | 0.99900    | 0.00    | 0.99909    | 0.00    | 0.99918    | 0.00    | 0.99926    | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(3.95 + i \underline{0.95}) = 1.00073 + i \underline{0.00012}$ .  
 $\tanh(3.95 + i \underline{1.05}) = 1.00073 - i \underline{0.00012}$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$

| $q$  | $x = 0.0$ |          | $x = 0.05$ |          | $x = 0.1$ |          | $x = 0.15$ |          | $x = 0.2$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
|      | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 0.000     | 90       | 0.05002    | 0.000    | 0.10017   | 0.000    | 0.15056    | 0.000    | 0.20134   | 0.000    |
| 0.05 | 0.07846   | 90       | 0.09305    | 57.593   | 0.12724   | 38.300   | 0.16978    | 27.861   | 0.21608   | 21.739   |
| 0.1  | 0.15643   | 90       | 0.16424    | 72.493   | 0.18576   | 57.819   | 0.21712    | 46.771   | 0.25497   | 38.746   |
| 0.15 | 0.23345   | 90       | 0.23874    | 78.245   | 0.25403   | 67.455   | 0.27779    | 58.195   | 0.30827   | 50.576   |
| 0.2  | 0.30902   | 90       | 0.31304    | 81.259   | 0.32485   | 72.947   | 0.34375    | 65.382   | 0.36882   | 58.723   |
| 0.25 | 0.38268   | 90       | 0.38594    | 83.123   | 0.39558   | 76.471   | 0.41124    | 70.229   | 0.43422   | 64.522   |
| 0.3  | 0.45399   | 90       | 0.45672    | 84.400   | 0.46491   | 78.932   | 0.47831    | 73.711   | 0.49663   | 68.825   |
| 0.35 | 0.52250   | 90       | 0.52487    | 85.339   | 0.53201   | 80.762   | 0.54376    | 76.344   | 0.55995   | 72.147   |
| 0.4  | 0.58778   | 90       | 0.58989    | 86.066   | 0.59626   | 82.189   | 0.60676    | 78.419   | 0.62131   | 74.802   |
| 0.45 | 0.64944   | 90       | 0.65135    | 86.652   | 0.65713   | 83.344   | 0.66667    | 80.111   | 0.67994   | 76.988   |
| 0.5  | 0.70711   | 90       | 0.70803    | 87.139   | 0.71417   | 84.308   | 0.72296    | 81.532   | 0.73521   | 78.835   |
| 0.55 | 0.76041   | 90       | 0.76202    | 87.557   | 0.76698   | 85.134   | 0.77527    | 82.753   | 0.78661   | 80.431   |
| 0.6  | 0.80902   | 90       | 0.81053    | 87.921   | 0.81520   | 85.858   | 0.82291    | 83.826   | 0.83370   | 81.839   |
| 0.65 | 0.85264   | 90       | 0.85407    | 88.246   | 0.85851   | 86.505   | 0.86583    | 84.787   | 0.87609   | 83.103   |
| 0.7  | 0.89101   | 90       | 0.89237    | 88.542   | 0.89662   | 87.093   | 0.90364    | 85.662   | 0.91347   | 84.257   |
| 0.75 | 0.92388   | 90       | 0.92523    | 88.814   | 0.92930   | 87.636   | 0.93607    | 86.471   | 0.94556   | 85.326   |
| 0.8  | 0.95106   | 90       | 0.95237    | 89.070   | 0.95632   | 88.145   | 0.96290    | 87.231   | 0.97213   | 86.331   |
| 0.85 | 0.97237   | 90       | 0.97366    | 89.313   | 0.97752   | 88.629   | 0.98396    | 87.953   | 0.99300   | 87.287   |
| 0.9  | 0.98769   | 90       | 0.98895    | 89.547   | 0.99275   | 89.095   | 0.99909    | 88.649   | 1.00800   | 88.209   |
| 0.95 | 0.99692   | 90       | 0.99817    | 89.775   | 1.00194   | 89.550   | 1.00822    | 89.329   | 1.01704   | 89.110   |
| 1.0  | 1.00000   | 90       | 1.00125    | 90.000   | 1.00500   | 90.000   | 1.01127    | 90.000   | 1.02007   | 90.000   |
| 1.05 | 0.99692   | 90       | 0.99817    | 90.225   | 1.00194   | 90.450   | 1.00822    | 90.671   | 1.01704   | 90.890   |
| 1.1  | 0.98769   | 90       | 0.98895    | 90.453   | 0.99275   | 90.905   | 0.99909    | 91.351   | 1.00800   | 91.791   |
| 1.15 | 0.97237   | 90       | 0.97366    | 90.687   | 0.97752   | 91.371   | 0.98396    | 92.047   | 0.99300   | 92.713   |
| 1.2  | 0.95106   | 90       | 0.95237    | 90.930   | 0.95632   | 91.855   | 0.96290    | 92.769   | 0.97213   | 93.669   |
| 1.25 | 0.92388   | 90       | 0.92523    | 91.186   | 0.92930   | 92.364   | 0.93607    | 93.529   | 0.94556   | 94.674   |
| 1.3  | 0.89101   | 90       | 0.89237    | 91.458   | 0.89662   | 92.907   | 0.90364    | 94.338   | 0.91347   | 95.743   |
| 1.35 | 0.85264   | 90       | 0.85407    | 91.754   | 0.85851   | 93.495   | 0.86583    | 95.213   | 0.87609   | 96.897   |
| 1.4  | 0.80902   | 90       | 0.81053    | 92.079   | 0.81520   | 94.142   | 0.82291    | 96.174   | 0.83370   | 98.161   |
| 1.45 | 0.76041   | 90       | 0.76202    | 92.443   | 0.76698   | 94.866   | 0.77527    | 97.247   | 0.78661   | 99.569   |
| 1.5  | 0.70711   | 90       | 0.70803    | 92.860   | 0.71417   | 95.692   | 0.72296    | 98.468   | 0.73521   | 101.165  |
| 1.55 | 0.64944   | 90       | 0.65135    | 93.348   | 0.65713   | 96.656   | 0.66667    | 99.889   | 0.67994   | 103.012  |
| 1.6  | 0.58778   | 90       | 0.58989    | 93.934   | 0.59626   | 97.811   | 0.60676    | 101.581  | 0.62131   | 105.198  |
| 1.65 | 0.52250   | 90       | 0.52487    | 94.661   | 0.53201   | 99.238   | 0.54376    | 103.656  | 0.55995   | 107.853  |
| 1.7  | 0.45399   | 90       | 0.45672    | 95.600   | 0.46491   | 101.068  | 0.47831    | 106.289  | 0.49663   | 111.175  |
| 1.75 | 0.38268   | 90       | 0.38594    | 96.877   | 0.39558   | 103.529  | 0.41124    | 109.771  | 0.43422   | 115.478  |
| 1.8  | 0.30902   | 90       | 0.31304    | 98.741   | 0.32485   | 107.053  | 0.34375    | 114.618  | 0.36882   | 121.277  |
| 1.85 | 0.23345   | 90       | 0.23874    | 101.755  | 0.25403   | 112.545  | 0.27779    | 121.805  | 0.30827   | 129.424  |
| 1.9  | 0.15643   | 90       | 0.16424    | 107.507  | 0.18576   | 122.181  | 0.21712    | 133.229  | 0.25497   | 141.254  |
| 1.95 | 0.07846   | 90       | 0.09305    | 122.407  | 0.12724   | 141.700  | 0.16978    | 152.139  | 0.21608   | 158.261  |
| 2.0  | 0.00      | 90       | 0.05002    | 180.000  | 0.10017   | 180.000  | 0.15056    | 180.000  | 0.20134   | 180.000  |

Example.  $\sinh(0.15 + i 0.15) = 0.27779 / 58^{\circ}.195 = 0.27779 / 58^{\circ}.11'.42''.$

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 0.25$ |          | $x = 0.3$ |          | $x = 0.35$ |          | $x = 0.4$ |          | $x = 0.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 0.25261    | 0.000    | 0.30452   | 0.000    | 0.35719    | 0.000    | 0.41075   | 0.000    | 0.46534    | 0.000    |
| 0.05 | 0.26452    | 17.814   | 0.31447   | 15.118   | 0.36571    | 13.169   | 0.41818   | 11.703   | 0.47191    | 10.567   |
| 0.1  | 0.29782    | 32.890   | 0.34235   | 28.533   | 0.38094    | 25.214   | 0.43953   | 22.629   | 0.49093    | 20.576   |
| 0.15 | 0.34396    | 44.943   | 0.38370   | 39.493   | 0.42671    | 35.516   | 0.47246   | 32.288   | 0.52062    | 29.642   |
| 0.2  | 0.39913    | 52.992   | 0.43385   | 48.122   | 0.47231    | 44.008   | 0.51401   | 40.536   | 0.55860    | 37.601   |
| 0.25 | 0.45854    | 59.405   | 0.48906   | 54.882   | 0.52348    | 50.921   | 0.56140   | 47.471   | 0.60249    | 44.473   |
| 0.3  | 0.51954    | 64.327   | 0.54666   | 60.242   | 0.57766    | 56.843   | 0.61223   | 53.288   | 0.65012    | 50.374   |
| 0.35 | 0.58036    | 68.215   | 0.60476   | 64.575   | 0.63292    | 61.237   | 0.66462   | 58.200   | 0.69968    | 55.544   |
| 0.4  | 0.63977    | 71.371   | 0.66199   | 68.151   | 0.68781    | 65.157   | 0.71708   | 62.393   | 0.74969    | 59.856   |
| 0.45 | 0.69685    | 73.999   | 0.71730   | 71.166   | 0.74119    | 68.503   | 0.76844   | 66.018   | 0.79895    | 63.712   |
| 0.5  | 0.75088    | 76.238   | 0.76989   | 73.759   | 0.79220    | 71.408   | 0.81775   | 69.196   | 0.84649    | 67.125   |
| 0.55 | 0.80127    | 78.185   | 0.81912   | 76.028   | 0.84012    | 73.971   | 0.86426   | 72.021   | 0.89149    | 70.184   |
| 0.6  | 0.84754    | 79.911   | 0.86443   | 78.049   | 0.88436    | 76.267   | 0.90732   | 74.568   | 0.93330    | 72.958   |
| 0.65 | 0.88927    | 81.464   | 0.90539   | 79.787   | 0.92444    | 78.353   | 0.94642   | 76.893   | 0.97136    | 75.504   |
| 0.7  | 0.92612    | 82.887   | 0.94161   | 81.557   | 0.95994    | 80.274   | 0.98113   | 79.043   | 1.00521    | 77.868   |
| 0.75 | 0.95779    | 84.207   | 0.97277   | 83.119   | 0.99052    | 82.068   | 1.01107   | 81.056   | 1.03446    | 80.087   |
| 0.8  | 0.98403    | 85.450   | 0.99862   | 84.593   | 1.01592    | 83.762   | 1.03597   | 82.902   | 1.05880    | 82.194   |
| 0.85 | 1.00464    | 86.635   | 1.01894   | 85.999   | 1.03590    | 85.383   | 1.05557   | 84.788   | 1.07798    | 84.216   |
| 0.9  | 1.01949    | 87.779   | 1.03357   | 87.358   | 1.05029    | 86.951   | 1.06970   | 86.556   | 1.09182    | 86.177   |
| 0.95 | 1.02843    | 88.896   | 1.04239   | 88.687   | 1.05898    | 88.484   | 1.07822   | 88.287   | 1.10017    | 88.098   |
| 1.0  | 1.03141    | 90.000   | 1.04534   | 90.000   | 1.06188    | 90.000   | 1.08107   | 90.000   | 1.10297    | 90.000   |
| 1.05 | 1.02843    | 91.104   | 1.04239   | 91.313   | 1.05898    | 91.516   | 1.07822   | 91.713   | 1.10017    | 91.002   |
| 1.1  | 1.01949    | 92.221   | 1.03357   | 92.642   | 1.05029    | 93.050   | 1.06970   | 93.444   | 1.09182    | 93.823   |
| 1.15 | 1.00464    | 93.365   | 1.01894   | 94.001   | 1.03590    | 94.617   | 1.05557   | 95.212   | 1.07798    | 95.784   |
| 1.2  | 0.98403    | 94.550   | 0.99862   | 95.407   | 1.01592    | 96.238   | 1.03597   | 97.038   | 1.05880    | 97.806   |
| 1.25 | 0.95779    | 95.793   | 0.97277   | 96.881   | 0.99052    | 97.932   | 1.01107   | 98.944   | 1.03446    | 99.913   |
| 1.3  | 0.92612    | 97.113   | 0.94161   | 98.443   | 0.95994    | 99.726   | 0.98113   | 100.957  | 1.00521    | 102.132  |
| 1.35 | 0.88927    | 98.536   | 0.90539   | 100.122  | 0.92444    | 101.647  | 0.94642   | 103.107  | 0.97136    | 104.496  |
| 1.4  | 0.84754    | 100.090  | 0.86443   | 101.951  | 0.88436    | 103.733  | 0.90732   | 105.432  | 0.93330    | 107.042  |
| 1.45 | 0.80127    | 101.815  | 0.81912   | 103.972  | 0.84012    | 106.029  | 0.86426   | 107.979  | 0.89149    | 109.816  |
| 1.5  | 0.75088    | 103.762  | 0.76989   | 106.241  | 0.79220    | 108.592  | 0.81775   | 110.804  | 0.84649    | 112.875  |
| 1.55 | 0.69685    | 106.001  | 0.71730   | 108.834  | 0.74119    | 111.497  | 0.76844   | 113.982  | 0.79895    | 116.288  |
| 1.6  | 0.63977    | 108.629  | 0.66199   | 111.849  | 0.68781    | 114.843  | 0.71708   | 117.607  | 0.74969    | 120.144  |
| 1.65 | 0.58036    | 111.785  | 0.60476   | 115.425  | 0.63292    | 118.763  | 0.66462   | 121.800  | 0.69968    | 124.546  |
| 1.7  | 0.51954    | 115.673  | 0.54666   | 119.758  | 0.57766    | 123.157  | 0.61223   | 126.712  | 0.65012    | 129.626  |
| 1.75 | 0.45854    | 120.595  | 0.48906   | 125.118  | 0.52348    | 129.079  | 0.56140   | 132.529  | 0.60249    | 135.527  |
| 1.8  | 0.39913    | 127.008  | 0.43385   | 131.878  | 0.47231    | 135.992  | 0.51401   | 139.464  | 0.55860    | 142.399  |
| 1.85 | 0.34396    | 135.057  | 0.38370   | 140.507  | 0.42671    | 144.484  | 0.47246   | 147.712  | 0.52062    | 150.358  |
| 1.9  | 0.29782    | 147.110  | 0.34235   | 151.467  | 0.38994    | 154.786  | 0.43953   | 157.371  | 0.49093    | 159.424  |
| 1.95 | 0.26452    | 162.186  | 0.31447   | 164.882  | 0.36571    | 166.831  | 0.41818   | 168.297  | 0.47191    | 169.433  |
| 2.0  | 0.25261    | 180.000  | 0.30452   | 180.000  | 0.35719    | 180.000  | 0.41075   | 180.000  | 0.46534    | 180.000  |

Example.  $\sinh(0.40 + i0.25) = 0.56140 / \underline{47^\circ.471} = 0.56140 / \underline{47^\circ.28'.16''}$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 0.5$ |          | $x = 0.55$ |          | $x = 0.6$ |          | $x = 0.65$ |          | $x = 0.7$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 0.52110   | 0.000    | 0.57815    | 0.000    | 0.63665   | 0.000    | 0.69675    | 0.000    | 0.75858   | 0.000    |
| 0.05 | 0.52697   | 9.665    | 0.58345    | 8.936    | 0.64147   | 8.337    | 0.70115    | 7.839    | 0.76263   | 7.419    |
| 0.1  | 0.54407   | 18.918   | 0.59894    | 17.559   | 0.65559   | 16.432   | 0.71409    | 15.486   | 0.77455   | 14.685   |
| 0.15 | 0.57100   | 27.453   | 0.62350    | 25.625   | 0.67810   | 24.086   | 0.73482    | 22.781   | 0.79369   | 21.665   |
| 0.2  | 0.60583   | 35.111   | 0.65555    | 32.990   | 0.70769   | 31.174   | 0.76220    | 29.613   | 0.81911   | 28.263   |
| 0.25 | 0.64652   | 41.871   | 0.69333    | 39.610   | 0.74282   | 37.642   | 0.79492    | 35.926   | 0.84965   | 34.426   |
| 0.3  | 0.69112   | 47.793   | 0.73510    | 45.511   | 0.78194   | 43.494   | 0.83160    | 41.710   | 0.88406   | 40.133   |
| 0.35 | 0.73793   | 52.980   | 0.77927    | 50.759   | 0.82361   | 48.769   | 0.87090    | 46.989   | 0.92112   | 45.397   |
| 0.4  | 0.78552   | 57.542   | 0.82447    | 55.437   | 0.86650   | 53.529   | 0.91156    | 51.803   | 0.95966   | 50.245   |
| 0.45 | 0.83266   | 61.584   | 0.86951    | 59.628   | 0.90946   | 57.838   | 0.95249    | 56.204   | 0.99861   | 54.716   |
| 0.5  | 0.87837   | 65.198   | 0.91338    | 63.411   | 0.95149   | 61.762   | 0.99270    | 60.245   | 1.03704   | 58.853   |
| 0.55 | 0.92182   | 68.462   | 0.95524    | 66.854   | 0.99171   | 65.360   | 1.03134    | 63.976   | 1.07409   | 62.698   |
| 0.6  | 0.96232   | 71.441   | 0.99437    | 70.016   | 1.02948   | 68.685   | 1.06769    | 67.445   | 1.10904   | 66.294   |
| 0.65 | 0.99927   | 74.189   | 1.03004    | 72.948   | 1.06411   | 71.784   | 1.10111    | 70.694   | 1.14125   | 69.677   |
| 0.7  | 1.03220   | 76.751   | 1.06214    | 75.693   | 1.09509   | 74.696   | 1.13108    | 73.760   | 1.17019   | 72.884   |
| 0.75 | 1.06070   | 79.164   | 1.08987    | 78.287   | 1.12200   | 77.459   | 1.15715    | 76.678   | 1.19541   | 75.946   |
| 0.8  | 1.08446   | 81.461   | 1.11300    | 80.763   | 1.14448   | 80.102   | 1.17892    | 79.477   | 1.21653   | 78.890   |
| 0.85 | 1.10320   | 83.669   | 1.13127    | 83.148   | 1.16226   | 82.653   | 1.19623    | 82.185   | 1.23327   | 81.744   |
| 0.9  | 1.11672   | 85.814   | 1.14446    | 85.467   | 1.17510   | 85.121   | 1.20871    | 84.826   | 1.24538   | 84.532   |
| 0.95 | 1.12489   | 87.917   | 1.15244    | 87.744   | 1.18287   | 87.580   | 1.21626    | 87.424   | 1.25271   | 87.277   |
| 1.0  | 1.12763   | 90.000   | 1.15510    | 90.000   | 1.18547   | 90.000   | 1.21879    | 90.000   | 1.25517   | 90.000   |
| 1.05 | 1.12489   | 92.083   | 1.15244    | 92.256   | 1.18287   | 92.420   | 1.21626    | 92.576   | 1.25271   | 92.723   |
| 1.1  | 1.11672   | 94.186   | 1.14446    | 94.533   | 1.17510   | 94.879   | 1.20871    | 95.174   | 1.24538   | 95.468   |
| 1.15 | 1.10320   | 96.331   | 1.13127    | 96.852   | 1.16226   | 97.347   | 1.19623    | 97.815   | 1.23327   | 98.256   |
| 1.2  | 1.08446   | 98.539   | 1.11300    | 99.237   | 1.14448   | 99.898   | 1.17892    | 100.523  | 1.21653   | 101.110  |
| 1.25 | 1.06070   | 100.836  | 1.08987    | 101.713  | 1.12200   | 102.541  | 1.15715    | 103.322  | 1.19541   | 104.054  |
| 1.3  | 1.03220   | 103.249  | 1.06214    | 104.307  | 1.09509   | 105.304  | 1.13108    | 106.240  | 1.17019   | 107.116  |
| 1.35 | 0.99927   | 105.811  | 1.03004    | 107.052  | 1.06411   | 108.216  | 1.10111    | 109.306  | 1.14125   | 110.323  |
| 1.4  | 0.96232   | 108.559  | 0.99437    | 109.984  | 1.02948   | 111.315  | 1.06769    | 112.555  | 1.10904   | 113.706  |
| 1.45 | 0.92182   | 111.538  | 0.95524    | 113.146  | 0.99174   | 114.640  | 1.03134    | 116.024  | 1.07409   | 117.302  |
| 1.5  | 0.87837   | 114.803  | 0.91338    | 116.589  | 0.95149   | 118.238  | 0.99270    | 119.755  | 1.03704   | 121.148  |
| 1.55 | 0.83266   | 118.416  | 0.86951    | 120.372  | 0.90946   | 122.162  | 0.95249    | 123.796  | 0.99861   | 125.284  |
| 1.6  | 0.78552   | 122.458  | 0.82447    | 124.563  | 0.86650   | 126.471  | 0.91156    | 128.197  | 0.95966   | 129.755  |
| 1.65 | 0.73793   | 127.020  | 0.77927    | 129.241  | 0.82361   | 131.231  | 0.87090    | 133.011  | 0.92112   | 134.603  |
| 1.7  | 0.69112   | 132.207  | 0.73510    | 134.489  | 0.78194   | 136.506  | 0.83160    | 138.290  | 0.88406   | 139.867  |
| 1.75 | 0.64652   | 138.129  | 0.69333    | 140.390  | 0.74282   | 142.358  | 0.79492    | 144.074  | 0.84965   | 145.574  |
| 1.8  | 0.60583   | 144.889  | 0.65555    | 147.010  | 0.70769   | 148.826  | 0.76220    | 150.387  | 0.81911   | 151.737  |
| 1.85 | 0.57100   | 152.547  | 0.62350    | 154.375  | 0.67810   | 155.914  | 0.73482    | 157.219  | 0.79369   | 158.335  |
| 1.9  | 0.54407   | 161.082  | 0.59894    | 162.441  | 0.65559   | 163.568  | 0.71409    | 164.514  | 0.77455   | 165.315  |
| 1.95 | 0.52697   | 170.335  | 0.58345    | 171.064  | 0.64147   | 171.663  | 0.70115    | 172.161  | 0.76263   | 172.581  |
| 2.0  | 0.52110   | 180.000  | 0.57815    | 180.000  | 0.63665   | 180.000  | 0.69675    | 180.000  | 0.75858   | 180.000  |

Example.  $\sinh(0.70 + i 1.70) = 0.88406 / 139^{\circ}.867 = 0.88406 / 130^{\circ}.52'.01''$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 0.75$ |          | $x = 0.8$ |          | $x = 0.85$ |          | $x = 0.9$ |          | $x = 0.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
|      | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0.00 | 0.82232    | 0.000    | 0.88811   | 0.000    | 0.95612    | 0.000    | 1.02652   | 0.000    | 1.09948    | 0.000    |
| 0.05 | 0.82605    | 7.064    | 0.89157   | 6.759    | 0.95933    | 6.497    | 1.02951   | 6.270    | 1.10227    | 6.073    |
| 0.10 | 0.83706    | 14.002   | 0.90178   | 13.415   | 0.96883    | 12.909   | 1.03837   | 12.468   | 1.11056    | 12.085   |
| 0.15 | 0.85481    | 20.706   | 0.91828   | 19.877   | 0.98420    | 19.158   | 1.05273   | 18.529   | 1.12400    | 17.980   |
| 0.2  | 0.87846    | 27.093   | 0.94033   | 26.073   | 1.00481    | 25.181   | 1.07202   | 24.399   | 1.14209    | 23.712   |
| 0.25 | 0.90700    | 33.111   | 0.96705   | 31.955   | 1.02985    | 30.938   | 1.09553   | 30.039   | 1.16418    | 29.245   |
| 0.3  | 0.93932    | 38.737   | 0.99742   | 37.500   | 1.05843    | 36.401   | 1.12243   | 35.426   | 1.19227    | 34.557   |
| 0.35 | 0.97427    | 43.974   | 1.03041   | 42.702   | 1.08957    | 41.565   | 1.15184   | 40.547   | 1.21732    | 39.637   |
| 0.4  | 1.01079    | 48.840   | 1.06500   | 47.574   | 1.12234    | 40.433   | 1.18289   | 45.407   | 1.24674    | 44.483   |
| 0.45 | 1.04785    | 53.363   | 1.10024   | 52.135   | 1.15583    | 51.022   | 1.21471   | 50.014   | 1.27697    | 49.102   |
| 0.5  | 1.08453    | 57.578   | 1.13522   | 56.414   | 1.18918    | 55.353   | 1.24649   | 54.386   | 1.30723    | 53.507   |
| 0.55 | 1.12001    | 61.522   | 1.16917   | 60.441   | 1.22163    | 59.450   | 1.27748   | 58.543   | 1.33682    | 57.714   |
| 0.6  | 1.15356    | 65.228   | 1.20135   | 64.245   | 1.25246    | 63.399   | 1.30700   | 62.507   | 1.36505    | 61.743   |
| 0.65 | 1.18457    | 68.733   | 1.23115   | 67.857   | 1.28107    | 67.048   | 1.33444   | 66.301   | 1.39136    | 65.613   |
| 0.7  | 1.21248    | 72.067   | 1.25802   | 71.307   | 1.30693    | 70.602   | 1.35927   | 69.949   | 1.41519    | 69.347   |
| 0.75 | 1.23689    | 75.260   | 1.28152   | 74.621   | 1.32955    | 74.026   | 1.38105   | 73.474   | 1.43611    | 72.963   |
| 0.8  | 1.25726    | 78.339   | 1.30124   | 77.825   | 1.34858    | 77.344   | 1.39937   | 76.898   | 1.45371    | 76.484   |
| 0.85 | 1.27346    | 81.330   | 1.31691   | 80.942   | 1.36369    | 80.580   | 1.41395   | 80.242   | 1.46778    | 79.929   |
| 0.9  | 1.28520    | 84.255   | 1.32825   | 83.996   | 1.37466    | 83.754   | 1.42452   | 83.527   | 1.47797    | 83.316   |
| 0.95 | 1.29230    | 87.138   | 1.33513   | 87.008   | 1.38130    | 86.887   | 1.43094   | 86.773   | 1.48415    | 86.668   |
| 1.0  | 1.29468    | 90.000   | 1.33743   | 90.000   | 1.38353    | 90.000   | 1.43309   | 90.000   | 1.48623    | 90.000   |
| 1.05 | 1.29230    | 92.862   | 1.33513   | 92.992   | 1.38130    | 93.113   | 1.43094   | 93.227   | 1.48415    | 93.332   |
| 1.1  | 1.28520    | 95.745   | 1.32825   | 96.004   | 1.37466    | 96.246   | 1.42452   | 96.473   | 1.47797    | 96.684   |
| 1.15 | 1.27346    | 98.670   | 1.31691   | 99.058   | 1.36369    | 99.420   | 1.41395   | 99.758   | 1.46778    | 100.071  |
| 1.2  | 1.25726    | 101.661  | 1.30124   | 102.175  | 1.34858    | 102.656  | 1.39937   | 103.102  | 1.45371    | 103.516  |
| 1.25 | 1.23689    | 104.740  | 1.28152   | 105.379  | 1.32955    | 105.974  | 1.38105   | 106.526  | 1.43611    | 107.937  |
| 1.3  | 1.21248    | 107.933  | 1.25802   | 108.693  | 1.30693    | 109.398  | 1.35927   | 110.051  | 1.41519    | 110.653  |
| 1.35 | 1.18457    | 111.267  | 1.23115   | 112.143  | 1.28107    | 112.952  | 1.33444   | 113.699  | 1.39136    | 114.387  |
| 1.4  | 1.15356    | 114.772  | 1.20135   | 115.755  | 1.25246    | 116.601  | 1.30700   | 117.493  | 1.36505    | 118.257  |
| 1.45 | 1.12001    | 118.478  | 1.16917   | 119.559  | 1.22163    | 120.550  | 1.27748   | 121.457  | 1.33682    | 122.286  |
| 1.5  | 1.08453    | 122.422  | 1.13522   | 123.586  | 1.18918    | 124.647  | 1.24649   | 125.614  | 1.30723    | 126.493  |
| 1.55 | 1.04785    | 126.637  | 1.10024   | 127.865  | 1.15583    | 128.978  | 1.21471   | 129.986  | 1.27697    | 130.898  |
| 1.6  | 1.01079    | 131.160  | 1.06500   | 132.426  | 1.12234    | 133.567  | 1.18289   | 134.593  | 1.24674    | 135.517  |
| 1.65 | 0.97427    | 136.026  | 1.03041   | 137.298  | 1.08957    | 138.435  | 1.15184   | 139.453  | 1.21732    | 140.363  |
| 1.7  | 0.93932    | 141.263  | 0.99742   | 142.500  | 1.05843    | 143.599  | 1.12243   | 144.574  | 1.19227    | 145.443  |
| 1.75 | 0.90700    | 146.889  | 0.96705   | 148.045  | 1.02985    | 149.062  | 1.09553   | 149.961  | 1.16418    | 150.755  |
| 1.8  | 0.87846    | 152.907  | 0.94033   | 153.927  | 1.00481    | 154.819  | 1.07202   | 155.601  | 1.14209    | 156.288  |
| 1.85 | 0.85481    | 159.294  | 0.91828   | 160.123  | 0.98420    | 160.842  | 1.05273   | 161.471  | 1.12400    | 162.020  |
| 1.9  | 0.83706    | 165.998  | 0.90178   | 166.585  | 0.96883    | 167.091  | 1.03837   | 167.532  | 1.11056    | 167.915  |
| 1.95 | 0.82605    | 172.936  | 0.89157   | 173.241  | 0.95933    | 173.503  | 1.02951   | 173.730  | 1.10227    | 173.927  |
| 2.0  | 0.82232    | 180.000  | 0.88811   | 180.000  | 0.95612    | 180.000  | 1.02652   | 180.000  | 1.09948    | 180.000  |

Example.  $\sinh(0.90 + i\ 1.0) = 1.43309/90^\circ$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 1.0$ |          | $x = 1.05$ |          | $x = 1.1$ |          | $x = 1.15$ |          | $x = 1.2$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 1.17520   | 0.000    | 1.25386    | 0.000    | 1.33565   | 0.000    | 1.42078    | 0.000    | 1.50946   | 0.000    |
| 0.05 | 1.17782   | 5.900    | 1.25031    | 5.748    | 1.33795   | 5.615    | 1.42294    | 5.497    | 1.51150   | 5.393    |
| 0.1  | 1.18552   | 11.748   | 1.26358    | 11.453   | 1.34478   | 11.192   | 1.42936    | 10.901   | 1.51755   | 10.757   |
| 0.15 | 1.19816   | 17.496   | 1.27540    | 17.071   | 1.35590   | 16.694   | 1.43983    | 16.361   | 1.52741   | 16.065   |
| 0.2  | 1.21515   | 23.105   | 1.29137    | 22.568   | 1.37093   | 22.092   | 1.45399    | 21.670   | 1.54077   | 21.294   |
| 0.25 | 1.23594   | 28.541   | 1.31095    | 27.915   | 1.38939   | 27.359   | 1.47141    | 26.864   | 1.55721   | 26.421   |
| 0.3  | 1.25984   | 33.784   | 1.33351    | 33.093   | 1.41070   | 32.477   | 1.49155    | 31.926   | 1.57626   | 31.433   |
| 0.35 | 1.28612   | 38.821   | 1.35837    | 38.090   | 1.43421   | 37.435   | 1.51381    | 36.847   | 1.59733   | 36.319   |
| 0.4  | 1.31400   | 43.651   | 1.38479    | 42.902   | 1.45926   | 42.227   | 1.53756    | 41.620   | 1.61987   | 41.073   |
| 0.45 | 1.34272   | 48.276   | 1.41207    | 47.530   | 1.48517   | 46.855   | 1.56217    | 46.245   | 1.64325   | 45.693   |
| 0.5  | 1.37153   | 52.707   | 1.43950    | 51.981   | 1.51128   | 51.323   | 1.58701    | 50.725   | 1.66688   | 50.184   |
| 0.55 | 1.39976   | 56.957   | 1.46641    | 56.268   | 1.53694   | 55.640   | 1.61147    | 55.069   | 1.69018   | 54.549   |
| 0.6  | 1.42675   | 61.043   | 1.49220    | 60.403   | 1.56156   | 59.818   | 1.63497    | 59.284   | 1.71260   | 58.797   |
| 0.65 | 1.45193   | 64.981   | 1.51630    | 64.401   | 1.58460   | 63.870   | 1.65699    | 63.384   | 1.73363   | 62.939   |
| 0.7  | 1.47479   | 68.791   | 1.53820    | 68.280   | 1.60558   | 67.811   | 1.67705    | 67.380   | 1.75282   | 66.986   |
| 0.75 | 1.49487   | 72.491   | 1.55747    | 72.056   | 1.62404   | 71.656   | 1.69472    | 71.287   | 1.76975   | 70.950   |
| 0.8  | 1.51182   | 76.101   | 1.57374    | 75.747   | 1.63965   | 75.421   | 1.70971    | 75.120   | 1.78409   | 74.844   |
| 0.85 | 1.52532   | 79.638   | 1.58671    | 79.369   | 1.65211   | 79.121   | 1.72166    | 78.893   | 1.79555   | 78.682   |
| 0.9  | 1.53513   | 83.122   | 1.59614    | 82.941   | 1.66117   | 82.774   | 1.73036    | 82.620   | 1.80389   | 82.478   |
| 0.95 | 1.54108   | 86.724   | 1.60187    | 86.479   | 1.66667   | 86.395   | 1.73564    | 86.317   | 1.80895   | 86.246   |
| 1.0  | 1.54308   | 90.000   | 1.60379    | 90.000   | 1.66852   | 90.000   | 1.73741    | 90.000   | 1.81066   | 90.000   |
| 1.05 | 1.54108   | 93.276   | 1.60187    | 93.521   | 1.66667   | 93.605   | 1.73564    | 93.683   | 1.80895   | 93.754   |
| 1.1  | 1.53513   | 96.878   | 1.59614    | 97.059   | 1.66117   | 97.226   | 1.73036    | 97.380   | 1.80389   | 97.522   |
| 1.15 | 1.52532   | 100.362  | 1.58671    | 100.631  | 1.65211   | 100.879  | 1.72166    | 101.107  | 1.79555   | 101.318  |
| 1.2  | 1.51182   | 103.899  | 1.57374    | 104.253  | 1.63965   | 104.579  | 1.70971    | 104.880  | 1.78409   | 105.156  |
| 1.25 | 1.49487   | 107.509  | 1.55747    | 107.944  | 1.62404   | 108.344  | 1.69472    | 108.713  | 1.76975   | 109.050  |
| 1.3  | 1.47479   | 111.209  | 1.53820    | 111.720  | 1.60558   | 112.189  | 1.67705    | 112.620  | 1.75282   | 113.014  |
| 1.35 | 1.45193   | 115.019  | 1.51630    | 115.599  | 1.58460   | 116.130  | 1.65699    | 116.616  | 1.73363   | 117.061  |
| 1.4  | 1.42675   | 118.957  | 1.49220    | 119.597  | 1.56156   | 120.182  | 1.63497    | 120.716  | 1.71260   | 121.203  |
| 1.45 | 1.39976   | 123.043  | 1.46641    | 123.732  | 1.53694   | 124.360  | 1.61147    | 124.931  | 1.69018   | 125.451  |
| 1.5  | 1.37153   | 127.293  | 1.43950    | 128.019  | 1.51128   | 128.677  | 1.58701    | 129.275  | 1.66688   | 129.816  |
| 1.55 | 1.34272   | 131.724  | 1.41207    | 132.470  | 1.48517   | 133.145  | 1.56217    | 133.755  | 1.64325   | 134.307  |
| 1.6  | 1.31400   | 136.349  | 1.38479    | 137.098  | 1.45926   | 137.773  | 1.53756    | 138.380  | 1.61987   | 138.927  |
| 1.65 | 1.28612   | 141.179  | 1.35837    | 141.910  | 1.43421   | 142.565  | 1.51381    | 143.153  | 1.59733   | 143.681  |
| 1.7  | 1.25984   | 146.216  | 1.33351    | 146.907  | 1.41070   | 147.523  | 1.49155    | 148.074  | 1.57626   | 148.567  |
| 1.75 | 1.23594   | 151.459  | 1.31095    | 152.085  | 1.38939   | 152.641  | 1.47141    | 153.136  | 1.55721   | 153.579  |
| 1.8  | 1.21515   | 156.895  | 1.29137    | 157.432  | 1.37093   | 157.908  | 1.45399    | 158.330  | 1.54077   | 158.706  |
| 1.85 | 1.19816   | 162.504  | 1.27540    | 162.929  | 1.35590   | 163.306  | 1.43983    | 163.639  | 1.52741   | 163.935  |
| 1.9  | 1.18552   | 168.252  | 1.26358    | 168.547  | 1.34478   | 168.808  | 1.42936    | 169.039  | 1.51755   | 169.243  |
| 1.95 | 1.17782   | 174.100  | 1.25031    | 174.252  | 1.33795   | 174.385  | 1.42294    | 174.503  | 1.51150   | 174.607  |
| 2.00 | 1.17520   | 180.000  | 1.25386    | 180.000  | 1.33565   | 180.000  | 1.42078    | 180.000  | 1.50946   | 180.000  |

Example.  $\sinh(1.20 + i 1.25) = 1.76975 / 109^{\circ}.050 = 1.76995 / 109^{\circ}.03'.00''$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 1.25$ |          | $x = 1.3$ |          | $x = 1.35$ |          | $x = 1.4$ |          | $x = 1.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
|      | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 1.60192    | 0.000    | 1.69838   | 0.000    | 1.79909    | 0.000    | 1.90430   | 0.000    | 2.01427    | 0.000    |
| 0.05 | 1.60384    | 5.301    | 1.70019   | 5.218    | 1.80080    | 5.145    | 1.90592   | 5.080    | 2.01580    | 5.021    |
| 0.1  | 1.60954    | 10.576   | 1.70557   | 10.415   | 1.80588    | 10.271   | 1.91072   | 10.143   | 2.02034    | 10.028   |
| 0.15 | 1.61884    | 15.803   | 1.71435   | 15.568   | 1.81417    | 15.359   | 1.91856   | 15.172   | 2.02775    | 15.005   |
| 0.2  | 1.63145    | 20.958   | 1.72627   | 20.659   | 1.82544    | 20.392   | 1.92921   | 20.153   | 2.03784    | 19.939   |
| 0.25 | 1.64699    | 26.026   | 1.74096   | 25.673   | 1.83934    | 25.356   | 1.94237   | 25.073   | 2.05030    | 24.818   |
| 0.3  | 1.66501    | 30.991   | 1.75802   | 30.595   | 1.85549    | 30.240   | 1.95767   | 29.921   | 2.06479    | 29.634   |
| 0.35 | 1.68497    | 35.845   | 1.77694   | 35.418   | 1.87343    | 35.035   | 1.97468   | 34.689   | 2.08093    | 34.379   |
| 0.4  | 1.70635    | 40.580   | 1.79722   | 40.135   | 1.89268    | 39.735   | 1.99295   | 39.373   | 2.09828    | 39.047   |
| 0.45 | 1.72856    | 45.195   | 1.81832   | 44.745   | 1.91273    | 44.338   | 2.01200   | 43.970   | 2.11637    | 43.638   |
| 0.5  | 1.75104    | 49.693   | 1.83970   | 49.248   | 1.93306    | 48.845   | 2.03135   | 48.480   | 2.13478    | 48.150   |
| 0.55 | 1.77323    | 54.077   | 1.86084   | 53.648   | 1.95319    | 53.258   | 2.05051   | 52.905   | 2.15302    | 52.584   |
| 0.6  | 1.79462    | 58.354   | 1.88123   | 57.950   | 1.97262    | 57.583   | 2.06903   | 57.249   | 2.17067    | 56.945   |
| 0.65 | 1.81470    | 62.533   | 1.90040   | 62.163   | 1.99091    | 61.825   | 2.08647   | 61.518   | 2.18731    | 61.238   |
| 0.7  | 1.83304    | 66.625   | 1.91792   | 66.295   | 2.00764    | 65.994   | 2.10244   | 65.719   | 2.20254    | 65.469   |
| 0.75 | 1.84924    | 70.640   | 1.93341   | 70.356   | 2.02245    | 70.097   | 2.11658   | 69.861   | 2.21604    | 69.645   |
| 0.8  | 1.86297    | 74.590   | 1.94654   | 74.358   | 2.03500    | 74.145   | 2.12858   | 73.951   | 2.22751    | 73.774   |
| 0.85 | 1.87394    | 78.489   | 1.95704   | 78.311   | 2.04505    | 78.149   | 2.13819   | 78.000   | 2.23669    | 77.864   |
| 0.9  | 1.88193    | 82.348   | 1.96470   | 82.228   | 2.05238    | 82.118   | 2.14520   | 82.018   | 2.24334    | 81.925   |
| 0.95 | 1.88679    | 86.180   | 1.96935   | 86.120   | 2.05684    | 86.065   | 2.14947   | 86.014   | 2.24747    | 85.968   |
| 1.0  | 1.88842    | 90.000   | 1.97091   | 90.000   | 2.05833    | 90.000   | 2.15090   | 90.000   | 2.24884    | 90.000   |
| 1.05 | 1.88679    | 93.820   | 1.96935   | 93.880   | 2.05684    | 93.935   | 2.14947   | 93.986   | 2.24747    | 94.032   |
| 1.1  | 1.88193    | 97.652   | 1.96470   | 97.772   | 2.05238    | 97.882   | 2.14520   | 97.982   | 2.24334    | 98.075   |
| 1.15 | 1.87394    | 101.511  | 1.95704   | 101.689  | 2.04505    | 101.851  | 2.13819   | 102.000  | 2.23669    | 102.136  |
| 1.2  | 1.86297    | 105.410  | 1.94654   | 105.642  | 2.03500    | 105.855  | 2.12858   | 106.049  | 2.22751    | 106.226  |
| 1.25 | 1.84924    | 109.360  | 1.93341   | 109.644  | 2.02245    | 109.903  | 2.11658   | 110.139  | 2.21604    | 110.355  |
| 1.3  | 1.83304    | 113.375  | 1.91792   | 113.705  | 2.00764    | 114.006  | 2.10244   | 114.281  | 2.20254    | 114.531  |
| 1.35 | 1.81470    | 117.467  | 1.90040   | 117.837  | 1.99091    | 118.175  | 2.08647   | 118.482  | 2.18731    | 118.762  |
| 1.4  | 1.79462    | 121.646  | 1.88123   | 122.050  | 1.97262    | 122.417  | 2.06903   | 122.751  | 2.17067    | 123.055  |
| 1.45 | 1.77323    | 125.923  | 1.86084   | 126.352  | 1.95319    | 126.742  | 2.05051   | 127.095  | 2.15302    | 127.416  |
| 1.5  | 1.75104    | 130.308  | 1.83970   | 130.752  | 1.93306    | 131.155  | 2.03135   | 131.520  | 2.13478    | 131.851  |
| 1.55 | 1.72856    | 134.805  | 1.81832   | 135.255  | 1.91273    | 135.602  | 2.01200   | 136.030  | 2.11637    | 136.362  |
| 1.6  | 1.70635    | 139.420  | 1.79722   | 139.865  | 1.89268    | 140.265  | 1.99295   | 140.627  | 2.09828    | 140.953  |
| 1.65 | 1.68497    | 144.155  | 1.77694   | 144.582  | 1.87343    | 144.965  | 1.97468   | 145.311  | 2.08093    | 145.621  |
| 1.7  | 1.66501    | 149.009  | 1.75802   | 149.405  | 1.85549    | 149.760  | 1.95767   | 150.079  | 2.06479    | 150.366  |
| 1.75 | 1.64699    | 153.974  | 1.74096   | 154.327  | 1.83934    | 154.644  | 1.94237   | 154.927  | 2.05030    | 155.182  |
| 1.8  | 1.63145    | 159.042  | 1.72627   | 159.341  | 1.82544    | 159.608  | 1.92921   | 159.847  | 2.03784    | 160.061  |
| 1.85 | 1.61884    | 164.197  | 1.71435   | 164.432  | 1.81417    | 164.641  | 1.91856   | 164.828  | 2.02775    | 164.995  |
| 1.9  | 1.60954    | 169.424  | 1.70557   | 169.585  | 1.80588    | 169.729  | 1.91072   | 169.857  | 2.02034    | 169.972  |
| 1.95 | 1.60384    | 174.699  | 1.70019   | 174.782  | 1.80080    | 174.855  | 1.90592   | 174.920  | 2.01580    | 174.979  |
| 2.0  | 1.60192    | 180.000  | 1.69838   | 180.000  | 1.79909    | 180.000  | 1.90430   | 180.000  | 2.01427    | 180.000  |

Example.  $\sinh(1.45 + i 1.70) = 2.06479 / 150^{\circ}.366 = 2.06479 / 150^{\circ}.22'.01''$ .



TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 1.50$ |          | $x = 1.55$ |          | $x = 1.60$ |          | $x = 1.65$ |          | $x = 1.70$ |          |
|------|------------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$        | $\gamma$ | $r$        | $\gamma$ | $r$        | $\gamma$ | $r$        | $\gamma$ |
| 0    | 2.12928    | 0.000    | 2.24961    | 0.000    | 2.37557    | 0.000    | 2.50746    | 0.000    | 2.64563    | 0.000    |
| 0.05 | 2.13073    | 4.969    | 2.25098    | 4.923    | 2.37686    | 4.881    | 2.50869    | 4.843    | 2.64679    | 4.809    |
| 0.1  | 2.13502    | 9.925    | 2.25504    | 9.833    | 2.38071    | 9.751    | 2.51234    | 9.677    | 2.65025    | 9.610    |
| 0.15 | 2.14204    | 14.855   | 2.26169    | 14.721   | 2.38701    | 14.600   | 2.51831    | 14.492   | 2.65591    | 14.395   |
| 0.2  | 2.15159    | 19.746   | 2.27074    | 19.574   | 2.39558    | 19.419   | 2.52644    | 19.280   | 2.66361    | 19.155   |
| 0.25 | 2.16340    | 24.590   | 2.28193    | 24.385   | 2.40619    | 24.200   | 2.53650    | 24.034   | 2.67316    | 23.884   |
| 0.3  | 2.17714    | 29.376   | 2.29496    | 29.144   | 2.41856    | 28.935   | 2.54824    | 28.747   | 2.68430    | 28.578   |
| 0.35 | 2.19245    | 34.099   | 2.30949    | 33.846   | 2.43235    | 33.619   | 2.56133    | 33.415   | 2.69673    | 33.229   |
| 0.4  | 2.20892    | 38.753   | 2.32513    | 38.488   | 2.44721    | 38.248   | 2.57544    | 38.032   | 2.71014    | 37.837   |
| 0.45 | 2.22612    | 43.337   | 2.34148    | 43.066   | 2.46274    | 42.820   | 2.59021    | 42.599   | 2.72418    | 42.398   |
| 0.5  | 2.24362    | 47.850   | 2.35812    | 47.580   | 2.47857    | 47.334   | 2.60527    | 47.113   | 2.73850    | 46.911   |
| 0.55 | 2.26098    | 52.294   | 2.37465    | 52.030   | 2.49430    | 51.791   | 2.62024    | 51.574   | 2.75274    | 51.378   |
| 0.6  | 2.27779    | 56.670   | 2.39066    | 56.420   | 2.50955    | 56.192   | 2.63476    | 55.986   | 2.76657    | 55.799   |
| 0.65 | 2.29365    | 60.984   | 2.40577    | 60.752   | 2.52395    | 60.542   | 2.64847    | 60.351   | 2.77963    | 60.178   |
| 0.7  | 2.30819    | 65.241   | 2.41976    | 65.033   | 2.53717    | 64.845   | 2.66108    | 64.673   | 2.79164    | 64.517   |
| 0.75 | 2.32107    | 69.448   | 2.43193    | 69.268   | 2.54890    | 69.105   | 2.67226    | 68.956   | 2.80230    | 68.821   |
| 0.8  | 2.33202    | 73.611   | 2.44238    | 73.464   | 2.55887    | 73.329   | 2.68178    | 73.206   | 2.81138    | 73.094   |
| 0.85 | 2.34080    | 77.740   | 2.45076    | 77.626   | 2.56687    | 77.523   | 2.68941    | 77.429   | 2.81867    | 77.343   |
| 0.9  | 2.34720    | 81.842   | 2.45688    | 81.765   | 2.57272    | 81.695   | 2.69499    | 81.631   | 2.82399    | 81.573   |
| 0.95 | 2.35110    | 85.925   | 2.46061    | 85.887   | 2.57627    | 85.851   | 2.69839    | 85.819   | 2.82723    | 85.790   |
| 1.0  | 2.35241    | 90.000   | 2.46186    | 90.000   | 2.57746    | 90.000   | 2.69951    | 90.000   | 2.82832    | 90.000   |
| 1.05 | 2.35110    | 94.075   | 2.46061    | 94.113   | 2.57627    | 94.149   | 2.69839    | 94.181   | 2.82723    | 94.210   |
| 1.1  | 2.34720    | 98.158   | 2.45688    | 98.235   | 2.57272    | 98.305   | 2.69499    | 98.369   | 2.82399    | 98.427   |
| 1.15 | 2.34080    | 102.260  | 2.45076    | 102.374  | 2.56687    | 102.477  | 2.68941    | 102.571  | 2.81867    | 102.657  |
| 1.2  | 2.33202    | 106.389  | 2.44238    | 106.536  | 2.55887    | 106.671  | 2.68178    | 106.794  | 2.81138    | 106.906  |
| 1.25 | 2.32107    | 110.552  | 2.43193    | 110.732  | 2.54890    | 110.895  | 2.67226    | 111.044  | 2.80230    | 111.179  |
| 1.3  | 2.30819    | 114.759  | 2.41976    | 114.967  | 2.53717    | 115.155  | 2.66108    | 115.327  | 2.79164    | 115.483  |
| 1.35 | 2.29365    | 119.016  | 2.40577    | 119.248  | 2.52395    | 119.458  | 2.64847    | 119.649  | 2.77963    | 119.822  |
| 1.4  | 2.27779    | 123.330  | 2.39066    | 123.580  | 2.50955    | 123.808  | 2.63476    | 124.014  | 2.76657    | 124.201  |
| 1.45 | 2.26098    | 127.706  | 2.37465    | 127.970  | 2.49430    | 128.209  | 2.62024    | 128.426  | 2.75274    | 128.622  |
| 1.5  | 2.24362    | 132.150  | 2.35812    | 132.421  | 2.47857    | 132.666  | 2.60527    | 132.887  | 2.73850    | 133.089  |
| 1.55 | 2.22612    | 136.663  | 2.34148    | 136.934  | 2.46274    | 137.180  | 2.59021    | 137.401  | 2.72418    | 137.602  |
| 1.6  | 2.20892    | 141.247  | 2.32513    | 141.512  | 2.44721    | 141.752  | 2.57544    | 141.968  | 2.71014    | 142.163  |
| 1.65 | 2.19245    | 145.901  | 2.30949    | 146.154  | 2.43235    | 146.381  | 2.56133    | 146.585  | 2.69673    | 146.771  |
| 1.7  | 2.17714    | 150.624  | 2.29496    | 150.856  | 2.41856    | 151.065  | 2.54824    | 151.253  | 2.68430    | 151.422  |
| 1.75 | 2.16340    | 155.410  | 2.28193    | 155.615  | 2.40619    | 155.800  | 2.53650    | 155.966  | 2.67316    | 156.116  |
| 1.8  | 2.15159    | 160.254  | 2.27074    | 160.426  | 2.39558    | 160.581  | 2.52644    | 160.720  | 2.66361    | 160.845  |
| 1.85 | 2.14204    | 165.145  | 2.26169    | 165.279  | 2.38701    | 165.400  | 2.51831    | 165.508  | 2.65591    | 165.605  |
| 1.9  | 2.13502    | 170.075  | 2.25504    | 170.167  | 2.38071    | 170.249  | 2.51234    | 170.323  | 2.65025    | 170.390  |
| 1.95 | 2.13073    | 175.031  | 2.25098    | 175.077  | 2.37686    | 175.119  | 2.50869    | 175.157  | 2.64679    | 175.191  |
| 2.0  | 2.12928    | 180.000  | 2.24961    | 180.000  | 2.37557    | 180.000  | 2.50746    | 180.000  | 2.64563    | 180.000  |

Example.  $\sinh(1.55 + i \underline{0.60}) = 2.39066 / \underline{56^{\circ}.420} = 2.39066 / \underline{56^{\circ}.25'.12''}$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 1.75$ |          | $x = 1.8$ |          | $x = 1.85$ |          | $x = 1.9$ |          | $x = 1.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 2.79041    | 0.00     | 2.94217   | 0.00     | 3.10129    | 0.00     | 3.26816   | 0.00     | 3.44321    | 0.00     |
| 0.05 | 2.79151    | 4.779    | 2.94322   | 4.752    | 3.10228    | 4.727    | 3.26911   | 4.705    | 3.44410    | 4.685    |
| 0.1  | 2.79479    | 9.551    | 2.94033   | 9.497    | 3.10523    | 9.448    | 3.27191   | 9.405    | 3.44075    | 9.305    |
| 0.15 | 2.80016    | 14.307   | 2.95142   | 14.228   | 3.11006    | 14.158   | 3.27649   | 14.094   | 3.45112    | 14.036   |
| 0.2  | 2.80747    | 19.042   | 2.95835   | 18.941   | 3.11665    | 18.850   | 3.28274   | 18.767   | 3.45702    | 18.693   |
| 0.25 | 2.81653    | 23.750   | 2.96695   | 23.629   | 3.12481    | 23.520   | 3.29049   | 23.421   | 3.46441    | 23.332   |
| 0.3  | 2.82710    | 28.425   | 2.97699   | 28.287   | 3.13434    | 28.163   | 3.29955   | 28.051   | 3.47301    | 27.950   |
| 0.35 | 2.83891    | 33.063   | 2.98821   | 32.912   | 3.14500    | 32.776   | 3.30967   | 32.654   | 3.48263    | 32.543   |
| 0.4  | 2.85165    | 37.661   | 3.00031   | 37.501   | 3.15650    | 37.357   | 3.32060   | 37.227   | 3.49302    | 37.110   |
| 0.45 | 2.86499    | 42.216   | 3.01300   | 42.053   | 3.16856    | 41.904   | 3.33207   | 41.770   | 3.50393    | 41.649   |
| 0.5  | 2.87861    | 46.730   | 3.02595   | 46.565   | 3.18088    | 46.416   | 3.34378   | 46.281   | 3.51507    | 46.160   |
| 0.55 | 2.89217    | 51.200   | 3.03885   | 51.039   | 3.19315    | 50.893   | 3.35546   | 50.761   | 3.52617    | 50.642   |
| 0.6  | 2.90532    | 55.630   | 3.05138   | 55.476   | 3.20507    | 55.337   | 3.36681   | 55.211   | 3.53698    | 55.096   |
| 0.65 | 2.91777    | 60.020   | 3.06323   | 59.877   | 3.21636    | 59.748   | 3.37756   | 59.631   | 3.54721    | 59.524   |
| 0.7  | 2.92921    | 64.375   | 3.07413   | 64.246   | 3.22675    | 64.129   | 3.38745   | 64.023   | 3.55663    | 63.927   |
| 0.75 | 2.93938    | 68.698   | 3.08382   | 68.586   | 3.23598    | 68.484   | 3.39624   | 68.392   | 3.56500    | 68.308   |
| 0.8  | 2.94804    | 72.992   | 3.09206   | 72.900   | 3.24384    | 72.816   | 3.40373   | 72.740   | 3.57213    | 72.670   |
| 0.85 | 2.95498    | 77.265   | 3.09869   | 77.194   | 3.25015    | 77.129   | 3.40975   | 77.071   | 3.57788    | 77.017   |
| 0.9  | 2.96006    | 81.520   | 3.10353   | 81.471   | 3.25477    | 81.428   | 3.41415   | 81.388   | 3.58207    | 81.352   |
| 0.95 | 2.96315    | 85.763   | 3.10648   | 85.738   | 3.25759    | 85.716   | 3.41683   | 85.696   | 3.58462    | 85.678   |
| 1.0  | 2.96419    | 90.000   | 3.10747   | 90.000   | 3.25853    | 90.000   | 3.41773   | 90.000   | 3.58548    | 90.000   |
| 1.05 | 2.96315    | 94.237   | 3.10648   | 94.262   | 3.25759    | 94.284   | 3.41683   | 94.304   | 3.58462    | 94.322   |
| 1.1  | 2.96006    | 98.480   | 3.10353   | 98.529   | 3.25477    | 98.572   | 3.41415   | 98.612   | 3.58207    | 98.648   |
| 1.15 | 2.95498    | 102.735  | 3.09869   | 102.806  | 3.25015    | 102.871  | 3.40975   | 102.929  | 3.57788    | 102.983  |
| 1.2  | 2.94804    | 107.008  | 3.09206   | 107.100  | 3.24384    | 107.184  | 3.40373   | 107.260  | 3.57213    | 107.330  |
| 1.25 | 2.93938    | 111.302  | 3.08382   | 111.414  | 3.23598    | 111.516  | 3.39624   | 111.608  | 3.56500    | 111.692  |
| 1.3  | 2.92921    | 115.625  | 3.07413   | 115.754  | 3.22675    | 115.871  | 3.38745   | 115.977  | 3.55663    | 116.073  |
| 1.35 | 2.91777    | 119.980  | 3.06323   | 120.123  | 3.21636    | 120.252  | 3.37756   | 120.369  | 3.54721    | 120.476  |
| 1.4  | 2.90532    | 124.370  | 3.05138   | 124.524  | 3.20507    | 124.663  | 3.36681   | 124.789  | 3.53698    | 124.904  |
| 1.45 | 2.89217    | 128.800  | 3.03885   | 128.961  | 3.19315    | 129.107  | 3.35546   | 129.239  | 3.52617    | 129.358  |
| 1.5  | 2.87861    | 133.270  | 3.02595   | 133.435  | 3.18088    | 133.584  | 3.34378   | 133.719  | 3.51507    | 133.840  |
| 1.55 | 2.86499    | 137.784  | 3.01300   | 137.947  | 3.16856    | 138.096  | 3.33207   | 138.230  | 3.50393    | 138.351  |
| 1.6  | 2.85165    | 142.339  | 3.00031   | 142.499  | 3.15650    | 142.643  | 3.32060   | 142.773  | 3.49302    | 142.890  |
| 1.65 | 2.83891    | 146.937  | 2.98821   | 147.088  | 3.14500    | 147.224  | 3.30967   | 147.346  | 3.48263    | 147.457  |
| 1.7  | 2.82710    | 151.575  | 2.97699   | 151.713  | 3.13434    | 151.837  | 3.29955   | 151.949  | 3.47301    | 152.050  |
| 1.75 | 2.81653    | 156.250  | 2.96695   | 156.371  | 3.12481    | 156.480  | 3.29049   | 156.579  | 3.46441    | 156.668  |
| 1.8  | 2.80747    | 160.958  | 2.95835   | 161.059  | 3.11665    | 161.150  | 3.28274   | 161.233  | 3.45702    | 161.307  |
| 1.85 | 2.80016    | 165.693  | 2.95142   | 165.772  | 3.11006    | 165.842  | 3.27649   | 165.906  | 3.45112    | 165.964  |
| 1.9  | 2.79479    | 170.449  | 2.94033   | 170.503  | 3.10523    | 170.552  | 3.27191   | 170.595  | 3.44675    | 170.635  |
| 1.95 | 2.79151    | 175.221  | 2.94322   | 175.248  | 3.10228    | 175.273  | 3.26911   | 175.295  | 3.44410    | 175.315  |
| 2.0  | 2.79041    | 180.000  | 2.94217   | 180.000  | 3.10129    | 180.000  | 3.26816   | 180.000  | 3.44321    | 180.000  |

Example.  $\sinh(1.90 + i 2.0) = 3.26816 / 180^\circ.0 = 3.26816 \sqrt{180^\circ.0}$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 2.0$ |          | $x = 2.05$ |          | $x = 2.1$ |          | $x = 2.15$ |          | $x = 2.2$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
|      | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 3.62686   | 0.000    | 2.81950    | 0.000    | 4.02186   | 0.000    | 4.23419    | 0.000    | 4.45711   | 0.000    |
| 0.05 | 3.62771   | 4.667    | 3.82039    | 4.651    | 4.02263   | 4.636    | 4.23491    | 4.623    | 4.45779   | 4.611    |
| 0.1  | 3.63023   | 9.330    | 3.82279    | 9.298    | 4.02490   | 9.269    | 4.23707    | 9.243    | 4.45985   | 9.220    |
| 0.15 | 3.63437   | 13.984   | 3.82671    | 13.938   | 4.02863   | 13.895   | 4.24061    | 13.857   | 4.46322   | 13.823   |
| 0.2  | 3.64000   | 18.626   | 3.83206    | 18.566   | 4.03371   | 18.511   | 4.24544    | 18.462   | 4.46780   | 18.418   |
| 0.25 | 3.64699   | 23.252   | 3.83870    | 23.180   | 4.04002   | 23.114   | 4.25145    | 23.055   | 4.47350   | 23.002   |
| 0.3  | 3.65517   | 27.858   | 3.84648    | 27.776   | 4.04740   | 27.701   | 4.25846    | 27.634   | 4.48017   | 27.573   |
| 0.35 | 3.66430   | 32.443   | 3.85515    | 32.353   | 4.05566   | 32.271   | 4.26630    | 32.197   | 4.48763   | 32.130   |
| 0.4  | 3.67418   | 37.004   | 3.86454    | 36.908   | 4.06459   | 36.821   | 4.27479    | 36.743   | 4.49570   | 36.671   |
| 0.45 | 3.68455   | 41.539   | 3.87440    | 41.440   | 4.07396   | 41.351   | 4.28371    | 41.270   | 4.50417   | 41.196   |
| 0.5  | 3.69515   | 46.049   | 3.88448    | 45.950   | 4.08355   | 45.859   | 4.29282    | 45.778   | 4.51285   | 45.703   |
| 0.55 | 3.70572   | 50.533   | 3.89453    | 50.435   | 4.09311   | 50.347   | 4.30193    | 50.266   | 4.52150   | 50.193   |
| 0.6  | 3.71600   | 54.992   | 3.90432    | 54.898   | 4.10243   | 54.813   | 4.31079    | 54.736   | 4.52993   | 54.666   |
| 0.65 | 3.72574   | 59.427   | 3.91359    | 59.340   | 4.11125   | 59.260   | 4.31918    | 59.188   | 4.53793   | 59.123   |
| 0.7  | 3.73470   | 63.840   | 3.92213    | 63.761   | 4.11938   | 63.689   | 4.32693    | 63.624   | 4.54529   | 63.565   |
| 0.75 | 3.74268   | 68.232   | 3.92972    | 68.164   | 4.12661   | 68.101   | 4.33381    | 68.044   | 4.55184   | 67.993   |
| 0.8  | 3.74948   | 72.608   | 3.93620    | 72.551   | 4.13278   | 72.499   | 4.33968    | 72.452   | 4.55744   | 72.409   |
| 0.85 | 3.75495   | 76.969   | 3.94142    | 76.925   | 4.13774   | 76.885   | 4.34440    | 76.849   | 4.56167   | 76.816   |
| 0.9  | 3.75894   | 81.319   | 3.94521    | 81.289   | 4.14136   | 81.262   | 4.34785    | 81.237   | 4.56523   | 81.215   |
| 0.95 | 3.76137   | 85.661   | 3.94753    | 85.646   | 4.14357   | 85.632   | 4.34996    | 85.620   | 4.56723   | 85.609   |
| 1.0  | 3.76220   | 90.000   | 3.94832    | 90.000   | 4.14431   | 90.000   | 4.35067    | 90.000   | 4.56791   | 90.000   |
| 1.05 | 3.76137   | 94.339   | 3.94753    | 94.354   | 4.14357   | 94.368   | 4.34996    | 94.380   | 4.56723   | 94.391   |
| 1.1  | 3.75894   | 98.681   | 3.94521    | 98.711   | 4.14136   | 98.738   | 4.34785    | 98.763   | 4.56523   | 98.785   |
| 1.15 | 3.75495   | 103.031  | 3.94142    | 103.075  | 4.13774   | 103.115  | 4.34440    | 103.151  | 4.56167   | 103.184  |
| 1.2  | 3.74948   | 107.392  | 3.93620    | 107.449  | 4.13278   | 107.501  | 4.33968    | 107.548  | 4.55744   | 107.591  |
| 1.25 | 3.74268   | 111.768  | 3.92972    | 111.836  | 4.12661   | 111.899  | 4.33381    | 111.956  | 4.55184   | 112.007  |
| 1.3  | 3.73470   | 116.160  | 3.92213    | 116.239  | 4.11938   | 116.311  | 4.32693    | 116.376  | 4.54529   | 116.435  |
| 1.35 | 3.72574   | 120.573  | 3.91359    | 120.660  | 4.11125   | 120.740  | 4.31918    | 120.812  | 4.53793   | 120.877  |
| 1.4  | 3.71600   | 125.008  | 3.90432    | 125.102  | 4.10243   | 125.187  | 4.31079    | 125.264  | 4.52993   | 125.334  |
| 1.45 | 3.70572   | 129.467  | 3.89453    | 129.565  | 4.09311   | 129.653  | 4.30193    | 129.734  | 4.52150   | 129.807  |
| 1.5  | 3.69515   | 133.951  | 3.88448    | 134.051  | 4.08355   | 134.141  | 4.29282    | 134.223  | 4.51285   | 134.297  |
| 1.55 | 3.68455   | 138.461  | 3.87440    | 138.560  | 4.07396   | 138.649  | 4.28371    | 138.730  | 4.50417   | 138.804  |
| 1.6  | 3.67418   | 142.996  | 3.86454    | 143.092  | 4.06459   | 143.179  | 4.27479    | 143.257  | 4.49570   | 143.329  |
| 1.65 | 3.66430   | 147.557  | 3.85515    | 147.647  | 4.05566   | 147.729  | 4.26630    | 147.803  | 4.48763   | 147.870  |
| 1.7  | 3.65517   | 152.142  | 3.84648    | 152.224  | 4.04740   | 152.299  | 4.25846    | 152.366  | 4.48017   | 152.427  |
| 1.75 | 3.64699   | 156.748  | 3.83870    | 156.820  | 4.04002   | 156.886  | 4.25145    | 156.945  | 4.47350   | 156.998  |
| 1.8  | 3.64000   | 161.374  | 3.83206    | 161.434  | 4.03371   | 161.489  | 4.24544    | 161.538  | 4.46780   | 161.582  |
| 1.85 | 3.63437   | 166.016  | 3.82671    | 166.062  | 4.02863   | 166.105  | 4.24061    | 166.143  | 4.46322   | 166.177  |
| 1.9  | 3.63023   | 170.670  | 3.82279    | 170.702  | 4.02490   | 170.731  | 4.23707    | 170.757  | 4.45985   | 170.780  |
| 1.95 | 3.62771   | 175.333  | 3.82039    | 175.349  | 4.02263   | 175.364  | 4.23491    | 175.377  | 4.45779   | 175.389  |
| 2.0  | 3.62686   | 180.000  | 3.81958    | 180.000  | 4.02186   | 180.000  | 4.23419    | 180.000  | 4.45711   | 180.000  |

Example.  $\sinh(2.0 + i 1.0) = 3.76220/\underline{90^\circ}$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 2.25$ |          | $x = 2.3$ |          | $x = 2.35$ |          | $x = 2.4$ |          | $x = 2.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
|      | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 4.69117    | 0.000    | 4.93696   | 0.000    | 5.19510    | 0.000    | 5.46623   | 0.000    | 5.75103    | 0.000    |
| 0.05 | 4.69182    | 4.601    | 4.93759   | 4.591    | 5.19569    | 4.582    | 5.46679   | 4.574    | 5.75156    | 4.567    |
| 0.1  | 4.69377    | 9.109    | 4.93944   | 9.180    | 5.19745    | 9.163    | 5.46847   | 9.147    | 5.75316    | 9.133    |
| 0.15 | 4.69697    | 13.792   | 4.94249   | 13.764   | 5.20035    | 13.739   | 5.47122   | 13.716   | 5.75576    | 13.695   |
| 0.2  | 4.70134    | 18.378   | 4.94662   | 18.341   | 5.20428    | 18.309   | 5.47495   | 18.279   | 5.75932    | 18.252   |
| 0.25 | 4.70675    | 22.954   | 4.95177   | 22.910   | 5.20917    | 22.871   | 5.47961   | 22.835   | 5.76375    | 22.803   |
| 0.3  | 4.71308    | 27.518   | 4.95780   | 27.469   | 5.21490    | 27.424   | 5.48505   | 27.383   | 5.76892    | 27.347   |
| 0.35 | 4.72017    | 32.070   | 4.96454   | 32.016   | 5.22131    | 31.966   | 5.49115   | 31.922   | 5.77472    | 31.881   |
| 0.4  | 4.72784    | 36.608   | 4.97183   | 36.549   | 5.22825    | 36.497   | 5.49775   | 36.449   | 5.78099    | 36.407   |
| 0.45 | 4.73592    | 41.130   | 4.97950   | 41.070   | 5.23553    | 41.016   | 5.50468   | 40.966   | 5.78758    | 40.922   |
| 0.5  | 4.74415    | 45.636   | 4.98734   | 45.576   | 5.23728    | 45.521   | 5.51177   | 45.471   | 5.79434    | 45.427   |
| 0.55 | 4.75240    | 50.127   | 4.99518   | 50.068   | 5.25046    | 50.014   | 5.51887   | 49.965   | 5.80108    | 49.921   |
| 0.6  | 4.76042    | 54.603   | 5.00281   | 54.546   | 5.25772    | 54.494   | 5.52578   | 54.447   | 5.80765    | 54.405   |
| 0.65 | 4.76802    | 59.064   | 5.01005   | 59.011   | 5.26461    | 58.962   | 5.53233   | 58.919   | 5.81389    | 58.879   |
| 0.7  | 4.77503    | 63.512   | 5.01672   | 63.463   | 5.27096    | 63.419   | 5.53837   | 63.380   | 5.81964    | 63.344   |
| 0.75 | 4.78127    | 67.947   | 5.02265   | 67.904   | 5.27662    | 67.866   | 5.54375   | 67.831   | 5.82476    | 67.800   |
| 0.8  | 4.78661    | 72.371   | 5.02773   | 72.336   | 5.28143    | 72.304   | 5.54834   | 72.275   | 5.82913    | 72.249   |
| 0.85 | 4.79088    | 76.786   | 5.03181   | 76.759   | 5.28531    | 76.735   | 5.55204   | 76.712   | 5.83265    | 76.692   |
| 0.9  | 4.79402    | 81.195   | 5.03479   | 81.176   | 5.28816    | 81.160   | 5.55474   | 81.144   | 5.83522    | 81.131   |
| 0.95 | 4.79593    | 85.599   | 5.03661   | 85.589   | 5.28989    | 85.581   | 5.55639   | 85.573   | 5.83680    | 85.566   |
| 1.0  | 4.79657    | 90.000   | 5.03722   | 90.000   | 5.29047    | 90.000   | 5.55695   | 90.000   | 5.83732    | 90.000   |
| 1.05 | 4.79593    | 94.401   | 5.03661   | 94.411   | 5.28989    | 94.419   | 5.55639   | 94.427   | 5.83680    | 94.434   |
| 1.1  | 4.79402    | 98.805   | 5.03479   | 98.824   | 5.28816    | 98.840   | 5.55474   | 98.856   | 5.83522    | 98.869   |
| 1.15 | 4.79088    | 103.214  | 5.03181   | 103.241  | 5.28531    | 103.265  | 5.55204   | 103.288  | 5.83265    | 103.308  |
| 1.2  | 4.78661    | 107.629  | 5.02773   | 107.664  | 5.28143    | 107.696  | 5.54834   | 107.725  | 5.82913    | 107.751  |
| 1.25 | 4.78127    | 112.053  | 5.02265   | 112.096  | 5.27662    | 112.134  | 5.54375   | 112.169  | 5.82476    | 112.200  |
| 1.3  | 4.77503    | 116.488  | 5.01672   | 116.537  | 5.27096    | 116.581  | 5.53837   | 116.620  | 5.81964    | 116.656  |
| 1.35 | 4.76802    | 120.936  | 5.01005   | 120.989  | 5.26461    | 121.038  | 5.53233   | 121.081  | 5.81389    | 121.121  |
| 1.4  | 4.76042    | 125.397  | 5.00281   | 125.454  | 5.25772    | 125.506  | 5.52578   | 125.553  | 5.80765    | 125.595  |
| 1.45 | 4.75240    | 129.873  | 4.99518   | 129.932  | 5.25046    | 129.986  | 5.51887   | 130.035  | 5.80108    | 130.079  |
| 1.5  | 4.74415    | 134.364  | 4.98734   | 134.424  | 5.23728    | 134.479  | 5.51177   | 134.529  | 5.79434    | 134.573  |
| 1.55 | 4.73592    | 138.870  | 4.97950   | 138.930  | 5.23553    | 138.984  | 5.50468   | 139.034  | 5.78758    | 139.078  |
| 1.6  | 4.72784    | 143.392  | 4.97183   | 143.451  | 5.22825    | 143.503  | 5.49775   | 143.551  | 5.78099    | 143.593  |
| 1.65 | 4.72017    | 147.930  | 4.96454   | 147.984  | 5.22131    | 148.034  | 5.49115   | 148.078  | 5.77472    | 148.119  |
| 1.7  | 4.71308    | 152.482  | 4.95780   | 152.531  | 5.21490    | 152.576  | 5.48505   | 152.617  | 5.76982    | 152.653  |
| 1.75 | 4.70675    | 157.046  | 4.95177   | 157.090  | 5.20917    | 157.129  | 5.47961   | 157.165  | 5.76575    | 157.197  |
| 1.8  | 4.70134    | 161.622  | 4.94662   | 161.659  | 5.20428    | 161.691  | 5.47495   | 161.721  | 5.75932    | 161.748  |
| 1.85 | 4.69697    | 166.208  | 4.94249   | 166.236  | 5.20035    | 166.261  | 5.47122   | 166.284  | 5.75576    | 166.305  |
| 1.9  | 4.69377    | 170.801  | 4.93944   | 170.820  | 5.19745    | 170.837  | 5.46847   | 170.853  | 5.75316    | 170.867  |
| 1.95 | 4.69182    | 175.399  | 4.93759   | 175.409  | 5.19569    | 175.418  | 5.46679   | 175.426  | 5.75156    | 175.433  |
| 2.0  | 4.69117    | 180.000  | 4.93696   | 180.000  | 5.19510    | 180.000  | 5.46623   | 180.000  | 5.75103    | 180.000  |

Example.  $\sinh(2.40 + i0.4) = 5.49775 / \underline{36^\circ.449} = 5.49775 / \underline{36^\circ.26'.56''}$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 2.5$ |          | $x = 2.55$ |          | $x = 2.6$ |          | $x = 2.65$ |          | $x = 2.7$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
|      | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 6.05020   | 0.000    | 6.36451    | 0.000    | 6.69473   | 0.000    | 7.04169    | 0.000    | 7.40626   | 0.000    |
| 0.05 | 6.05071   | 4.561    | 6.36499    | 4.555    | 6.69518   | 4.550    | 7.04213    | 4.545    | 7.40668   | 4.540    |
| 0.1  | 6.05223   | 9.120    | 6.36644    | 9.109    | 6.69656   | 9.098    | 7.04343    | 9.089    | 7.40791   | 9.080    |
| 0.15 | 6.05471   | 13.676   | 6.36879    | 13.660   | 6.69880   | 13.644   | 7.04556    | 13.631   | 7.40994   | 13.618   |
| 0.2  | 6.05808   | 18.228   | 6.37201    | 18.206   | 6.70185   | 18.187   | 7.04847    | 18.169   | 7.41271   | 18.153   |
| 0.25 | 6.06229   | 22.774   | 6.37601    | 22.748   | 6.70565   | 22.725   | 7.05208    | 22.703   | 7.41614   | 22.684   |
| 0.3  | 6.06722   | 27.314   | 6.38068    | 27.284   | 6.71011   | 27.257   | 7.05631    | 27.232   | 7.42017   | 27.210   |
| 0.35 | 6.07273   | 31.845   | 6.38592    | 31.812   | 6.71509   | 31.782   | 7.06105    | 31.756   | 7.42467   | 31.731   |
| 0.4  | 6.07869   | 36.368   | 6.39160    | 36.333   | 6.72048   | 36.301   | 7.06618    | 36.273   | 7.42955   | 36.246   |
| 0.45 | 6.08496   | 40.882   | 6.39755    | 40.845   | 6.72616   | 40.813   | 7.07158    | 40.783   | 7.43468   | 40.756   |
| 0.5  | 6.09139   | 45.386   | 6.40367    | 45.350   | 6.73197   | 45.316   | 7.07711    | 45.286   | 7.43994   | 45.259   |
| 0.55 | 6.09781   | 49.881   | 6.40977    | 49.845   | 6.73778   | 49.812   | 7.08264    | 49.782   | 7.44519   | 49.755   |
| 0.6  | 6.10406   | 54.366   | 6.41572    | 54.332   | 6.74343   | 54.300   | 7.08802    | 54.272   | 7.45032   | 54.246   |
| 0.65 | 6.10999   | 58.843   | 6.42137    | 58.810   | 6.74881   | 58.781   | 7.09313    | 58.754   | 7.45518   | 58.730   |
| 0.7  | 6.11547   | 63.311   | 6.42658    | 63.282   | 6.75376   | 63.255   | 7.09784    | 63.230   | 7.45966   | 63.209   |
| 0.75 | 6.12034   | 67.772   | 6.43121    | 67.746   | 6.75818   | 67.722   | 7.10204    | 67.701   | 7.46366   | 67.682   |
| 0.8  | 6.12450   | 72.226   | 6.43518    | 72.204   | 6.76195   | 72.185   | 7.10563    | 72.167   | 7.46708   | 72.152   |
| 0.85 | 6.12785   | 76.674   | 6.43836    | 76.658   | 6.76499   | 76.643   | 7.10851    | 76.629   | 7.46982   | 76.617   |
| 0.9  | 6.13030   | 81.119   | 6.44069    | 81.107   | 6.76720   | 81.097   | 7.11062    | 81.088   | 7.47183   | 81.080   |
| 0.95 | 6.13179   | 85.560   | 6.44212    | 85.554   | 6.76855   | 85.549   | 7.11191    | 85.544   | 7.47306   | 85.540   |
| 1.00 | 6.13229   | 90.000   | 6.44259    | 90.000   | 6.76901   | 90.000   | 7.11234    | 90.000   | 7.47347   | 90.000   |
| 1.05 | 6.13179   | 94.440   | 6.44212    | 94.446   | 6.76855   | 94.451   | 7.11191    | 94.456   | 7.47306   | 94.460   |
| 1.1  | 6.13030   | 98.881   | 6.44069    | 98.893   | 6.76720   | 98.903   | 7.11062    | 98.912   | 7.47183   | 98.920   |
| 1.15 | 6.12785   | 103.326  | 6.43836    | 103.342  | 6.76499   | 103.357  | 7.10851    | 103.371  | 7.46982   | 103.383  |
| 1.2  | 6.12450   | 107.774  | 6.43518    | 107.796  | 6.76195   | 107.815  | 7.10563    | 107.833  | 7.46708   | 107.848  |
| 1.25 | 6.12034   | 112.228  | 6.43121    | 112.254  | 6.75818   | 112.278  | 7.10204    | 112.299  | 7.46366   | 112.318  |
| 1.3  | 6.11547   | 116.689  | 6.42658    | 116.718  | 6.75376   | 116.745  | 7.09784    | 116.770  | 7.45966   | 116.791  |
| 1.35 | 6.10999   | 121.157  | 6.42137    | 121.190  | 6.74881   | 121.219  | 7.09313    | 121.246  | 7.45518   | 121.270  |
| 1.4  | 6.10406   | 125.634  | 6.41572    | 125.668  | 6.74343   | 125.700  | 7.08802    | 125.728  | 7.45032   | 125.754  |
| 1.45 | 6.09781   | 130.119  | 6.40977    | 130.155  | 6.73778   | 130.188  | 7.08264    | 130.218  | 7.44519   | 130.245  |
| 1.5  | 6.09139   | 134.614  | 6.40367    | 134.651  | 6.73197   | 134.684  | 7.07711    | 134.713  | 7.43994   | 134.741  |
| 1.55 | 6.08496   | 139.118  | 6.39755    | 139.155  | 6.72616   | 139.187  | 7.07158    | 139.217  | 7.43468   | 139.244  |
| 1.6  | 6.07869   | 143.632  | 6.39160    | 143.667  | 6.72048   | 143.699  | 7.06618    | 143.727  | 7.42955   | 143.754  |
| 1.65 | 6.07273   | 148.155  | 6.38592    | 148.188  | 6.71509   | 148.218  | 7.06105    | 148.244  | 7.42467   | 148.269  |
| 1.7  | 6.06722   | 152.686  | 6.38068    | 152.716  | 6.71011   | 152.743  | 7.05631    | 152.768  | 7.42017   | 152.790  |
| 1.75 | 6.06229   | 157.226  | 6.37601    | 157.252  | 6.70565   | 157.275  | 7.05208    | 157.297  | 7.41614   | 157.316  |
| 1.8  | 6.05808   | 161.772  | 6.37201    | 161.794  | 6.70185   | 161.813  | 7.04847    | 161.831  | 7.41271   | 161.847  |
| 1.85 | 6.05471   | 166.324  | 6.36879    | 166.340  | 6.69880   | 166.356  | 7.04566    | 166.369  | 7.40994   | 166.382  |
| 1.9  | 6.05223   | 170.880  | 6.36644    | 170.891  | 6.69656   | 170.902  | 7.04343    | 170.911  | 7.40791   | 170.920  |
| 1.95 | 6.05071   | 175.439  | 6.36499    | 175.445  | 6.69518   | 175.450  | 7.04213    | 175.455  | 7.40668   | 175.460  |
| 2.0  | 6.05020   | 180.000  | 6.36451    | 180.000  | 6.69473   | 180.000  | 7.04169    | 180.000  | 7.40626   | 180.000  |

Example.  $\sinh(2.6 + i0.6) = 6.74343 / 54^\circ.300 = 6.74343 / 54^\circ.18'00''$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 2.75$ |          | $x = 2.8$ |          | $x = 2.85$ |          | $x = 2.9$ |          | $x = 2.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 7.78935    | 0.000    | 8.19192   | 0.000    | 8.61497    | 0.000    | 9.05956   | 0.000    | 9.52681    | 0.000    |
| 0.05 | 7.78975    | 4.537    | 8.19230   | 4.533    | 8.61532    | 4.530    | 9.05990   | 4.527    | 9.52713    | 4.525    |
| 0.1  | 7.79092    | 9.073    | 8.19341   | 9.066    | 8.61639    | 9.060    | 9.06091   | 9.054    | 9.52809    | 9.049    |
| 0.15 | 7.79285    | 13.607   | 8.19524   | 13.596   | 8.61813    | 13.587   | 9.06257   | 13.579   | 9.52967    | 13.571   |
| 0.2  | 7.79547    | 18.138   | 8.19774   | 18.125   | 8.62051    | 18.113   | 9.06483   | 18.102   | 9.53182    | 18.093   |
| 0.25 | 7.79875    | 22.666   | 8.20085   | 22.650   | 8.62347    | 22.636   | 9.06764   | 22.623   | 9.53450    | 22.611   |
| 0.3  | 7.80257    | 27.190   | 8.20449   | 27.172   | 8.62691    | 27.155   | 9.07093   | 27.141   | 9.53762    | 27.127   |
| 0.35 | 7.80685    | 31.709   | 8.20857   | 31.689   | 8.63080    | 31.671   | 9.07461   | 31.655   | 9.54113    | 31.640   |
| 0.4  | 7.81149    | 36.223   | 8.21298   | 36.202   | 8.63500    | 36.183   | 9.07861   | 36.165   | 9.54492    | 36.150   |
| 0.45 | 7.81637    | 40.731   | 8.21762   | 40.710   | 8.63941    | 40.690   | 9.08281   | 40.671   | 9.54892    | 40.655   |
| 0.5  | 7.82138    | 45.234   | 8.22238   | 45.212   | 8.64395    | 45.192   | 9.08711   | 45.173   | 9.55301    | 45.157   |
| 0.55 | 7.82638    | 49.731   | 8.22713   | 49.709   | 8.64846    | 49.689   | 9.09142   | 49.671   | 9.55711    | 49.655   |
| 0.6  | 7.83125    | 54.222   | 8.23177   | 54.201   | 8.65287    | 54.182   | 9.09561   | 54.165   | 9.56109    | 54.149   |
| 0.65 | 7.83588    | 58.708   | 8.23617   | 58.688   | 8.65706    | 58.670   | 9.09959   | 58.654   | 9.56488    | 58.640   |
| 0.7  | 7.84015    | 63.189   | 8.24024   | 63.171   | 8.66092    | 63.155   | 9.10327   | 63.140   | 9.56838    | 63.127   |
| 0.75 | 7.84395    | 67.665   | 8.24385   | 67.649   | 8.66436    | 67.636   | 9.10655   | 67.622   | 9.57150    | 67.611   |
| 0.8  | 7.84720    | 72.137   | 8.24694   | 72.124   | 8.66731    | 72.112   | 9.10934   | 72.102   | 9.57416    | 72.092   |
| 0.85 | 7.84980    | 76.606   | 8.24942   | 76.596   | 8.66967    | 76.587   | 9.11160   | 76.579   | 9.57630    | 76.571   |
| 0.9  | 7.85172    | 81.072   | 8.25124   | 81.065   | 8.67140    | 81.059   | 9.11324   | 81.053   | 9.57787    | 81.048   |
| 0.95 | 7.85288    | 85.536   | 8.25235   | 85.533   | 8.67246    | 85.530   | 9.11424   | 85.527   | 9.57882    | 85.524   |
| 1.0  | 7.85328    | 90.000   | 8.25273   | 90.000   | 8.67281    | 90.000   | 9.11458   | 90.000   | 9.57915    | 90.000   |
| 1.05 | 7.85288    | 94.464   | 8.25235   | 94.467   | 8.67246    | 94.470   | 9.11424   | 94.473   | 9.57882    | 94.476   |
| 1.1  | 7.85172    | 98.928   | 8.25124   | 98.935   | 8.67140    | 98.941   | 9.11324   | 98.947   | 9.57787    | 98.952   |
| 1.15 | 7.84980    | 103.394  | 8.24942   | 103.404  | 8.66967    | 103.413  | 9.11160   | 103.421  | 9.57630    | 103.429  |
| 1.2  | 7.84720    | 107.863  | 8.24694   | 107.876  | 8.66731    | 107.888  | 9.10934   | 107.898  | 9.57416    | 107.908  |
| 1.25 | 7.84395    | 112.335  | 8.24385   | 112.351  | 8.66436    | 112.364  | 9.10655   | 112.378  | 9.57150    | 112.389  |
| 1.3  | 7.84015    | 116.811  | 8.24024   | 116.829  | 8.66092    | 116.845  | 9.10327   | 116.860  | 9.56838    | 116.873  |
| 1.35 | 7.83588    | 121.292  | 8.23617   | 121.312  | 8.65706    | 121.330  | 9.09959   | 121.346  | 9.56488    | 121.360  |
| 1.4  | 7.83125    | 125.778  | 8.23177   | 125.799  | 8.65287    | 125.818  | 9.09561   | 125.835  | 9.56109    | 125.851  |
| 1.45 | 7.82638    | 130.269  | 8.22713   | 130.291  | 8.64846    | 130.311  | 9.09142   | 130.329  | 9.55711    | 130.345  |
| 1.5  | 7.82138    | 134.766  | 8.22238   | 134.788  | 8.64395    | 134.808  | 9.08711   | 134.827  | 9.55301    | 134.843  |
| 1.55 | 7.81637    | 139.269  | 8.21762   | 139.290  | 8.63941    | 139.310  | 9.08281   | 139.329  | 9.54892    | 139.345  |
| 1.6  | 7.81149    | 143.777  | 8.21298   | 143.798  | 8.63500    | 143.817  | 9.07861   | 143.835  | 9.54492    | 143.850  |
| 1.65 | 7.80685    | 148.291  | 8.20857   | 148.311  | 8.63080    | 148.329  | 9.07461   | 148.345  | 9.54113    | 148.360  |
| 1.7  | 7.80257    | 152.810  | 8.20449   | 152.828  | 8.62691    | 152.845  | 9.07093   | 152.859  | 9.53762    | 152.873  |
| 1.75 | 7.79875    | 157.334  | 8.20085   | 157.350  | 8.62347    | 157.364  | 9.06764   | 157.377  | 9.53450    | 157.389  |
| 1.8  | 7.79547    | 161.862  | 8.19774   | 161.875  | 8.62051    | 161.887  | 9.06483   | 161.898  | 9.53182    | 161.907  |
| 1.85 | 7.79285    | 166.393  | 8.19524   | 166.404  | 8.61813    | 166.413  | 9.06257   | 166.421  | 9.52967    | 166.429  |
| 1.9  | 7.79092    | 170.927  | 8.19341   | 170.934  | 8.61639    | 170.940  | 9.06091   | 170.946  | 9.52809    | 170.951  |
| 1.95 | 7.78975    | 175.463  | 8.19230   | 175.467  | 8.61532    | 175.470  | 9.05990   | 175.473  | 9.52713    | 175.475  |
| 2.0  | 7.78935    | 180.000  | 8.19192   | 180.000  | 8.61497    | 180.000  | 9.05956   | 180.000  | 9.52681    | 180.000  |

Example.  $\sinh(2.95 + i\ 1.95) = 9.52713 / 175^{\circ}.475 = 9.52713 / 175^{\circ}.28'.30''$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 3.0$ |          | $x = 3.05$ |          | $x = 3.1$ |          | $x = 3.15$ |          | $x = 3.2$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
|      | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 10.01787  | 0.000    | 10.53399   | 0.000    | 11.07645  | 0.000    | 11.64661   | 0.000    | 12.24588  | 0.000    |
| 0.05 | 10.01820  | 4.522    | 10.53430   | 4.520    | 11.07670  | 4.518    | 11.64690   | 4.516    | 12.24610  | 4.515    |
| 0.1  | 10.01910  | 9.044    | 10.53520   | 9.040    | 11.07750  | 9.036    | 11.64770   | 9.033    | 12.24690  | 9.030    |
| 0.15 | 10.02060  | 13.565   | 10.53660   | 13.559   | 11.07890  | 13.553   | 11.64890   | 13.548   | 12.24810  | 13.543   |
| 0.2  | 10.02260  | 18.084   | 10.53850   | 18.076   | 11.08080  | 18.068   | 11.65070   | 18.062   | 12.24980  | 18.056   |
| 0.25 | 10.02520  | 22.601   | 10.54090   | 22.591   | 11.08310  | 22.583   | 11.65290   | 22.575   | 12.25180  | 22.568   |
| 0.3  | 10.02820  | 27.115   | 10.54380   | 27.104   | 11.08580  | 27.094   | 11.65540   | 27.085   | 12.25430  | 27.077   |
| 0.35 | 10.03150  | 31.627   | 10.54690   | 31.615   | 11.08880  | 31.604   | 11.65830   | 31.594   | 12.25700  | 31.585   |
| 0.4  | 10.03510  | 36.135   | 10.55040   | 36.122   | 11.09200  | 36.111   | 11.66140   | 36.100   | 12.26000  | 36.091   |
| 0.45 | 10.03890  | 40.640   | 10.55400   | 40.627   | 11.09540  | 40.615   | 11.66470   | 40.604   | 12.26310  | 40.594   |
| 0.5  | 10.04280  | 45.142   | 10.55770   | 45.129   | 11.09900  | 45.116   | 11.66810   | 45.105   | 12.26630  | 45.095   |
| 0.55 | 10.04670  | 49.640   | 10.56140   | 49.627   | 11.10250  | 49.615   | 11.67140   | 49.604   | 12.26950  | 49.594   |
| 0.6  | 10.05050  | 54.135   | 10.56500   | 54.122   | 11.10600  | 54.110   | 11.67470   | 54.100   | 12.27260  | 54.090   |
| 0.65 | 10.05410  | 58.626   | 10.56840   | 58.614   | 11.10920  | 58.604   | 11.67780   | 58.594   | 12.27550  | 58.585   |
| 0.7  | 10.05743  | 63.114   | 10.57160   | 63.104   | 11.11220  | 63.094   | 11.68060   | 63.085   | 12.27820  | 63.077   |
| 0.75 | 10.06040  | 67.600   | 10.57440   | 67.591   | 11.11490  | 67.582   | 11.68320   | 67.574   | 12.28070  | 67.567   |
| 0.8  | 10.06292  | 72.083   | 10.57680   | 72.075   | 11.11720  | 72.068   | 11.68540   | 72.062   | 12.28280  | 72.056   |
| 0.85 | 10.06500  | 76.564   | 10.57880   | 76.558   | 11.11900  | 76.553   | 11.68710   | 76.548   | 12.28440  | 76.543   |
| 0.9  | 10.06645  | 81.044   | 10.58020   | 81.040   | 11.12040  | 81.036   | 11.68840   | 81.032   | 12.28560  | 81.029   |
| 0.95 | 10.06737  | 85.522   | 10.58110   | 85.520   | 11.12120  | 85.518   | 11.68920   | 85.516   | 12.28640  | 85.515   |
| 1.0  | 10.06766  | 90.000   | 10.58135   | 90.000   | 11.12150  | 90.000   | 11.68946   | 90.000   | 12.28665  | 90.000   |
| 1.05 | 10.06737  | 94.478   | 10.58110   | 94.480   | 11.12120  | 94.482   | 11.68920   | 94.484   | 12.28640  | 94.485   |
| 1.1  | 10.06645  | 98.956   | 10.58020   | 98.960   | 11.12040  | 98.964   | 11.68840   | 98.968   | 12.28560  | 98.971   |
| 1.15 | 10.06500  | 103.436  | 10.57880   | 103.442  | 11.11900  | 103.447  | 11.68710   | 103.452  | 12.28440  | 103.457  |
| 1.2  | 10.06292  | 107.917  | 10.57680   | 107.925  | 11.11720  | 107.932  | 11.68540   | 107.938  | 12.28280  | 107.944  |
| 1.25 | 10.06040  | 112.400  | 10.57440   | 112.409  | 11.11490  | 112.418  | 11.68320   | 112.426  | 12.28070  | 112.433  |
| 1.3  | 10.05743  | 116.886  | 10.57160   | 116.896  | 11.11220  | 116.906  | 11.68060   | 116.915  | 12.27820  | 116.923  |
| 1.35 | 10.05410  | 121.374  | 10.56840   | 121.386  | 11.10920  | 121.396  | 11.67780   | 121.406  | 12.27550  | 121.415  |
| 1.4  | 10.05050  | 125.856  | 10.56500   | 125.878  | 11.10600  | 125.890  | 11.67470   | 125.900  | 12.27260  | 125.910  |
| 1.45 | 10.04670  | 130.360  | 10.56140   | 130.373  | 11.10250  | 130.385  | 11.67140   | 130.396  | 12.26950  | 130.406  |
| 1.5  | 10.04280  | 134.858  | 10.55770   | 134.871  | 11.09900  | 134.884  | 11.66810   | 134.895  | 12.26630  | 134.905  |
| 1.55 | 10.03890  | 139.360  | 10.55400   | 139.373  | 11.09540  | 139.385  | 11.66470   | 139.396  | 12.26310  | 139.406  |
| 1.6  | 10.03510  | 143.865  | 10.55040   | 143.878  | 11.09200  | 143.889  | 11.66140   | 143.900  | 12.26000  | 143.909  |
| 1.65 | 10.03150  | 148.373  | 10.54690   | 148.385  | 11.08880  | 148.396  | 11.65830   | 148.406  | 12.25700  | 148.415  |
| 1.7  | 10.02820  | 152.885  | 10.54380   | 152.896  | 11.08580  | 152.906  | 11.65540   | 152.915  | 12.25430  | 152.923  |
| 1.75 | 10.02520  | 157.399  | 10.54090   | 157.409  | 11.08310  | 157.417  | 11.65290   | 157.425  | 12.25180  | 157.432  |
| 1.8  | 10.02260  | 161.916  | 10.53850   | 161.924  | 11.08080  | 161.931  | 11.65070   | 161.938  | 12.24980  | 161.944  |
| 1.85 | 10.02060  | 166.435  | 10.53660   | 166.441  | 11.07890  | 166.447  | 11.64890   | 166.452  | 12.24810  | 166.457  |
| 1.9  | 10.01910  | 170.956  | 10.53520   | 170.960  | 11.07750  | 170.964  | 11.64770   | 170.967  | 12.24690  | 170.970  |
| 1.95 | 10.01820  | 175.478  | 10.53430   | 175.480  | 11.07670  | 175.482  | 11.64690   | 175.484  | 12.24610  | 175.485  |
| 2.0  | 10.01787  | 180.000  | 10.53399   | 180.000  | 11.07645  | 180.000  | 11.64661   | 180.000  | 12.24588  | 180.000  |

Example.  $\sinh(3.2 + i\ 1.1) = 12.28560 / 98^\circ.971 = 12.28560 / 98^\circ.58'.16''.$

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 3.25$ |          | $x = 3.3$ |          | $x = 3.35$ |          | $x = 3.4$ |          | $x = 3.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
|      | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 12.87578   | 0.000    | 13.53788  | 0.000    | 14.23382   | 0.000    | 14.96536  | 0.000    | 15.73432   | 0.000    |
| 0.05 | 12.87600   | 4.514    | 13.53810  | 4.512    | 14.23403   | 4.511    | 14.96556  | 4.510    | 15.73450   | 4.509    |
| 0.1  | 12.87670   | 9.027    | 13.53878  | 9.024    | 14.23470   | 9.022    | 14.96616  | 9.020    | 15.73510   | 9.018    |
| 0.15 | 12.87790   | 13.539   | 13.53991  | 13.536   | 14.23573   | 13.532   | 14.96716  | 13.529   | 15.73603   | 13.526   |
| 0.2  | 12.87949   | 18.051   | 13.54141  | 18.046   | 14.23718   | 18.041   | 14.96855  | 18.038   | 15.73736   | 18.034   |
| 0.25 | 12.88146   | 22.561   | 13.54322  | 22.555   | 14.23897   | 22.550   | 14.97023  | 22.545   | 15.73898   | 22.541   |
| 0.3  | 12.88380   | 27.070   | 13.54550  | 27.063   | 14.24106   | 27.057   | 14.97225  | 27.052   | 15.74088   | 27.047   |
| 0.35 | 12.88637   | 31.577   | 13.54796  | 31.569   | 14.24339   | 31.563   | 14.97448  | 31.557   | 15.74300   | 31.551   |
| 0.4  | 12.88910   | 36.082   | 13.55062  | 36.074   | 14.24595   | 36.067   | 14.97690  | 36.061   | 15.74529   | 36.055   |
| 0.45 | 12.89215   | 40.585   | 13.55346  | 40.577   | 14.24863   | 40.570   | 14.97945  | 40.563   | 15.74772   | 40.557   |
| 0.5  | 12.89518   | 45.086   | 13.55633  | 45.078   | 14.25137   | 45.071   | 14.98205  | 45.064   | 15.75020   | 45.058   |
| 0.55 | 12.89820   | 49.585   | 13.55918  | 49.577   | 14.25412   | 49.570   | 14.98466  | 49.563   | 15.75270   | 49.557   |
| 0.6  | 12.90117   | 54.082   | 13.56203  | 54.074   | 14.25614   | 54.067   | 14.98721  | 54.061   | 15.75510   | 54.055   |
| 0.65 | 12.90398   | 58.577   | 13.56470  | 58.569   | 14.25933   | 58.563   | 14.98960  | 58.557   | 15.75740   | 58.551   |
| 0.7  | 12.90658   | 63.070   | 13.56718  | 63.063   | 14.26167   | 63.057   | 14.99186  | 63.052   | 15.75950   | 63.047   |
| 0.75 | 12.90888   | 67.561   | 13.56936  | 67.555   | 14.26377   | 67.550   | 14.99385  | 67.545   | 15.76144   | 67.541   |
| 0.8  | 12.91085   | 72.051   | 13.57123  | 72.046   | 14.26556   | 72.041   | 14.99555  | 72.038   | 15.76303   | 72.034   |
| 0.85 | 12.91240   | 76.539   | 13.57275  | 76.535   | 14.26700   | 76.532   | 14.99692  | 76.529   | 15.76433   | 76.526   |
| 0.9  | 12.91360   | 81.027   | 13.57387  | 81.024   | 14.26805   | 81.022   | 14.99790  | 81.020   | 15.76530   | 81.018   |
| 0.95 | 12.91430   | 85.514   | 13.57455  | 85.512   | 14.26870   | 85.511   | 14.99853  | 85.510   | 15.76587   | 85.509   |
| 1.0  | 12.91456   | 90.000   | 13.57476  | 90.000   | 14.26891   | 90.000   | 14.99874  | 90.000   | 15.76607   | 90.000   |
| 1.05 | 12.91430   | 94.486   | 13.57455  | 94.488   | 14.26870   | 94.489   | 14.99853  | 94.490   | 15.76587   | 94.491   |
| 1.1  | 12.91360   | 98.973   | 13.57387  | 98.976   | 14.26805   | 98.978   | 14.99790  | 98.980   | 15.76530   | 98.982   |
| 1.15 | 12.91240   | 103.461  | 13.57275  | 103.465  | 14.26700   | 103.468  | 14.99692  | 103.471  | 15.76433   | 103.474  |
| 1.2  | 12.91085   | 107.949  | 13.57123  | 107.954  | 14.26556   | 107.959  | 14.99555  | 107.963  | 15.76303   | 107.966  |
| 1.25 | 12.90888   | 112.439  | 13.56936  | 112.445  | 14.26377   | 112.450  | 14.99385  | 112.455  | 15.76144   | 112.459  |
| 1.3  | 12.90658   | 116.930  | 13.56718  | 116.937  | 14.26167   | 116.943  | 14.99186  | 116.948  | 15.75950   | 116.953  |
| 1.35 | 12.90398   | 121.423  | 13.56470  | 121.431  | 14.25933   | 121.437  | 14.98960  | 121.443  | 15.75740   | 121.449  |
| 1.4  | 12.90117   | 125.918  | 13.56203  | 125.926  | 14.25614   | 125.933  | 14.98721  | 125.939  | 15.75510   | 125.945  |
| 1.45 | 12.89820   | 130.415  | 13.55918  | 130.423  | 14.25412   | 130.430  | 14.98466  | 130.437  | 15.75270   | 130.443  |
| 1.5  | 12.89518   | 134.914  | 13.55633  | 134.922  | 14.25137   | 134.929  | 14.98205  | 134.936  | 15.75020   | 134.942  |
| 1.55 | 12.89215   | 139.415  | 13.55346  | 139.423  | 14.24863   | 139.430  | 14.97945  | 139.437  | 15.74772   | 139.443  |
| 1.6  | 12.88910   | 143.918  | 13.55062  | 143.926  | 14.24595   | 143.933  | 14.97690  | 143.939  | 15.74529   | 143.945  |
| 1.65 | 12.88637   | 148.423  | 13.54796  | 148.431  | 14.24339   | 148.437  | 14.97448  | 148.443  | 15.74300   | 148.449  |
| 1.7  | 12.88380   | 152.930  | 13.54550  | 152.937  | 14.24106   | 152.943  | 14.97225  | 152.948  | 15.74088   | 152.953  |
| 1.75 | 12.88146   | 157.439  | 13.54322  | 157.445  | 14.23897   | 157.450  | 14.97023  | 157.455  | 15.73898   | 157.459  |
| 1.8  | 12.87949   | 161.949  | 13.54141  | 161.954  | 14.23718   | 161.959  | 14.96855  | 161.962  | 15.73736   | 161.966  |
| 1.85 | 12.87790   | 166.461  | 13.53991  | 166.464  | 14.23573   | 166.468  | 14.96716  | 166.471  | 15.73603   | 166.474  |
| 1.9  | 12.87670   | 170.973  | 13.53878  | 170.976  | 14.23470   | 170.978  | 14.96616  | 170.980  | 15.73510   | 170.982  |
| 1.95 | 12.87600   | 175.486  | 13.53810  | 175.488  | 14.23403   | 175.489  | 14.96556  | 175.490  | 15.73450   | 175.491  |
| 2.0  | 12.87578   | 180.000  | 13.53788  | 180.000  | 14.23382   | 180.000  | 14.96536  | 180.000  | 15.73432   | 180.000  |

Example.  $\sinh(3.3 + i \underline{1.3}) = 13.56718 / \underline{116^{\circ}.937} = 13.56718 / \underline{116^{\circ}.56'.13''}$ .



TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

| $x = 3.5$ |          |          | $x = 3.55$ |          | $x = 3.6$ |          | $x = 3.65$ |          | $x = 3.7$ |          |
|-----------|----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$       | $r$      | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
|           |          | °        |            | °        |           | °        |            | °        |           | °        |
| 0         | 16.54263 | 0.000    | 17.39230   | 0.000    | 18.28546  | 0.000    | 19.22434   | 0.000    | 20.21129  | 0.000    |
| 0.05      | 16.54281 | 4.508    | 17.39248   | 4.507    | 18.28560  | 4.507    | 19.22448   | 4.506    | 20.21140  | 4.506    |
| 0.1       | 16.54337 | 9.016    | 17.39300   | 9.015    | 18.28611  | 9.013    | 19.22496   | 9.012    | 20.21189  | 9.011    |
| 0.15      | 16.54428 | 13.524   | 17.39386   | 13.522   | 18.28694  | 13.519   | 19.22577   | 13.518   | 20.21268  | 13.516   |
| 0.2       | 16.54550 | 18.031   | 17.39504   | 18.028   | 18.28805  | 18.025   | 19.22681   | 18.023   | 20.21365  | 18.021   |
| 0.25      | 16.54702 | 22.537   | 17.39650   | 22.534   | 18.28942  | 22.530   | 19.22813   | 22.527   | 20.21490  | 22.525   |
| 0.3       | 16.54885 | 27.042   | 17.39822   | 27.038   | 18.29109  | 27.035   | 19.22968   | 27.031   | 20.21639  | 27.028   |
| 0.35      | 16.55087 | 31.547   | 17.40015   | 31.542   | 18.29292  | 31.538   | 19.23142   | 31.535   | 20.21804  | 31.531   |
| 0.4       | 16.55307 | 36.050   | 17.40222   | 36.045   | 18.29490  | 36.041   | 19.23331   | 36.037   | 20.21981  | 36.033   |
| 0.45      | 16.55538 | 40.552   | 17.40441   | 40.547   | 18.29699  | 40.542   | 19.23530   | 40.538   | 20.22170  | 40.535   |
| 0.5       | 16.55774 | 45.052   | 17.40665   | 45.047   | 18.29912  | 45.043   | 19.23732   | 45.039   | 20.22365  | 45.035   |
| 0.55      | 16.56010 | 49.552   | 17.40900   | 49.547   | 18.30126  | 49.542   | 19.23940   | 49.538   | 20.22560  | 49.535   |
| 0.6       | 16.56240 | 54.050   | 17.41110   | 54.045   | 18.30335  | 54.041   | 19.24134   | 54.037   | 20.22750  | 54.033   |
| 0.65      | 16.56460 | 58.547   | 17.41310   | 58.542   | 18.30530  | 58.538   | 19.24323   | 58.535   | 20.22930  | 58.531   |
| 0.7       | 16.56660 | 63.042   | 17.41510   | 63.038   | 18.30715  | 63.035   | 19.24496   | 63.031   | 20.23090  | 63.028   |
| 0.75      | 16.56840 | 67.537   | 17.41680   | 67.533   | 18.30880  | 67.530   | 19.24650   | 67.527   | 20.23240  | 67.525   |
| 0.8       | 16.56996 | 72.031   | 17.41830   | 72.028   | 18.31020  | 72.025   | 19.24790   | 72.023   | 20.23365  | 72.021   |
| 0.85      | 16.57120 | 76.524   | 17.41945   | 76.521   | 18.31130  | 76.519   | 19.24890   | 76.518   | 20.23460  | 76.516   |
| 0.9       | 16.57210 | 81.016   | 17.42030   | 81.015   | 18.31210  | 81.013   | 19.24970   | 81.012   | 20.23540  | 81.011   |
| 0.95      | 16.57260 | 85.508   | 17.42083   | 85.507   | 18.31260  | 85.507   | 19.25015   | 85.506   | 20.23585  | 85.506   |
| 1.0       | 16.57282 | 90.000   | 17.42102   | 90.000   | 18.31278  | 90.000   | 19.25033   | 90.000   | 20.23601  | 90.000   |
| 1.05      | 16.57260 | 94.492   | 17.42083   | 94.493   | 18.31260  | 94.493   | 19.25015   | 94.494   | 20.23585  | 94.494   |
| 1.1       | 16.57210 | 98.984   | 17.42030   | 98.985   | 18.31210  | 98.987   | 19.24970   | 98.988   | 20.23540  | 98.989   |
| 1.15      | 16.57120 | 103.476  | 17.41945   | 103.479  | 18.31130  | 103.480  | 19.24890   | 103.482  | 20.23460  | 103.484  |
| 1.2       | 16.56996 | 107.969  | 17.41830   | 107.972  | 18.31020  | 107.975  | 19.24790   | 107.977  | 20.23365  | 107.979  |
| 1.25      | 16.56840 | 112.463  | 17.41680   | 112.467  | 18.30880  | 112.470  | 19.24650   | 112.473  | 20.23240  | 112.475  |
| 1.3       | 16.56660 | 116.958  | 17.41510   | 116.962  | 18.30715  | 116.965  | 19.24496   | 116.969  | 20.23090  | 116.972  |
| 1.35      | 16.56460 | 121.453  | 17.41310   | 121.458  | 18.30530  | 121.462  | 19.24323   | 121.465  | 20.22930  | 121.469  |
| 1.4       | 16.56240 | 125.950  | 17.41110   | 125.955  | 18.30335  | 125.959  | 19.24134   | 125.963  | 20.22750  | 125.967  |
| 1.45      | 16.56010 | 130.448  | 17.40900   | 130.453  | 18.30126  | 130.458  | 19.23940   | 130.462  | 20.22560  | 130.465  |
| 1.5       | 16.55774 | 134.948  | 17.40665   | 134.953  | 18.29912  | 134.957  | 19.23732   | 134.961  | 20.22365  | 134.965  |
| 1.55      | 16.55538 | 139.448  | 17.40441   | 139.453  | 18.29699  | 139.458  | 19.23530   | 139.462  | 20.22170  | 139.465  |
| 1.6       | 16.55307 | 143.950  | 17.40222   | 143.955  | 18.29490  | 143.959  | 19.23331   | 143.963  | 20.21981  | 143.967  |
| 1.65      | 16.55087 | 148.453  | 17.40015   | 148.458  | 18.29292  | 148.462  | 19.23142   | 148.465  | 20.21804  | 148.469  |
| 1.7       | 16.54885 | 152.958  | 17.39822   | 152.962  | 18.29109  | 152.965  | 19.22968   | 152.969  | 20.21639  | 152.972  |
| 1.75      | 16.54702 | 157.463  | 17.39650   | 157.466  | 18.28942  | 157.470  | 19.22813   | 157.473  | 20.21490  | 157.475  |
| 1.8       | 16.54550 | 161.969  | 17.38504   | 161.972  | 18.28805  | 161.975  | 19.22681   | 161.977  | 20.21365  | 161.979  |
| 1.85      | 16.54428 | 166.476  | 17.39386   | 166.478  | 18.28694  | 166.481  | 19.22577   | 166.482  | 20.21268  | 166.484  |
| 1.9       | 16.54337 | 170.984  | 17.39300   | 170.985  | 18.28611  | 170.987  | 19.22496   | 170.988  | 20.21189  | 170.989  |
| 1.95      | 16.54281 | 175.492  | 17.39248   | 175.493  | 18.28560  | 175.493  | 19.22448   | 175.494  | 20.21140  | 175.494  |
| 2.0       | 16.54263 | 180.000  | 17.39230   | 180.000  | 18.28546  | 180.000  | 19.22434   | 180.000  | 20.21129  | 180.000  |

Example.  $\sinh(3.65 + i 0.25) = 19.22813 / 22^\circ.527 = 19.22813 / 22^\circ.31'.37''$ .

TABLE X. HYPERBOLIC SINES.  $\sinh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 3.75$ |          | $x = 3.8$ |          | $x = 3.85$ |          | $x = 3.9$ |          | $x = 3.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
|      | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 21.24878   | 0.000    | 22.33941  | 0.000    | 23.48589   | 0.000    | 24.69110  | 0.000    | 25.95806   | 0.000    |
| 0.05 | 21.24891   | 4.505    | 22.33952  | 4.504    | 23.48601   | 4.504    | 24.69120  | 4.504    | 25.95820   | 4.503    |
| 0.1  | 21.24935   | 9.010    | 22.33995  | 9.009    | 23.48640   | 9.008    | 24.69159  | 9.007    | 25.95854   | 9.007    |
| 0.15 | 21.25006   | 13.514   | 22.34061  | 13.513   | 23.48704   | 13.512   | 24.69223  | 13.511   | 25.95911   | 13.510   |
| 0.2  | 21.25102   | 18.019   | 22.34153  | 18.017   | 23.48791   | 18.015   | 24.69302  | 18.014   | 25.95991   | 18.013   |
| 0.25 | 21.25221   | 22.522   | 22.34270  | 22.520   | 23.48900   | 22.518   | 24.69406  | 22.517   | 25.96090   | 22.515   |
| 0.3  | 21.25362   | 27.026   | 22.34401  | 27.023   | 23.49028   | 27.021   | 24.69528  | 27.019   | 25.96205   | 27.017   |
| 0.35 | 21.25520   | 31.528   | 22.34550  | 31.526   | 23.49115   | 31.523   | 24.69662  | 31.521   | 25.96333   | 31.519   |
| 0.4  | 21.25685   | 36.030   | 22.34712  | 36.027   | 23.49322   | 36.025   | 24.69809  | 36.022   | 25.96471   | 36.020   |
| 0.45 | 21.25869   | 40.531   | 22.34883  | 40.528   | 23.49486   | 40.526   | 24.69964  | 40.523   | 25.96618   | 40.521   |
| 0.5  | 21.26052   | 45.032   | 22.35060  | 45.029   | 23.49652   | 45.026   | 24.70121  | 45.023   | 25.96770   | 45.021   |
| 0.55 | 21.26236   | 49.531   | 22.35230  | 49.528   | 23.49820   | 49.526   | 24.70280  | 49.523   | 25.96920   | 49.521   |
| 0.6  | 21.26415   | 54.030   | 22.35403  | 54.027   | 23.49980   | 54.025   | 24.70440  | 54.022   | 25.97066   | 54.020   |
| 0.65 | 21.26586   | 58.528   | 22.35565  | 58.526   | 23.50136   | 58.523   | 24.70580  | 58.521   | 25.97206   | 58.519   |
| 0.7  | 21.26745   | 63.026   | 22.35716  | 63.023   | 23.50277   | 63.021   | 24.70720  | 63.019   | 25.97337   | 63.017   |
| 0.75 | 21.26885   | 67.522   | 22.35850  | 67.520   | 23.50404   | 67.518   | 24.70840  | 67.517   | 25.97450   | 67.515   |
| 0.8  | 21.27000   | 72.019   | 22.35962  | 72.017   | 23.50512   | 72.015   | 24.70940  | 72.014   | 25.97550   | 72.013   |
| 0.85 | 21.27100   | 76.514   | 22.36055  | 76.513   | 23.50600   | 76.512   | 24.71024  | 76.511   | 25.97630   | 76.510   |
| 0.9  | 21.27170   | 81.010   | 22.36122  | 81.009   | 23.50664   | 81.008   | 24.71090  | 81.007   | 25.97680   | 81.007   |
| 0.95 | 21.27212   | 85.505   | 22.36163  | 85.504   | 23.50702   | 85.504   | 24.71120  | 85.504   | 25.97720   | 85.503   |
| 1.0  | 21.27230   | 90.000   | 22.36178  | 90.000   | 23.50717   | 90.000   | 24.71135  | 90.000   | 25.97731   | 90.000   |
| 1.05 | 21.27212   | 94.495   | 22.36163  | 94.496   | 23.50702   | 94.496   | 24.71120  | 94.496   | 25.97720   | 94.497   |
| 1.1  | 21.27170   | 98.990   | 22.36122  | 98.991   | 23.50664   | 98.992   | 24.71090  | 98.993   | 25.97680   | 98.993   |
| 1.15 | 21.27100   | 103.486  | 22.36055  | 103.487  | 23.50600   | 103.488  | 24.71024  | 103.489  | 25.97630   | 103.490  |
| 1.2  | 21.27000   | 107.981  | 22.35962  | 107.983  | 23.50512   | 107.985  | 24.70940  | 107.986  | 25.97550   | 107.987  |
| 1.25 | 21.26885   | 112.478  | 22.35850  | 112.480  | 23.50404   | 112.482  | 24.70840  | 112.483  | 25.97450   | 112.485  |
| 1.3  | 21.26745   | 116.974  | 22.35716  | 116.977  | 23.50277   | 116.979  | 24.70720  | 116.981  | 25.97337   | 116.983  |
| 1.35 | 21.26586   | 121.472  | 22.35565  | 121.474  | 23.50136   | 121.477  | 24.70580  | 121.479  | 25.97206   | 121.481  |
| 1.4  | 21.26415   | 125.970  | 22.35403  | 125.973  | 23.49980   | 125.975  | 24.70440  | 125.977  | 25.97066   | 125.980  |
| 1.45 | 21.26236   | 130.469  | 22.35230  | 130.472  | 23.49820   | 130.474  | 24.70280  | 130.477  | 25.96920   | 130.479  |
| 1.5  | 21.26052   | 134.968  | 22.35060  | 134.971  | 23.49652   | 134.974  | 24.70121  | 134.976  | 25.96770   | 134.979  |
| 1.55 | 21.25869   | 139.469  | 22.34883  | 139.472  | 23.49486   | 139.474  | 24.69964  | 139.477  | 25.96618   | 139.479  |
| 1.6  | 21.25685   | 143.970  | 22.34712  | 143.973  | 23.49322   | 143.975  | 24.69809  | 143.978  | 25.96471   | 143.980  |
| 1.65 | 21.25520   | 148.472  | 22.34550  | 148.474  | 23.49115   | 148.477  | 24.69662  | 148.479  | 25.96333   | 148.481  |
| 1.7  | 21.25362   | 152.974  | 22.34401  | 152.977  | 23.49028   | 152.979  | 24.69528  | 152.981  | 25.96205   | 152.983  |
| 1.75 | 21.25221   | 157.478  | 22.34270  | 157.480  | 23.48900   | 157.482  | 24.69406  | 157.483  | 25.96090   | 157.485  |
| 1.8  | 21.25102   | 161.981  | 22.34153  | 161.983  | 23.48791   | 161.985  | 24.69302  | 161.986  | 25.95991   | 161.987  |
| 1.85 | 21.25006   | 166.486  | 22.34061  | 166.487  | 23.48704   | 166.488  | 24.69223  | 166.489  | 25.95911   | 166.490  |
| 1.9  | 21.24935   | 170.990  | 22.33995  | 170.991  | 23.48640   | 170.992  | 24.69159  | 170.993  | 25.95854   | 170.993  |
| 1.95 | 21.24891   | 175.495  | 22.33952  | 175.496  | 23.48601   | 175.496  | 24.69120  | 175.496  | 25.95820   | 175.497  |
| 2.0  | 21.24878   | 180.000  | 22.33941  | 180.000  | 23.48589   | 180.000  | 24.69110  | 180.000  | 25.95806   | 180.000  |

Example.  $\sinh(3.90 + i 1.90) = 24.69159 / \underline{170^\circ.993} = 24.69159 / \underline{170^\circ.59'.35''}$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ 

| $q$  | $x = 0.0$ |          | $x = 0.05$ |          | $x = 0.1$ |          | $x = 0.15$ |          | $x = 0.2$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
|      | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 1.00000   | 0.00     | 1.00125    | 0.00     | 1.00500   | 0.00     | 1.01127    | 0.00     | 1.02007   | 0.00     |
| 0.05 | 0.99692   | 0.00     | 0.99817    | 0.225    | 1.00194   | 0.450    | 1.00822    | 0.671    | 1.01704   | 0.890    |
| 0.1  | 0.98769   | 0.00     | 0.98895    | 0.453    | 0.99275   | 0.905    | 0.99909    | 1.351    | 1.00800   | 1.791    |
| 0.15 | 0.97237   | 0.00     | 0.97366    | 0.687    | 0.97752   | 1.371    | 0.98396    | 2.047    | 0.99300   | 2.713    |
| 0.2  | 0.95106   | 0.00     | 0.95237    | 0.930    | 0.95632   | 1.855    | 0.96290    | 2.769    | 0.97213   | 3.669    |
| 0.25 | 0.92388   | 0.00     | 0.92523    | 1.186    | 0.92930   | 2.364    | 0.93607    | 3.529    | 0.94556   | 4.674    |
| 0.3  | 0.89101   | 0.00     | 0.89237    | 1.458    | 0.89662   | 2.907    | 0.90364    | 4.338    | 0.91347   | 5.743    |
| 0.35 | 0.85264   | 0.00     | 0.85407    | 1.754    | 0.85851   | 3.495    | 0.86583    | 5.213    | 0.87609   | 6.807    |
| 0.4  | 0.80902   | 0.00     | 0.81053    | 2.079    | 0.81520   | 4.142    | 0.82291    | 6.174    | 0.83370   | 8.161    |
| 0.45 | 0.76041   | 0.00     | 0.76202    | 2.443    | 0.76698   | 4.866    | 0.77527    | 7.247    | 0.78661   | 9.569    |
| 0.5  | 0.70711   | 0.00     | 0.70803    | 2.860    | 0.71417   | 5.692    | 0.72206    | 8.468    | 0.73521   | 11.165   |
| 0.55 | 0.64945   | 0.00     | 0.65135    | 3.348    | 0.65713   | 6.656    | 0.66667    | 9.889    | 0.67994   | 13.012   |
| 0.6  | 0.58779   | 0.00     | 0.58989    | 3.934    | 0.59626   | 7.811    | 0.60676    | 11.581   | 0.62131   | 15.198   |
| 0.65 | 0.52250   | 0.00     | 0.52487    | 4.661    | 0.53201   | 9.238    | 0.54376    | 13.656   | 0.55995   | 17.853   |
| 0.7  | 0.45399   | 0.00     | 0.45672    | 5.600    | 0.46491   | 11.068   | 0.47831    | 16.289   | 0.49663   | 21.175   |
| 0.75 | 0.38268   | 0.00     | 0.38594    | 6.877    | 0.39558   | 13.529   | 0.41124    | 19.771   | 0.43242   | 25.478   |
| 0.8  | 0.30902   | 0.00     | 0.31304    | 8.741    | 0.32485   | 17.053   | 0.34375    | 24.618   | 0.36882   | 31.277   |
| 0.85 | 0.23345   | 0.00     | 0.23874    | 11.755   | 0.25403   | 22.545   | 0.27779    | 31.805   | 0.30827   | 39.424   |
| 0.9  | 0.15643   | 0.00     | 0.16424    | 17.507   | 0.18576   | 32.181   | 0.21712    | 43.229   | 0.25497   | 51.254   |
| 0.95 | 0.07846   | 0.00     | 0.09305    | 32.407   | 0.12724   | 51.700   | 0.16978    | 62.139   | 0.21608   | 68.261   |
| 1.0  | 0.00      | 180      | 0.05002    | 90.000   | 0.10017   | 90.000   | 0.15056    | 90.000   | 0.20134   | 90.000   |
| 1.05 | 0.07846   | 180      | 0.09305    | 147.593  | 0.12724   | 128.300  | 0.16978    | 117.861  | 0.21608   | 111.739  |
| 1.1  | 0.15643   | 180      | 0.16424    | 162.493  | 0.18576   | 147.819  | 0.21712    | 136.771  | 0.25497   | 128.746  |
| 1.15 | 0.23345   | 180      | 0.23874    | 168.245  | 0.25403   | 157.455  | 0.27779    | 148.195  | 0.30827   | 140.576  |
| 1.2  | 0.30902   | 180      | 0.31304    | 171.259  | 0.32485   | 162.947  | 0.34375    | 155.382  | 0.36882   | 148.723  |
| 1.25 | 0.38268   | 180      | 0.38594    | 173.123  | 0.39558   | 166.471  | 0.41124    | 160.229  | 0.43242   | 154.522  |
| 1.3  | 0.45399   | 180      | 0.45672    | 174.400  | 0.46491   | 168.032  | 0.47831    | 163.711  | 0.49663   | 158.825  |
| 1.35 | 0.52250   | 180      | 0.52487    | 175.339  | 0.53201   | 170.762  | 0.54376    | 166.344  | 0.55995   | 162.147  |
| 1.4  | 0.58779   | 180      | 0.58989    | 176.066  | 0.59626   | 172.189  | 0.60676    | 168.419  | 0.62131   | 164.802  |
| 1.45 | 0.64945   | 180      | 0.65135    | 176.652  | 0.65713   | 173.344  | 0.66667    | 170.111  | 0.67994   | 166.988  |
| 1.5  | 0.70711   | 180      | 0.70803    | 177.139  | 0.71417   | 174.308  | 0.72206    | 171.532  | 0.73521   | 168.835  |
| 1.55 | 0.76041   | 180      | 0.76202    | 177.557  | 0.76698   | 175.134  | 0.77527    | 172.753  | 0.78661   | 170.431  |
| 1.6  | 0.80902   | 180      | 0.81053    | 177.921  | 0.81520   | 175.858  | 0.82291    | 173.826  | 0.83370   | 171.839  |
| 1.65 | 0.85264   | 180      | 0.85407    | 178.246  | 0.85851   | 176.505  | 0.86583    | 174.787  | 0.87609   | 173.103  |
| 1.7  | 0.89101   | 180      | 0.89237    | 178.542  | 0.89662   | 177.093  | 0.90304    | 175.662  | 0.91347   | 174.257  |
| 1.75 | 0.92388   | 180      | 0.92523    | 178.814  | 0.92930   | 177.636  | 0.93607    | 176.471  | 0.94556   | 175.326  |
| 1.8  | 0.95106   | 180      | 0.95237    | 179.070  | 0.95632   | 178.145  | 0.96290    | 177.231  | 0.97213   | 176.331  |
| 1.85 | 0.97237   | 180      | 0.97366    | 179.313  | 0.97752   | 178.629  | 0.98396    | 177.953  | 0.99300   | 177.287  |
| 1.9  | 0.98769   | 180      | 0.98895    | 179.547  | 0.99275   | 179.095  | 0.99909    | 178.649  | 1.00800   | 178.209  |
| 1.95 | 0.99692   | 180      | 0.99817    | 179.775  | 1.00194   | 179.550  | 1.00822    | 179.329  | 1.01704   | 179.110  |
| 2.0  | 1.00000   | 180      | 1.00125    | 180.000  | 1.00500   | 180.000  | 1.01127    | 180.000  | 1.02007   | 180.000  |

Example.  $\cosh(0.10 + i 0.55) = 0.65713 / 6^{\circ}.656 = 0.65713 / 6^{\circ}.39'.22''$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 0.25$ |               | $x = 0.3$ |               | $x = 0.35$ |               | $x = 0.4$ |               | $x = 0.45$ |               |
|------|------------|---------------|-----------|---------------|------------|---------------|-----------|---------------|------------|---------------|
|      | $r$        | $\gamma$<br>° | $r$       | $\gamma$<br>° | $r$        | $\gamma$<br>° | $r$       | $\gamma$<br>° | $r$        | $\gamma$<br>° |
| 0    | 1.03141    | 0.000         | 1.04534   | 0.000         | 1.06188    | 0.000         | 1.08107   | 0.000         | 1.10297    | 0.000         |
| 0.05 | 1.02843    | 1.104         | 1.04239   | 1.313         | 1.05898    | 1.516         | 1.07822   | 1.713         | 1.10017    | 1.902         |
| 0.1  | 1.01949    | 2.221         | 1.03357   | 2.642         | 1.05029    | 3.050         | 1.06970   | 3.444         | 1.09182    | 3.823         |
| 0.15 | 1.00464    | 3.365         | 1.01894   | 4.001         | 1.03590    | 4.617         | 1.05557   | 5.212         | 1.07798    | 5.784         |
| 0.2  | 0.98403    | 4.550         | 0.99862   | 5.407         | 1.01592    | 6.238         | 1.03597   | 7.038         | 1.05880    | 7.806         |
| 0.25 | 0.95779    | 5.793         | 0.97277   | 6.881         | 0.99052    | 7.932         | 1.01107   | 8.944         | 1.03446    | 9.913         |
| 0.3  | 0.92612    | 7.113         | 0.94161   | 8.443         | 0.95994    | 9.726         | 0.98113   | 10.957        | 1.00521    | 12.132        |
| 0.35 | 0.88927    | 8.536         | 0.90539   | 10.122        | 0.92444    | 11.647        | 0.94642   | 13.107        | 0.97136    | 14.496        |
| 0.4  | 0.84754    | 10.090        | 0.86443   | 11.951        | 0.88436    | 13.733        | 0.90732   | 15.432        | 0.93330    | 17.042        |
| 0.45 | 0.80127    | 11.815        | 0.81912   | 13.972        | 0.84012    | 16.029        | 0.86426   | 17.979        | 0.89149    | 19.816        |
| 0.5  | 0.75088    | 13.762        | 0.76989   | 16.241        | 0.79220    | 18.592        | 0.81775   | 20.804        | 0.84469    | 22.875        |
| 0.55 | 0.69685    | 16.001        | 0.71730   | 18.834        | 0.74119    | 21.497        | 0.76844   | 23.982        | 0.79895    | 26.288        |
| 0.6  | 0.63977    | 18.629        | 0.66199   | 21.849        | 0.68781    | 24.843        | 0.71708   | 27.607        | 0.74969    | 30.144        |
| 0.65 | 0.58036    | 21.785        | 0.60476   | 25.425        | 0.63292    | 28.763        | 0.66462   | 31.800        | 0.69968    | 34.546        |
| 0.7  | 0.51954    | 25.673        | 0.54666   | 29.758        | 0.57766    | 33.157        | 0.61223   | 36.712        | 0.65012    | 39.626        |
| 0.75 | 0.45854    | 30.595        | 0.48906   | 35.118        | 0.52348    | 39.079        | 0.56140   | 42.529        | 0.60249    | 45.527        |
| 0.8  | 0.39913    | 37.008        | 0.43385   | 41.878        | 0.47231    | 45.992        | 0.51401   | 49.464        | 0.55860    | 52.399        |
| 0.85 | 0.34396    | 45.057        | 0.38370   | 50.507        | 0.42671    | 54.484        | 0.47246   | 57.712        | 0.52062    | 60.358        |
| 0.9  | 0.29782    | 57.110        | 0.34235   | 61.467        | 0.38994    | 64.786        | 0.43953   | 67.371        | 0.49093    | 69.424        |
| 0.95 | 0.26452    | 72.186        | 0.31447   | 74.882        | 0.36571    | 76.831        | 0.41818   | 78.297        | 0.47191    | 79.433        |
| 1.0  | 0.25261    | 90.000        | 0.30452   | 90.000        | 0.35719    | 90.000        | 0.41075   | 90.000        | 0.46534    | 90.000        |
| 1.05 | 0.26452    | 107.814       | 0.31447   | 105.118       | 0.36571    | 103.169       | 0.41818   | 101.703       | 0.47191    | 100.567       |
| 1.1  | 0.29782    | 122.890       | 0.34235   | 118.533       | 0.38994    | 115.214       | 0.43953   | 112.629       | 0.49093    | 110.576       |
| 1.15 | 0.34396    | 134.943       | 0.38370   | 129.493       | 0.42671    | 125.516       | 0.47246   | 122.288       | 0.52062    | 119.642       |
| 1.2  | 0.39913    | 142.992       | 0.43385   | 138.122       | 0.47231    | 134.008       | 0.51401   | 130.536       | 0.55860    | 127.601       |
| 1.25 | 0.45854    | 149.405       | 0.48906   | 144.882       | 0.52348    | 140.921       | 0.56140   | 137.471       | 0.60249    | 134.473       |
| 1.3  | 0.51954    | 154.327       | 0.54666   | 150.242       | 0.57766    | 146.843       | 0.61223   | 143.288       | 0.65012    | 140.374       |
| 1.35 | 0.58036    | 158.215       | 0.60476   | 154.575       | 0.63292    | 151.237       | 0.66462   | 148.200       | 0.69968    | 145.454       |
| 1.4  | 0.63977    | 161.371       | 0.66199   | 158.151       | 0.68781    | 155.157       | 0.71708   | 152.393       | 0.74969    | 149.856       |
| 1.45 | 0.69685    | 163.999       | 0.71730   | 161.166       | 0.74119    | 158.503       | 0.76844   | 156.018       | 0.79895    | 153.712       |
| 1.5  | 0.75088    | 166.238       | 0.76989   | 163.759       | 0.79220    | 161.408       | 0.81775   | 159.196       | 0.84469    | 157.125       |
| 1.55 | 0.80127    | 168.185       | 0.81912   | 166.028       | 0.84012    | 163.971       | 0.86426   | 162.021       | 0.89149    | 160.184       |
| 1.6  | 0.84754    | 169.910       | 0.86443   | 168.049       | 0.88436    | 166.267       | 0.90732   | 164.568       | 0.93330    | 162.958       |
| 1.65 | 0.88927    | 171.464       | 0.90539   | 169.878       | 0.92444    | 168.353       | 0.94642   | 166.893       | 0.97136    | 165.504       |
| 1.7  | 0.92612    | 172.887       | 0.94161   | 171.557       | 0.95994    | 170.274       | 0.98113   | 169.043       | 1.00521    | 167.868       |
| 1.75 | 0.95779    | 174.207       | 0.97277   | 173.119       | 0.99052    | 172.068       | 1.01107   | 171.056       | 1.03446    | 170.087       |
| 1.8  | 0.98403    | 175.450       | 0.99862   | 174.593       | 1.01592    | 173.762       | 1.03597   | 172.962       | 1.05880    | 172.194       |
| 1.85 | 1.00464    | 176.635       | 1.01894   | 175.999       | 1.03590    | 175.383       | 1.05557   | 174.788       | 1.07798    | 174.216       |
| 1.9  | 1.01949    | 177.779       | 1.03357   | 177.358       | 1.05029    | 176.950       | 1.06970   | 176.556       | 1.09182    | 176.177       |
| 1.95 | 1.02843    | 178.896       | 1.04239   | 178.687       | 1.05898    | 178.484       | 1.07822   | 178.287       | 1.10017    | 178.098       |
| 2.0  | 1.03141    | 180.000       | 1.04534   | 180.000       | 1.06188    | 180.000       | 1.08107   | 180.000       | 1.10297    | 180.000       |

Example.  $\cosh(0.40 + i0.5) = 0.81775 / 20^{\circ}.804 = 0.81775 / 20^{\circ}.48'.14''$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 0.5$ |          | $x = 0.55$ |          | $x = 0.6$ |          | $x = 0.65$ |          | $x = 0.7$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
|      | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 1.12763   | 0.000    | 1.15510    | 0.000    | 1.18547   | 0.000    | 1.21879    | 0.000    | 1.25517   | 0.000    |
| 0.05 | 1.12489   | 2.083    | 1.15244    | 2.256    | 1.18287   | 2.420    | 1.21626    | 2.576    | 1.25271   | 2.723    |
| 0.1  | 1.11672   | 4.186    | 1.14446    | 4.533    | 1.17510   | 4.879    | 1.20871    | 5.174    | 1.24538   | 5.468    |
| 0.15 | 1.10320   | 6.331    | 1.13127    | 6.852    | 1.16226   | 7.347    | 1.19623    | 7.815    | 1.23327   | 8.256    |
| 0.2  | 1.08446   | 8.539    | 1.11300    | 9.237    | 1.14448   | 9.898    | 1.17892    | 10.523   | 1.21653   | 11.110   |
| 0.25 | 1.06070   | 10.836   | 1.08987    | 11.713   | 1.12200   | 12.541   | 1.15715    | 13.322   | 1.19541   | 14.054   |
| 0.3  | 1.03220   | 13.249   | 1.06214    | 14.307   | 1.09509   | 15.304   | 1.13108    | 16.240   | 1.17019   | 17.116   |
| 0.35 | 0.99927   | 15.811   | 1.03004    | 17.052   | 1.06411   | 18.216   | 1.10111    | 19.306   | 1.14125   | 20.323   |
| 0.4  | 0.96232   | 18.559   | 0.99437    | 19.984   | 1.02948   | 21.315   | 1.06769    | 22.555   | 1.10904   | 23.706   |
| 0.45 | 0.92182   | 21.538   | 0.95524    | 23.146   | 0.99174   | 24.640   | 1.03134    | 26.024   | 1.07409   | 27.302   |
| 0.5  | 0.87837   | 24.803   | 0.91338    | 26.589   | 0.95149   | 28.238   | 0.99270    | 29.755   | 1.03704   | 31.148   |
| 0.55 | 0.83266   | 28.416   | 0.86951    | 30.372   | 0.90946   | 32.162   | 0.95249    | 33.796   | 0.99861   | 35.284   |
| 0.6  | 0.78552   | 32.458   | 0.82447    | 34.563   | 0.86650   | 36.471   | 0.91156    | 38.197   | 0.95966   | 39.755   |
| 0.65 | 0.73793   | 37.020   | 0.77927    | 39.241   | 0.82361   | 41.231   | 0.87090    | 43.011   | 0.92112   | 44.603   |
| 0.7  | 0.69112   | 42.207   | 0.73510    | 44.489   | 0.78194   | 46.506   | 0.83160    | 48.290   | 0.88406   | 49.867   |
| 0.75 | 0.64652   | 48.129   | 0.69333    | 50.390   | 0.74282   | 52.358   | 0.79492    | 54.074   | 0.84965   | 55.574   |
| 0.8  | 0.60583   | 54.889   | 0.65555    | 57.010   | 0.70769   | 58.826   | 0.76220    | 60.387   | 0.81911   | 61.737   |
| 0.85 | 0.57100   | 62.547   | 0.62350    | 64.375   | 0.67810   | 65.914   | 0.73482    | 67.219   | 0.79369   | 68.335   |
| 0.9  | 0.54407   | 71.082   | 0.59894    | 72.441   | 0.65559   | 73.568   | 0.71409    | 74.514   | 0.77455   | 75.315   |
| 0.95 | 0.52697   | 80.335   | 0.58345    | 81.064   | 0.64147   | 81.663   | 0.70115    | 82.161   | 0.76263   | 82.581   |
| 1.0  | 0.52110   | 90.000   | 0.57815    | 90.000   | 0.63665   | 90.000   | 0.69675    | 90.000   | 0.75858   | 90.000   |
| 1.05 | 0.52697   | 99.665   | 0.58345    | 98.936   | 0.64147   | 98.337   | 0.70115    | 97.839   | 0.76263   | 97.419   |
| 1.1  | 0.54407   | 108.918  | 0.59894    | 107.559  | 0.65559   | 106.432  | 0.71409    | 105.486  | 0.77455   | 104.685  |
| 1.15 | 0.57100   | 117.453  | 0.62350    | 115.625  | 0.67810   | 114.086  | 0.73482    | 112.781  | 0.79369   | 111.665  |
| 1.2  | 0.60583   | 125.111  | 0.65555    | 122.990  | 0.70769   | 121.174  | 0.76220    | 119.613  | 0.81911   | 118.263  |
| 1.25 | 0.64652   | 131.871  | 0.69333    | 129.610  | 0.74282   | 127.642  | 0.79492    | 125.926  | 0.84965   | 124.426  |
| 1.3  | 0.69112   | 137.793  | 0.73510    | 135.511  | 0.78194   | 133.494  | 0.83160    | 131.710  | 0.88406   | 130.133  |
| 1.35 | 0.73793   | 142.980  | 0.77927    | 140.759  | 0.82361   | 138.769  | 0.87090    | 136.989  | 0.92112   | 135.397  |
| 1.4  | 0.78552   | 147.542  | 0.82447    | 145.437  | 0.86650   | 143.529  | 0.91156    | 141.803  | 0.95966   | 140.245  |
| 1.45 | 0.83266   | 151.584  | 0.86951    | 149.628  | 0.90946   | 147.838  | 0.95249    | 146.204  | 0.99861   | 144.716  |
| 1.50 | 0.87837   | 155.198  | 0.91338    | 153.411  | 0.95149   | 151.762  | 0.99270    | 150.245  | 1.03704   | 148.853  |
| 1.55 | 0.92182   | 158.462  | 0.95524    | 156.854  | 0.99174   | 155.360  | 1.03134    | 153.976  | 1.07409   | 152.698  |
| 1.60 | 0.96232   | 161.441  | 0.99437    | 160.016  | 1.02948   | 158.685  | 1.06769    | 157.445  | 1.10904   | 156.294  |
| 1.65 | 0.99927   | 164.189  | 1.03004    | 162.948  | 1.06411   | 161.784  | 1.10111    | 160.694  | 1.14125   | 159.677  |
| 1.70 | 1.03220   | 166.751  | 1.06214    | 165.693  | 1.09509   | 164.696  | 1.13108    | 163.760  | 1.17019   | 162.884  |
| 1.75 | 1.06070   | 169.164  | 1.08987    | 168.287  | 1.12200   | 167.459  | 1.15715    | 166.678  | 1.19541   | 165.946  |
| 1.8  | 1.08446   | 171.461  | 1.11300    | 170.763  | 1.14448   | 170.102  | 1.17892    | 169.477  | 1.21653   | 168.890  |
| 1.85 | 1.10320   | 173.669  | 1.13127    | 173.148  | 1.16226   | 172.653  | 1.19623    | 172.185  | 1.23327   | 171.744  |
| 1.9  | 1.11672   | 175.814  | 1.14446    | 175.467  | 1.17510   | 175.121  | 1.20871    | 174.826  | 1.24538   | 174.532  |
| 1.95 | 1.12489   | 177.917  | 1.15244    | 177.744  | 1.18287   | 177.580  | 1.21626    | 177.424  | 1.25271   | 177.277  |
| 2.0  | 1.12763   | 180.000  | 1.15510    | 180.000  | 1.18547   | 180.000  | 1.21879    | 180.000  | 1.25517   | 180.000  |

Example.  $\cosh(0.65 + i 1.0) = 0.69675 / 90^\circ$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 0.75$ |          | $x = 0.8$ |          | $x = 0.85$ |          | $x = 0.9$ |          | $x = 0.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
|      | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 1.29468    | 0.000    | 1.33743   | 0.000    | 1.38353    | 0.000    | 1.43309   | 0.000    | 1.48623    | 0.000    |
| 0.05 | 1.29230    | 2.862    | 1.33513   | 2.992    | 1.38130    | 3.113    | 1.43094   | 3.227    | 1.48415    | 3.332    |
| 0.1  | 1.28520    | 5.745    | 1.32825   | 6.004    | 1.37466    | 6.246    | 1.42452   | 6.473    | 1.47797    | 6.684    |
| 0.15 | 1.27346    | 8.670    | 1.31691   | 9.058    | 1.36369    | 9.420    | 1.41395   | 9.758    | 1.46778    | 10.071   |
| 0.2  | 1.25726    | 11.661   | 1.30124   | 12.175   | 1.34858    | 12.656   | 1.39937   | 13.102   | 1.45371    | 13.516   |
| 0.25 | 1.23689    | 14.740   | 1.28152   | 15.379   | 1.32955    | 15.974   | 1.38105   | 16.526   | 1.43611    | 17.037   |
| 0.3  | 1.21248    | 17.933   | 1.25802   | 18.693   | 1.30693    | 19.398   | 1.35927   | 20.051   | 1.41519    | 20.653   |
| 0.35 | 1.18457    | 21.267   | 1.23115   | 22.143   | 1.28107    | 22.952   | 1.33444   | 23.699   | 1.39136    | 24.387   |
| 0.4  | 1.15356    | 24.772   | 1.20135   | 25.755   | 1.25246    | 26.601   | 1.30700   | 27.493   | 1.36505    | 28.257   |
| 0.45 | 1.12001    | 28.478   | 1.16917   | 29.559   | 1.22163    | 30.550   | 1.27748   | 31.457   | 1.33682    | 32.286   |
| 0.5  | 1.08453    | 32.442   | 1.13522   | 33.586   | 1.18918    | 34.647   | 1.24649   | 35.614   | 1.30723    | 36.493   |
| 0.55 | 1.04785    | 36.637   | 1.10024   | 37.865   | 1.15583    | 38.978   | 1.21471   | 39.986   | 1.27697    | 40.898   |
| 0.6  | 1.01079    | 41.160   | 1.06500   | 42.426   | 1.12234    | 43.567   | 1.18289   | 44.593   | 1.24674    | 45.517   |
| 0.65 | 0.97427    | 46.026   | 1.03041   | 47.298   | 1.08957    | 48.435   | 1.15184   | 49.453   | 1.21732    | 50.363   |
| 0.7  | 0.93932    | 51.263   | 0.99742   | 52.500   | 1.05843    | 53.599   | 1.12243   | 54.574   | 1.19227    | 55.443   |
| 0.75 | 0.90700    | 56.889   | 0.96705   | 58.045   | 1.02985    | 59.062   | 1.09553   | 59.961   | 1.16418    | 60.755   |
| 0.8  | 0.87846    | 62.907   | 0.94033   | 63.927   | 1.00481    | 64.819   | 1.07202   | 65.601   | 1.14209    | 66.288   |
| 0.85 | 0.85481    | 69.294   | 0.91828   | 70.123   | 0.98420    | 70.842   | 1.05273   | 71.471   | 1.12400    | 72.020   |
| 0.9  | 0.83706    | 75.998   | 0.90178   | 76.585   | 0.96883    | 77.091   | 1.03837   | 77.532   | 1.11056    | 77.915   |
| 0.95 | 0.82605    | 82.936   | 0.89157   | 83.241   | 0.95933    | 83.503   | 1.02951   | 83.730   | 1.10227    | 83.927   |
| 1.0  | 0.82232    | 90.000   | 0.88811   | 90.000   | 0.95612    | 90.000   | 1.02652   | 90.000   | 1.09948    | 90.000   |
| 1.05 | 0.82605    | 97.064   | 0.89157   | 96.759   | 0.95933    | 96.497   | 1.02951   | 96.270   | 1.10227    | 96.073   |
| 1.1  | 0.83706    | 104.002  | 0.90178   | 103.415  | 0.96883    | 102.909  | 1.03837   | 102.468  | 1.11056    | 102.085  |
| 1.15 | 0.85481    | 110.706  | 0.91828   | 109.877  | 0.98420    | 109.158  | 1.05273   | 108.529  | 1.12400    | 107.980  |
| 1.2  | 0.87846    | 117.093  | 0.94033   | 116.073  | 1.00481    | 115.181  | 1.07202   | 114.399  | 1.14209    | 113.712  |
| 1.25 | 0.90700    | 123.111  | 0.96705   | 121.955  | 1.02985    | 120.938  | 1.09553   | 120.039  | 1.16418    | 119.245  |
| 1.3  | 0.93932    | 128.737  | 0.99742   | 127.500  | 1.05843    | 126.401  | 1.12243   | 125.426  | 1.19227    | 124.557  |
| 1.35 | 0.97427    | 133.974  | 1.03041   | 132.702  | 1.08957    | 131.565  | 1.15184   | 130.547  | 1.21732    | 129.637  |
| 1.4  | 1.01079    | 138.840  | 1.06500   | 137.574  | 1.12234    | 136.433  | 1.18289   | 135.407  | 1.24674    | 134.483  |
| 1.45 | 1.04785    | 143.363  | 1.10024   | 142.135  | 1.15583    | 141.022  | 1.21471   | 140.014  | 1.27697    | 139.102  |
| 1.5  | 1.08453    | 147.578  | 1.13522   | 146.414  | 1.18918    | 145.353  | 1.24649   | 144.386  | 1.30723    | 143.507  |
| 1.55 | 1.12001    | 151.522  | 1.16917   | 150.441  | 1.22163    | 149.450  | 1.27748   | 148.543  | 1.33682    | 147.714  |
| 1.6  | 1.15356    | 155.228  | 1.20135   | 154.245  | 1.25246    | 153.399  | 1.30700   | 152.507  | 1.36505    | 151.743  |
| 1.65 | 1.18457    | 158.733  | 1.23115   | 157.857  | 1.28107    | 157.048  | 1.33444   | 156.301  | 1.39136    | 155.613  |
| 1.7  | 1.21248    | 162.067  | 1.25802   | 161.307  | 1.30693    | 160.602  | 1.35927   | 159.949  | 1.41519    | 159.347  |
| 1.75 | 1.23689    | 165.260  | 1.28152   | 164.621  | 1.32955    | 164.026  | 1.38105   | 163.474  | 1.43611    | 162.963  |
| 1.8  | 1.25726    | 168.339  | 1.30124   | 167.825  | 1.34858    | 167.344  | 1.39937   | 166.898  | 1.45371    | 166.484  |
| 1.85 | 1.27346    | 171.330  | 1.31691   | 170.942  | 1.36369    | 170.580  | 1.41395   | 170.242  | 1.46778    | 169.929  |
| 1.9  | 1.28520    | 174.255  | 1.32825   | 173.996  | 1.37466    | 173.754  | 1.42452   | 173.527  | 1.47797    | 173.316  |
| 1.95 | 1.29230    | 177.138  | 1.33513   | 177.008  | 1.38130    | 176.887  | 1.43094   | 176.773  | 1.48415    | 176.668  |
| 2.0  | 1.29468    | 180.000  | 1.33743   | 180.000  | 1.38353    | 180.000  | 1.43309   | 180.000  | 1.48623    | 180.000  |

Example.  $\cosh(0.90 + i0.5) = 1.24649 / 35^\circ.614 = 1.24649 / 35^\circ.36'.50''$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 1.0$ |          | $x = 1.05$ |          | $x = 1.1$ |          | $x = 1.15$ |          | $x = 1.2$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
|      |           | °        |            | °        |           | °        |            | °        |           | °        |
| 0    | 1.54308   | 0.000    | 1.60379    | 0.000    | 1.66852   | 0.000    | 1.73741    | 0.000    | 1.81066   | 0.000    |
| 0.05 | 1.54108   | 3.276    | 1.60187    | 3.521    | 1.66667   | 3.605    | 1.73564    | 3.683    | 1.80895   | 3.754    |
| 0.1  | 1.53513   | 6.878    | 1.59614    | 7.059    | 1.66117   | 7.226    | 1.73036    | 7.380    | 1.80389   | 7.522    |
| 0.15 | 1.52532   | 10.362   | 1.58671    | 10.631   | 1.65211   | 10.879   | 1.72166    | 11.107   | 1.79555   | 11.318   |
| 0.2  | 1.51182   | 13.899   | 1.57374    | 14.253   | 1.63965   | 14.579   | 1.70971    | 14.880   | 1.78409   | 15.156   |
| 0.25 | 1.49487   | 17.509   | 1.55747    | 17.944   | 1.62404   | 18.344   | 1.69472    | 18.713   | 1.76975   | 19.050   |
| 0.3  | 1.47479   | 21.209   | 1.53820    | 21.720   | 1.60558   | 22.189   | 1.67705    | 22.620   | 1.75282   | 23.014   |
| 0.35 | 1.45193   | 25.019   | 1.51630    | 25.599   | 1.58460   | 26.130   | 1.65699    | 26.616   | 1.73363   | 27.061   |
| 0.4  | 1.42675   | 28.957   | 1.49220    | 29.597   | 1.56156   | 30.182   | 1.63497    | 30.716   | 1.71260   | 31.203   |
| 0.45 | 1.39976   | 33.043   | 1.46641    | 33.732   | 1.53694   | 34.360   | 1.61147    | 34.931   | 1.69018   | 35.451   |
| 0.5  | 1.37153   | 37.293   | 1.43950    | 38.019   | 1.51128   | 38.677   | 1.58701    | 39.275   | 1.66688   | 39.816   |
| 0.55 | 1.34272   | 41.724   | 1.41207    | 42.470   | 1.48517   | 43.145   | 1.56217    | 43.755   | 1.64325   | 44.397   |
| 0.6  | 1.31400   | 46.349   | 1.38479    | 47.098   | 1.45926   | 47.773   | 1.53756    | 48.380   | 1.61987   | 48.927   |
| 0.65 | 1.28612   | 51.179   | 1.35837    | 51.910   | 1.43421   | 52.565   | 1.51381    | 53.153   | 1.59733   | 53.681   |
| 0.7  | 1.25984   | 56.216   | 1.33351    | 56.907   | 1.41070   | 57.523   | 1.49155    | 58.074   | 1.57626   | 58.567   |
| 0.75 | 1.23594   | 61.459   | 1.31095    | 62.085   | 1.38939   | 62.641   | 1.47141    | 63.136   | 1.55721   | 63.579   |
| 0.8  | 1.21515   | 66.895   | 1.29137    | 67.432   | 1.37093   | 67.908   | 1.45399    | 68.330   | 1.54077   | 68.706   |
| 0.85 | 1.19816   | 72.504   | 1.27540    | 72.929   | 1.35590   | 73.306   | 1.43983    | 73.639   | 1.52471   | 73.935   |
| 0.9  | 1.18552   | 78.252   | 1.26358    | 78.547   | 1.34478   | 78.808   | 1.42936    | 79.039   | 1.51755   | 79.243   |
| 0.95 | 1.17782   | 84.100   | 1.25631    | 84.252   | 1.33795   | 84.385   | 1.42294    | 84.503   | 1.51150   | 84.607   |
| 1.0  | 1.17520   | 90.000   | 1.25386    | 90.000   | 1.33565   | 90.000   | 1.42078    | 90.000   | 1.50946   | 90.000   |
| 1.05 | 1.17782   | 95.900   | 1.25631    | 95.748   | 1.33795   | 95.615   | 1.42294    | 95.497   | 1.51150   | 95.393   |
| 1.1  | 1.18552   | 101.748  | 1.26358    | 101.453  | 1.34478   | 101.192  | 1.42936    | 100.961  | 1.51755   | 100.757  |
| 1.15 | 1.19816   | 107.496  | 1.27540    | 107.071  | 1.35590   | 106.694  | 1.43983    | 106.361  | 1.52741   | 106.065  |
| 1.2  | 1.21515   | 113.105  | 1.29137    | 112.568  | 1.37093   | 112.092  | 1.45399    | 111.670  | 1.54077   | 111.294  |
| 1.25 | 1.23594   | 118.541  | 1.31095    | 117.915  | 1.38939   | 117.359  | 1.47141    | 116.864  | 1.55721   | 116.421  |
| 1.3  | 1.25984   | 123.784  | 1.33351    | 123.093  | 1.41070   | 122.477  | 1.49155    | 121.926  | 1.57626   | 121.433  |
| 1.35 | 1.28612   | 128.821  | 1.35837    | 128.090  | 1.43421   | 127.435  | 1.51381    | 126.847  | 1.59733   | 126.319  |
| 1.4  | 1.31400   | 133.651  | 1.38479    | 132.902  | 1.45926   | 132.227  | 1.53756    | 131.620  | 1.61987   | 131.073  |
| 1.45 | 1.34272   | 138.276  | 1.41207    | 137.530  | 1.48517   | 136.855  | 1.56217    | 136.245  | 1.64325   | 135.693  |
| 1.5  | 1.37153   | 142.707  | 1.43950    | 141.981  | 1.51128   | 141.323  | 1.58701    | 140.725  | 1.66688   | 140.184  |
| 1.55 | 1.39976   | 146.957  | 1.46641    | 146.268  | 1.53694   | 145.640  | 1.61147    | 145.069  | 1.69018   | 144.549  |
| 1.6  | 1.42675   | 151.043  | 1.49220    | 150.403  | 1.56156   | 149.818  | 1.63497    | 149.284  | 1.71260   | 148.797  |
| 1.65 | 1.45193   | 154.981  | 1.51630    | 154.401  | 1.58460   | 153.870  | 1.65699    | 153.384  | 1.73363   | 152.939  |
| 1.7  | 1.47479   | 158.791  | 1.53820    | 158.280  | 1.60558   | 157.811  | 1.67705    | 157.380  | 1.75282   | 156.986  |
| 1.75 | 1.49487   | 162.491  | 1.55747    | 162.056  | 1.62404   | 161.656  | 1.69472    | 161.287  | 1.76975   | 160.950  |
| 1.8  | 1.51182   | 166.101  | 1.57374    | 165.747  | 1.63965   | 165.421  | 1.70971    | 165.120  | 1.78409   | 164.844  |
| 1.85 | 1.52532   | 169.638  | 1.58671    | 169.369  | 1.65211   | 169.121  | 1.72166    | 168.893  | 1.79555   | 168.682  |
| 1.9  | 1.53513   | 173.122  | 1.59614    | 172.941  | 1.66117   | 172.774  | 1.73036    | 172.620  | 1.80389   | 172.478  |
| 1.95 | 1.54108   | 176.724  | 1.60187    | 176.479  | 1.66667   | 176.395  | 1.73564    | 176.317  | 1.80895   | 176.246  |
| 2.0  | 1.54308   | 180.000  | 1.60379    | 180.000  | 1.66852   | 180.000  | 1.73741    | 180.000  | 1.81066   | 180.000  |

Example.  $\cosh(1.20 + i0) = 1.81066 / \angle 0^\circ$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 1.25$ |          | $x = 1.3$ |          | $x = 1.35$ |          | $x = 1.4$ |          | $x = 1.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 1.88842    | 0.000    | 1.97091   | 0.000    | 2.05833    | 0.000    | 2.15090   | 0.000    | 2.24884    | 0.000    |
| 0.05 | 1.88679    | 3.820    | 1.96935   | 3.880    | 2.05684    | 3.935    | 2.14947   | 3.986    | 2.24747    | 4.032    |
| 0.1  | 1.88193    | 7.652    | 1.96470   | 7.772    | 2.05238    | 7.882    | 2.14520   | 7.982    | 2.24334    | 8.075    |
| 0.15 | 1.87394    | 11.511   | 1.95704   | 11.689   | 2.04505    | 11.851   | 2.13819   | 12.000   | 2.23669    | 12.136   |
| 0.2  | 1.86297    | 15.410   | 1.94654   | 15.642   | 2.03500    | 15.855   | 2.12858   | 16.049   | 2.22751    | 16.226   |
| 0.25 | 1.84924    | 19.360   | 1.93341   | 19.644   | 2.02245    | 19.903   | 2.11658   | 20.139   | 2.21604    | 20.355   |
| 0.3  | 1.83304    | 23.375   | 1.91792   | 23.705   | 2.00764    | 24.006   | 2.10244   | 24.281   | 2.20254    | 24.531   |
| 0.35 | 1.81470    | 27.467   | 1.90040   | 27.837   | 1.99091    | 28.175   | 2.08647   | 28.482   | 2.18731    | 28.762   |
| 0.4  | 1.79462    | 31.646   | 1.88123   | 32.050   | 1.97262    | 32.417   | 2.06903   | 32.751   | 2.17067    | 33.055   |
| 0.45 | 1.77323    | 35.923   | 1.86084   | 36.352   | 1.95319    | 36.742   | 2.05051   | 37.095   | 2.15302    | 37.416   |
| 0.5  | 1.75104    | 40.308   | 1.83970   | 40.752   | 1.93306    | 41.155   | 2.03135   | 41.520   | 2.13478    | 41.851   |
| 0.55 | 1.72856    | 44.805   | 1.81832   | 45.255   | 1.91273    | 45.662   | 2.01200   | 46.030   | 2.11637    | 46.362   |
| 0.6  | 1.70635    | 49.420   | 1.79722   | 49.865   | 1.89268    | 50.265   | 1.99295   | 50.627   | 2.09828    | 50.953   |
| 0.65 | 1.68497    | 54.155   | 1.77694   | 54.582   | 1.87343    | 54.965   | 1.97468   | 55.311   | 2.08093    | 55.621   |
| 0.7  | 1.66501    | 59.009   | 1.75802   | 59.405   | 1.85549    | 59.760   | 1.95767   | 60.079   | 2.06479    | 60.366   |
| 0.75 | 1.64699    | 63.974   | 1.74096   | 64.327   | 1.83934    | 64.644   | 1.94237   | 64.927   | 2.05030    | 65.182   |
| 0.8  | 1.63145    | 69.042   | 1.72627   | 69.341   | 1.82544    | 69.608   | 1.92921   | 69.847   | 2.03784    | 70.061   |
| 0.85 | 1.61884    | 74.197   | 1.71435   | 74.432   | 1.81417    | 74.641   | 1.91856   | 74.828   | 2.02775    | 74.995   |
| 0.9  | 1.60954    | 79.424   | 1.70557   | 79.585   | 1.80588    | 79.729   | 1.91072   | 79.857   | 2.02034    | 79.972   |
| 0.95 | 1.60384    | 84.699   | 1.70019   | 84.782   | 1.80080    | 84.855   | 1.90592   | 84.920   | 2.01580    | 84.979   |
| 1.0  | 1.60192    | 90.000   | 1.69838   | 90.000   | 1.79909    | 90.000   | 1.90430   | 90.000   | 2.01427    | 90.000   |
| 1.05 | 1.60384    | 95.301   | 1.70019   | 95.218   | 1.80080    | 95.145   | 1.90592   | 95.080   | 2.01580    | 95.021   |
| 1.1  | 1.60954    | 100.576  | 1.70557   | 100.415  | 1.80588    | 100.271  | 1.91072   | 100.143  | 2.02034    | 100.028  |
| 1.15 | 1.61884    | 105.803  | 1.71435   | 105.568  | 1.81417    | 105.359  | 1.91856   | 105.172  | 2.02775    | 105.005  |
| 1.2  | 1.63145    | 110.958  | 1.72627   | 110.659  | 1.82544    | 110.392  | 1.92921   | 110.153  | 2.03784    | 109.939  |
| 1.25 | 1.64699    | 116.026  | 1.74096   | 115.673  | 1.83934    | 115.356  | 1.94237   | 115.073  | 2.05030    | 114.818  |
| 1.3  | 1.66501    | 120.991  | 1.75802   | 120.595  | 1.85549    | 120.240  | 1.95767   | 119.921  | 2.06479    | 119.634  |
| 1.35 | 1.68497    | 125.845  | 1.77694   | 125.418  | 1.87343    | 125.035  | 1.97468   | 124.689  | 2.08093    | 124.379  |
| 1.4  | 1.70635    | 130.580  | 1.79722   | 130.135  | 1.89268    | 129.735  | 1.99295   | 129.373  | 2.09828    | 129.047  |
| 1.45 | 1.72856    | 135.195  | 1.81832   | 134.745  | 1.91273    | 134.338  | 2.01200   | 133.970  | 2.11637    | 133.638  |
| 1.5  | 1.75104    | 139.693  | 1.83970   | 139.248  | 1.93306    | 138.845  | 2.03135   | 138.480  | 2.13478    | 138.150  |
| 1.55 | 1.77323    | 144.077  | 1.86084   | 143.648  | 1.95319    | 143.258  | 2.05051   | 142.905  | 2.15302    | 142.584  |
| 1.6  | 1.79462    | 148.354  | 1.88123   | 147.950  | 1.97262    | 147.583  | 2.06903   | 147.249  | 2.17067    | 146.945  |
| 1.65 | 1.81470    | 152.533  | 1.90040   | 152.163  | 1.99091    | 151.825  | 2.08647   | 151.518  | 2.18731    | 151.238  |
| 1.7  | 1.83304    | 156.625  | 1.91792   | 156.295  | 2.00764    | 155.994  | 2.10244   | 155.719  | 2.20254    | 155.469  |
| 1.75 | 1.84924    | 160.640  | 1.93341   | 160.356  | 2.02245    | 160.097  | 2.11658   | 159.861  | 2.21604    | 159.645  |
| 1.8  | 1.86297    | 164.590  | 1.94654   | 164.358  | 2.03500    | 164.145  | 2.12858   | 163.951  | 2.22751    | 163.774  |
| 1.85 | 1.87394    | 168.489  | 1.95704   | 168.311  | 2.04505    | 168.149  | 2.13819   | 168.000  | 2.23669    | 167.864  |
| 1.9  | 1.88193    | 172.348  | 1.96470   | 172.228  | 2.05238    | 172.118  | 2.14520   | 172.018  | 2.24334    | 171.925  |
| 1.95 | 1.88679    | 176.180  | 1.96935   | 176.120  | 2.05684    | 176.065  | 2.14947   | 176.014  | 2.24747    | 175.968  |
| 2.0  | 1.88842    | 180.000  | 1.97091   | 180.000  | 2.05833    | 180.000  | 2.15090   | 180.000  | 2.24884    | 180.000  |

Example.  $\cosh(1.35 + i 1.30) = 1.85549 / 120^\circ.240 = 1.85549 / 120^\circ.14'.24''$ .



TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 1.5$ |          | $x = 1.55$ |          | $x = 1.6$ |          | $x = 1.65$ |          | $x = 1.7$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 2.35241   | 0.000    | 2.46186    | 0.000    | 2.57746   | 0.000    | 2.69952    | 0.000    | 2.82832   | 0.000    |
| 0.05 | 2.35110   | 4.075    | 2.46061    | 4.113    | 2.57627   | 4.149    | 2.69839    | 4.181    | 2.82723   | 4.210    |
| 0.1  | 2.34720   | 8.158    | 2.45688    | 8.235    | 2.57272   | 8.305    | 2.69499    | 8.369    | 2.82399   | 8.427    |
| 0.15 | 2.34080   | 12.260   | 2.45076    | 12.374   | 2.56687   | 12.477   | 2.68941    | 12.571   | 2.81867   | 12.657   |
| 0.2  | 2.33202   | 16.389   | 2.44238    | 16.536   | 2.55887   | 16.671   | 2.68178    | 16.794   | 2.81138   | 16.906   |
| 0.25 | 2.32107   | 20.552   | 2.43193    | 20.732   | 2.54890   | 20.895   | 2.67226    | 21.044   | 2.80230   | 21.179   |
| 0.3  | 2.30819   | 24.759   | 2.41976    | 24.967   | 2.53717   | 25.155   | 2.66108    | 25.327   | 2.79164   | 25.483   |
| 0.35 | 2.29365   | 29.016   | 2.40577    | 29.248   | 2.52395   | 29.458   | 2.64847    | 29.649   | 2.77963   | 29.822   |
| 0.4  | 2.27779   | 33.330   | 2.39066    | 33.580   | 2.50955   | 33.808   | 2.63476    | 34.014   | 2.76657   | 34.201   |
| 0.45 | 2.26098   | 37.706   | 2.37465    | 37.970   | 2.49430   | 38.209   | 2.62024    | 38.426   | 2.75274   | 38.622   |
| 0.5  | 2.24362   | 42.150   | 2.35812    | 42.421   | 2.47857   | 42.666   | 2.60527    | 42.887   | 2.73850   | 43.089   |
| 0.55 | 2.22612   | 46.663   | 2.34148    | 46.934   | 2.46274   | 47.180   | 2.59021    | 47.401   | 2.72418   | 47.602   |
| 0.6  | 2.20892   | 51.247   | 2.32513    | 51.512   | 2.44721   | 51.752   | 2.57544    | 51.968   | 2.71014   | 52.163   |
| 0.65 | 2.19245   | 55.901   | 2.30949    | 56.154   | 2.43235   | 56.381   | 2.56133    | 56.585   | 2.69673   | 56.771   |
| 0.7  | 2.17714   | 60.624   | 2.29496    | 60.856   | 2.41856   | 61.065   | 2.54824    | 61.253   | 2.68430   | 61.422   |
| 0.75 | 2.16340   | 65.410   | 2.28193    | 65.615   | 2.40619   | 65.800   | 2.53650    | 65.966   | 2.67316   | 66.116   |
| 0.8  | 2.15159   | 70.254   | 2.27074    | 70.426   | 2.39558   | 70.581   | 2.52644    | 70.720   | 2.66361   | 70.845   |
| 0.85 | 2.14204   | 75.145   | 2.26169    | 75.279   | 2.38701   | 75.400   | 2.51831    | 75.508   | 2.65591   | 75.605   |
| 0.9  | 2.13502   | 80.075   | 2.25504    | 80.167   | 2.38071   | 80.249   | 2.51234    | 80.323   | 2.65025   | 80.390   |
| 0.95 | 2.13073   | 85.031   | 2.25098    | 85.077   | 2.37686   | 85.119   | 2.50869    | 85.157   | 2.64679   | 85.191   |
| 1.0  | 2.12928   | 90.000   | 2.24961    | 90.000   | 2.37557   | 90.000   | 2.50747    | 90.000   | 2.64563   | 90.000   |
| 1.05 | 2.13073   | 94.969   | 2.25098    | 94.923   | 2.37686   | 94.881   | 2.50869    | 94.843   | 2.64679   | 94.809   |
| 1.1  | 2.13502   | 99.925   | 2.25504    | 99.833   | 2.38071   | 99.751   | 2.51234    | 99.677   | 2.65025   | 99.610   |
| 1.15 | 2.14204   | 104.855  | 2.26169    | 104.721  | 2.38701   | 104.600  | 2.51831    | 104.492  | 2.65591   | 104.395  |
| 1.2  | 2.15159   | 109.746  | 2.27074    | 109.574  | 2.39558   | 109.419  | 2.52644    | 109.280  | 2.66361   | 109.155  |
| 1.25 | 2.16340   | 114.590  | 2.28193    | 114.385  | 2.40619   | 114.200  | 2.53650    | 114.034  | 2.67316   | 113.884  |
| 1.3  | 2.17714   | 119.376  | 2.29496    | 119.144  | 2.41856   | 118.935  | 2.54824    | 118.747  | 2.68430   | 118.578  |
| 1.35 | 2.19245   | 124.099  | 2.30949    | 123.846  | 2.43235   | 123.619  | 2.56133    | 123.415  | 2.69673   | 123.229  |
| 1.4  | 2.20892   | 128.753  | 2.32513    | 128.488  | 2.44721   | 128.248  | 2.57544    | 128.032  | 2.71014   | 127.837  |
| 1.45 | 2.22612   | 133.337  | 2.34148    | 133.066  | 2.46274   | 132.820  | 2.59021    | 132.599  | 2.72418   | 132.398  |
| 1.5  | 2.24362   | 137.850  | 2.35812    | 137.580  | 2.47857   | 137.334  | 2.60527    | 137.113  | 2.73850   | 136.911  |
| 1.55 | 2.26098   | 142.294  | 2.37465    | 142.030  | 2.49430   | 141.791  | 2.62024    | 141.574  | 2.75274   | 141.378  |
| 1.6  | 2.27779   | 146.670  | 2.39066    | 146.420  | 2.50955   | 146.192  | 2.63476    | 145.986  | 2.76657   | 145.799  |
| 1.65 | 2.29365   | 150.984  | 2.40577    | 150.752  | 2.52395   | 150.542  | 2.64847    | 150.351  | 2.77963   | 150.178  |
| 1.7  | 2.30819   | 155.241  | 2.41976    | 155.033  | 2.53717   | 154.845  | 2.66108    | 154.673  | 2.79164   | 154.517  |
| 1.75 | 2.32107   | 159.448  | 2.43193    | 159.268  | 2.54890   | 159.105  | 2.67226    | 158.956  | 2.80230   | 158.821  |
| 1.8  | 2.33202   | 163.611  | 2.44238    | 163.464  | 2.55887   | 163.329  | 2.68178    | 163.206  | 2.81138   | 163.094  |
| 1.85 | 2.34080   | 167.740  | 2.45076    | 167.626  | 2.56687   | 167.523  | 2.68941    | 167.429  | 2.81867   | 167.343  |
| 1.9  | 2.34720   | 171.842  | 2.45688    | 171.765  | 2.57272   | 171.695  | 2.69499    | 171.631  | 2.82399   | 171.573  |
| 1.95 | 2.35110   | 175.925  | 2.46061    | 175.887  | 2.57627   | 175.851  | 2.69839    | 175.819  | 2.82723   | 175.790  |
| 2.0  | 2.35241   | 180.000  | 2.46186    | 180.000  | 2.57746   | 180.000  | 2.69952    | 180.000  | 2.82832   | 180.000  |

Example.  $\cosh(1.6 + i 1.6) = 2.50955 / 146^{\circ}.192 = 2.50955 / 146^{\circ}.11'.31''$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 1.75$ |          | $x = 1.8$ |          | $x = 1.85$ |          | $x = 1.9$ |          | $x = 1.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 2.96419    | 0.000    | 3.10747   | 0.000    | 3.25853    | 0.000    | 3.41773   | 0.000    | 3.58548    | 0.000    |
| 0.05 | 2.96315    | 4.237    | 3.10648   | 4.262    | 3.25759    | 4.284    | 3.41683   | 4.304    | 3.58462    | 4.322    |
| 0.1  | 2.96006    | 8.480    | 3.10353   | 8.529    | 3.25477    | 8.572    | 3.41415   | 8.612    | 3.58207    | 8.648    |
| 0.15 | 2.95498    | 12.735   | 3.09869   | 12.806   | 3.25015    | 12.871   | 3.40975   | 12.929   | 3.57788    | 12.983   |
| 0.2  | 2.94804    | 17.008   | 3.09206   | 17.100   | 3.24384    | 17.184   | 3.40373   | 17.260   | 3.57213    | 17.330   |
| 0.25 | 2.93938    | 21.302   | 3.08382   | 21.414   | 3.23598    | 21.516   | 3.39624   | 21.608   | 3.56500    | 21.692   |
| 0.3  | 2.92921    | 25.625   | 3.07413   | 25.754   | 3.22675    | 25.871   | 3.38745   | 25.977   | 3.55663    | 26.073   |
| 0.35 | 2.91777    | 29.980   | 3.06323   | 30.123   | 3.21636    | 30.252   | 3.37750   | 30.369   | 3.54721    | 30.476   |
| 0.4  | 2.90532    | 34.370   | 3.05138   | 34.524   | 3.20507    | 34.663   | 3.36681   | 34.789   | 3.53698    | 34.904   |
| 0.45 | 2.89217    | 38.800   | 3.03885   | 38.961   | 3.19315    | 39.107   | 3.35546   | 39.239   | 3.52617    | 39.358   |
| 0.5  | 2.87861    | 43.270   | 3.02595   | 43.435   | 3.18088    | 43.584   | 3.34378   | 43.719   | 3.51507    | 43.840   |
| 0.55 | 2.86499    | 47.784   | 3.01300   | 47.947   | 3.16856    | 48.096   | 3.33207   | 48.230   | 3.50393    | 48.351   |
| 0.6  | 2.85165    | 52.339   | 3.00031   | 52.499   | 3.15650    | 52.643   | 3.32060   | 52.773   | 3.49302    | 52.890   |
| 0.65 | 2.83891    | 56.937   | 2.98821   | 57.088   | 3.14500    | 57.224   | 3.30967   | 57.346   | 3.48263    | 57.457   |
| 0.7  | 2.82710    | 61.575   | 2.97699   | 61.713   | 3.13434    | 61.837   | 3.29955   | 61.949   | 3.47301    | 62.050   |
| 0.75 | 2.81653    | 66.250   | 2.96695   | 66.371   | 3.12481    | 66.480   | 3.29049   | 66.579   | 3.46441    | 66.668   |
| 0.8  | 2.80747    | 70.958   | 2.95835   | 71.059   | 3.11665    | 71.150   | 3.28274   | 71.233   | 3.45702    | 71.307   |
| 0.85 | 2.80016    | 75.693   | 2.95142   | 75.772   | 3.11006    | 75.842   | 3.27649   | 75.906   | 3.45112    | 75.964   |
| 0.9  | 2.79479    | 80.449   | 2.94633   | 80.503   | 3.10523    | 80.552   | 3.27191   | 80.595   | 3.44675    | 80.635   |
| 0.95 | 2.79151    | 85.221   | 2.94322   | 85.248   | 3.10228    | 85.273   | 3.26911   | 85.295   | 3.44419    | 85.315   |
| 1.0  | 2.79041    | 90.000   | 2.94217   | 90.000   | 3.10129    | 90.000   | 3.26816   | 90.000   | 3.44321    | 90.000   |
| 1.05 | 2.79151    | 94.779   | 2.94322   | 94.752   | 3.10228    | 94.727   | 3.26911   | 94.705   | 3.44410    | 94.685   |
| 1.1  | 2.79479    | 99.551   | 2.94633   | 99.497   | 3.10523    | 99.448   | 3.27191   | 99.405   | 3.44675    | 99.365   |
| 1.15 | 2.80016    | 104.307  | 2.95142   | 104.228  | 3.11006    | 104.158  | 3.27649   | 104.094  | 3.45112    | 104.036  |
| 1.2  | 2.80747    | 109.042  | 2.95835   | 108.941  | 3.11665    | 108.850  | 3.28274   | 108.767  | 3.45702    | 108.693  |
| 1.25 | 2.81653    | 113.750  | 2.96695   | 113.629  | 3.12481    | 113.520  | 3.29049   | 113.421  | 3.46441    | 113.332  |
| 1.3  | 2.82710    | 118.425  | 2.97699   | 118.287  | 3.13434    | 118.163  | 3.29955   | 118.051  | 3.47301    | 117.950  |
| 1.35 | 2.83891    | 123.063  | 2.98821   | 122.912  | 3.14500    | 122.776  | 3.30967   | 122.654  | 3.48263    | 122.543  |
| 1.4  | 2.85165    | 127.661  | 3.00031   | 127.501  | 3.15650    | 127.357  | 3.32060   | 127.227  | 3.49302    | 127.110  |
| 1.45 | 2.86499    | 132.216  | 3.01300   | 132.053  | 3.16856    | 127.1904 | 3.33207   | 131.770  | 3.50393    | 131.649  |
| 1.5  | 2.87861    | 136.730  | 3.02595   | 136.565  | 3.18088    | 136.416  | 3.34378   | 136.281  | 3.51507    | 136.160  |
| 1.55 | 2.89217    | 141.200  | 3.03885   | 141.039  | 3.19315    | 140.893  | 3.35546   | 140.761  | 3.52617    | 140.642  |
| 1.6  | 2.90532    | 145.630  | 3.05138   | 145.476  | 3.20507    | 145.337  | 3.36681   | 145.211  | 3.53698    | 145.096  |
| 1.65 | 2.91777    | 150.020  | 3.06323   | 149.877  | 3.21636    | 149.748  | 3.37756   | 149.631  | 3.54721    | 149.524  |
| 1.7  | 2.92921    | 154.375  | 3.07413   | 154.246  | 3.22675    | 154.129  | 3.38745   | 154.023  | 3.55663    | 153.927  |
| 1.75 | 2.93938    | 158.698  | 3.08382   | 158.586  | 3.23598    | 158.484  | 3.39624   | 158.392  | 3.56500    | 158.308  |
| 1.8  | 2.94804    | 162.992  | 3.09206   | 162.900  | 3.24384    | 162.816  | 3.40373   | 162.740  | 3.57213    | 162.670  |
| 1.85 | 2.95498    | 167.265  | 3.09869   | 167.194  | 3.25015    | 167.129  | 3.40975   | 167.071  | 3.57788    | 167.017  |
| 1.9  | 2.96006    | 171.520  | 3.10353   | 171.471  | 3.25477    | 171.428  | 3.41415   | 171.388  | 3.58207    | 171.352  |
| 1.95 | 2.96315    | 175.763  | 3.10648   | 175.738  | 3.25759    | 175.716  | 3.41683   | 175.696  | 3.58462    | 175.678  |
| 2.0  | 2.96419    | 180.000  | 3.10747   | 180.000  | 3.25853    | 180.000  | 3.41773   | 180.000  | 3.58548    | 180.000  |

Example.  $\cosh(1.95 + i\ 1.25) = 3.46441 / 113^{\circ}.332 = 3.46441 / 113^{\circ}.10'.55''$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 2.0$ |          | $x = 2.05$ |          | $x = 2.1$ |          | $x = 2.15$ |          | $x = 2.2$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 3.76220   | 0.000    | 3.94832    | 0.000    | 4.14431   | 0.000    | 4.35067    | 0.000    | 4.56791   | 0.000    |
| 0.05 | 3.76137   | 4.339    | 3.94753    | 4.354    | 4.14357   | 4.368    | 4.34996    | 4.380    | 4.56723   | 4.391    |
| 0.1  | 3.75894   | 8.681    | 3.94521    | 8.711    | 4.14136   | 8.738    | 4.34785    | 8.763    | 4.56523   | 8.785    |
| 0.15 | 3.75495   | 13.031   | 3.94142    | 13.075   | 4.13774   | 13.115   | 4.34440    | 13.151   | 4.56167   | 13.184   |
| 0.2  | 3.74948   | 17.392   | 3.93620    | 17.449   | 4.13278   | 17.501   | 4.33968    | 17.548   | 4.55744   | 17.591   |
| 0.25 | 3.74268   | 21.768   | 3.92972    | 21.836   | 4.12661   | 21.899   | 4.33381    | 21.956   | 4.55184   | 22.007   |
| 0.3  | 3.73470   | 26.160   | 3.92213    | 26.239   | 4.11938   | 26.311   | 4.32693    | 26.376   | 4.54529   | 26.435   |
| 0.35 | 3.72574   | 30.573   | 3.91359    | 30.660   | 4.11125   | 30.740   | 4.31918    | 30.812   | 4.53793   | 30.877   |
| 0.4  | 3.71600   | 35.008   | 3.90432    | 35.102   | 4.10243   | 35.187   | 4.31079    | 35.264   | 4.52993   | 35.334   |
| 0.45 | 3.70572   | 39.467   | 3.89453    | 39.565   | 4.09311   | 39.653   | 4.30193    | 39.734   | 4.52150   | 39.807   |
| 0.5  | 3.69515   | 43.951   | 3.88448    | 44.051   | 4.08355   | 44.141   | 4.29282    | 44.223   | 4.51285   | 44.297   |
| 0.55 | 3.68455   | 48.461   | 3.87440    | 48.560   | 4.07396   | 48.649   | 4.28371    | 48.730   | 4.50417   | 48.804   |
| 0.6  | 3.67418   | 52.996   | 3.86454    | 53.092   | 4.06459   | 53.179   | 4.27479    | 53.257   | 4.49570   | 53.329   |
| 0.65 | 3.66430   | 57.557   | 3.85515    | 57.647   | 4.05566   | 57.729   | 4.26630    | 57.803   | 4.48763   | 57.870   |
| 0.7  | 3.65517   | 62.142   | 3.84648    | 62.224   | 4.04740   | 62.299   | 4.25846    | 62.366   | 4.48017   | 62.427   |
| 0.75 | 3.64699   | 66.748   | 3.83870    | 66.820   | 4.04002   | 66.886   | 4.25145    | 66.945   | 4.47350   | 66.998   |
| 0.8  | 3.64000   | 71.374   | 3.83206    | 71.434   | 4.03371   | 71.489   | 4.24544    | 71.538   | 4.46780   | 71.582   |
| 0.85 | 3.63437   | 76.016   | 3.82671    | 76.062   | 4.02863   | 76.105   | 4.24061    | 76.143   | 4.46322   | 76.177   |
| 0.9  | 3.63023   | 80.670   | 3.82279    | 80.702   | 4.02490   | 80.731   | 4.23707    | 80.757   | 4.45985   | 80.780   |
| 0.95 | 3.62771   | 85.333   | 3.82039    | 85.349   | 4.02263   | 85.364   | 4.23491    | 85.377   | 4.45779   | 85.389   |
| 1.0  | 3.62686   | 90.000   | 3.81958    | 90.000   | 4.02186   | 90.000   | 4.23419    | 90.000   | 4.45711   | 90.000   |
| 1.05 | 3.62771   | 94.667   | 3.82039    | 94.651   | 4.02263   | 94.636   | 4.23491    | 94.623   | 4.45779   | 94.611   |
| 1.1  | 3.63023   | 99.330   | 3.82279    | 99.298   | 4.02490   | 99.269   | 4.23707    | 99.243   | 4.45985   | 99.220   |
| 1.15 | 3.63437   | 103.984  | 3.82671    | 103.938  | 4.02863   | 103.895  | 4.24061    | 103.857  | 4.46322   | 103.823  |
| 1.2  | 3.64000   | 108.626  | 3.83206    | 108.566  | 4.03371   | 108.511  | 4.24544    | 108.462  | 4.46780   | 108.418  |
| 1.25 | 3.64699   | 113.252  | 3.83870    | 113.180  | 4.04002   | 113.114  | 4.25145    | 113.055  | 4.47350   | 113.002  |
| 1.3  | 3.65517   | 117.858  | 3.84648    | 117.776  | 4.04740   | 117.701  | 4.25846    | 117.634  | 4.48017   | 117.573  |
| 1.35 | 3.66430   | 122.443  | 3.85515    | 122.353  | 4.05566   | 122.271  | 4.26630    | 122.197  | 4.48763   | 122.130  |
| 1.4  | 3.67418   | 127.004  | 3.86454    | 126.908  | 4.06459   | 126.821  | 4.27479    | 126.743  | 4.49570   | 126.671  |
| 1.45 | 3.68455   | 131.539  | 3.87440    | 131.440  | 4.07396   | 131.351  | 4.28371    | 131.270  | 4.50417   | 131.196  |
| 1.5  | 3.69515   | 136.049  | 3.88448    | 135.950  | 4.08355   | 135.859  | 4.29282    | 135.778  | 4.51285   | 135.703  |
| 1.55 | 3.70572   | 140.533  | 3.89453    | 140.435  | 4.09311   | 140.347  | 4.30193    | 140.266  | 4.52150   | 140.193  |
| 1.6  | 3.71600   | 144.992  | 3.90432    | 144.898  | 4.10243   | 144.813  | 4.31079    | 144.736  | 4.52993   | 144.666  |
| 1.65 | 3.72574   | 149.427  | 3.91359    | 149.340  | 4.11125   | 149.260  | 4.31918    | 149.188  | 4.53793   | 149.123  |
| 1.7  | 3.73470   | 153.840  | 3.92213    | 153.761  | 4.11938   | 153.689  | 4.32693    | 153.624  | 4.54529   | 153.565  |
| 1.75 | 3.74268   | 158.232  | 3.92972    | 158.164  | 4.12661   | 158.101  | 4.33381    | 158.044  | 4.55184   | 157.993  |
| 1.8  | 3.74948   | 162.608  | 3.93620    | 162.551  | 4.13278   | 162.499  | 4.33968    | 162.452  | 4.55744   | 162.409  |
| 1.85 | 3.75495   | 166.969  | 3.94142    | 166.925  | 4.13774   | 166.885  | 4.34440    | 166.849  | 4.56167   | 166.816  |
| 1.9  | 3.75894   | 171.319  | 3.94521    | 171.289  | 4.14136   | 171.262  | 4.34785    | 171.237  | 4.56523   | 171.215  |
| 1.95 | 3.76137   | 175.661  | 3.94753    | 175.646  | 4.14357   | 175.632  | 4.34996    | 175.620  | 4.56723   | 175.609  |
| 2.0  | 3.76220   | 180.000  | 3.94832    | 180.000  | 4.14431   | 180.000  | 4.35067    | 180.000  | 4.56791   | 180.000  |

Example.  $\cosh(2.0 + i 0.5) = 3.69515 / 43^\circ.951' = 3.69515 / 43^\circ.57'.04''$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 2.25$ |          | $x = 2.3$ |          | $x = 2.35$ |          | $x = 2.4$ |          | $x = 2.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 4.79657    | 0.000    | 5.03722   | 0.000    | 5.29047    | 0.000    | 5.55695   | 0.000    | 5.83732    | 0.000    |
| 0.05 | 4.79593    | 4.401    | 5.03661   | 4.411    | 5.28989    | 4.419    | 5.55639   | 4.427    | 5.83680    | 4.434    |
| 0.1  | 4.79402    | 8.805    | 5.03479   | 8.824    | 5.28816    | 8.840    | 5.55474   | 8.856    | 5.83522    | 8.860    |
| 0.15 | 4.79088    | 13.214   | 5.03181   | 13.241   | 5.28531    | 13.265   | 5.55204   | 13.288   | 5.83265    | 13.308   |
| 0.2  | 4.78661    | 17.629   | 5.02773   | 17.664   | 5.28143    | 17.696   | 5.54834   | 17.725   | 5.82913    | 17.751   |
| 0.25 | 4.78127    | 22.053   | 5.02265   | 22.096   | 5.27662    | 22.134   | 5.54375   | 22.169   | 5.82476    | 22.200   |
| 0.3  | 4.77503    | 26.488   | 5.01672   | 26.537   | 5.27096    | 26.581   | 5.53837   | 26.620   | 5.81964    | 26.656   |
| 0.35 | 4.76802    | 30.936   | 5.01005   | 30.989   | 5.26461    | 31.038   | 5.53233   | 31.081   | 5.81389    | 31.121   |
| 0.4  | 4.76042    | 35.397   | 5.00281   | 35.454   | 5.25772    | 35.506   | 5.52578   | 35.553   | 5.80765    | 35.595   |
| 0.45 | 4.75240    | 39.873   | 4.99518   | 39.932   | 5.25046    | 39.986   | 5.51887   | 40.035   | 5.80108    | 40.079   |
| 0.5  | 4.74415    | 44.364   | 4.98734   | 44.424   | 5.23728    | 44.479   | 5.51177   | 44.529   | 5.79434    | 44.573   |
| 0.55 | 4.73592    | 48.870   | 4.97950   | 48.930   | 5.23553    | 48.984   | 5.50468   | 49.034   | 5.78758    | 49.078   |
| 0.6  | 4.72784    | 53.392   | 4.97183   | 53.451   | 5.22825    | 53.503   | 5.49775   | 53.551   | 5.78099    | 53.593   |
| 0.65 | 4.72017    | 57.930   | 4.96454   | 57.984   | 5.22131    | 58.034   | 5.49115   | 58.078   | 5.77472    | 58.119   |
| 0.7  | 4.71308    | 62.482   | 4.95780   | 62.531   | 5.21490    | 62.576   | 5.48505   | 62.617   | 5.76892    | 62.653   |
| 0.75 | 4.70675    | 67.046   | 4.95177   | 67.090   | 5.20917    | 67.129   | 5.47961   | 67.165   | 5.76375    | 67.197   |
| 0.8  | 4.70134    | 71.622   | 4.94662   | 71.659   | 5.20428    | 71.691   | 5.47495   | 71.721   | 5.75932    | 71.748   |
| 0.85 | 4.69697    | 76.208   | 4.94249   | 76.236   | 5.20035    | 76.261   | 5.47122   | 76.284   | 5.75576    | 76.305   |
| 0.9  | 4.69377    | 80.801   | 4.93944   | 80.820   | 5.19745    | 80.837   | 5.46847   | 80.853   | 5.75316    | 80.867   |
| 0.95 | 4.69182    | 85.399   | 4.93759   | 85.409   | 5.19569    | 85.418   | 5.46679   | 85.426   | 5.75156    | 85.433   |
| 1.0  | 4.69117    | 90.000   | 4.93696   | 90.000   | 5.19510    | 90.000   | 5.46623   | 90.000   | 5.75103    | 90.000   |
| 1.05 | 4.69182    | 94.601   | 4.93759   | 94.591   | 5.19569    | 94.582   | 5.46679   | 94.574   | 5.75156    | 94.567   |
| 1.1  | 4.69377    | 99.199   | 4.93944   | 99.180   | 5.19745    | 99.163   | 5.46847   | 99.147   | 5.75316    | 99.133   |
| 1.15 | 4.69697    | 103.792  | 4.94249   | 103.764  | 5.20035    | 103.739  | 5.47122   | 103.716  | 5.75576    | 103.695  |
| 1.2  | 4.70134    | 108.378  | 4.94662   | 108.341  | 5.20428    | 108.309  | 5.47495   | 108.279  | 5.75932    | 108.252  |
| 1.25 | 4.70675    | 112.954  | 4.95177   | 112.910  | 5.20917    | 112.871  | 5.47961   | 112.835  | 5.76375    | 112.803  |
| 1.3  | 4.71308    | 117.518  | 4.95780   | 117.469  | 5.21490    | 117.424  | 5.48505   | 117.383  | 5.76892    | 117.347  |
| 1.35 | 4.72017    | 122.070  | 4.96454   | 122.016  | 5.22131    | 121.966  | 5.49115   | 121.922  | 5.77472    | 121.881  |
| 1.4  | 4.72784    | 126.608  | 4.97183   | 126.549  | 5.22825    | 126.497  | 5.49775   | 126.449  | 5.78099    | 126.407  |
| 1.45 | 4.73592    | 131.130  | 4.97950   | 131.070  | 5.23553    | 131.016  | 5.50468   | 130.966  | 5.78758    | 130.922  |
| 1.5  | 4.74415    | 135.636  | 4.98734   | 135.576  | 5.23728    | 135.521  | 5.51177   | 135.471  | 5.79434    | 135.427  |
| 1.55 | 4.75240    | 140.127  | 4.99518   | 140.068  | 5.25046    | 140.014  | 5.51887   | 139.965  | 5.80108    | 139.921  |
| 1.6  | 4.76042    | 144.603  | 5.00281   | 144.546  | 5.25772    | 144.494  | 5.52578   | 144.447  | 5.80765    | 144.405  |
| 1.65 | 4.76802    | 149.064  | 5.01005   | 149.011  | 5.26461    | 148.962  | 5.53233   | 148.919  | 5.81389    | 148.879  |
| 1.7  | 4.77503    | 153.512  | 5.01672   | 153.463  | 5.27096    | 153.419  | 5.53837   | 153.380  | 5.81964    | 153.344  |
| 1.75 | 4.78127    | 157.947  | 5.02265   | 157.904  | 5.27662    | 157.866  | 5.54375   | 157.831  | 5.82476    | 157.800  |
| 1.8  | 4.78661    | 162.371  | 5.02773   | 162.336  | 5.28143    | 162.304  | 5.54834   | 162.275  | 5.82913    | 162.249  |
| 1.85 | 4.79088    | 166.786  | 5.03181   | 166.759  | 5.28531    | 166.735  | 5.55204   | 166.712  | 5.83265    | 166.692  |
| 1.9  | 4.79402    | 171.195  | 5.03479   | 171.176  | 5.28816    | 171.160  | 5.55474   | 171.144  | 5.83522    | 171.131  |
| 1.95 | 4.79593    | 175.599  | 5.03661   | 175.589  | 5.28989    | 175.581  | 5.55639   | 175.573  | 5.83680    | 175.566  |
| 2.0  | 4.79657    | 180.000  | 5.03722   | 180.000  | 5.29047    | 180.000  | 5.55695   | 180.000  | 5.83732    | 180.000  |

Example.  $\cosh(2.40 + i 2.0) = 5.55695/\underline{180^\circ} = 5.55695 \sqrt{180^\circ}$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 2.5$ |          | $x = 2.55$ |          | $x = 2.6$ |          | $x = 2.65$ |          | $x = 2.7$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
|      | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 6.13229   | 0.000    | 6.44259    | 0.000    | 6.76901   | 0.000    | 7.11234    | 0.000    | 7.47347   | 0.000    |
| 0.05 | 6.13179   | 4.440    | 6.44212    | 4.446    | 6.76855   | 4.451    | 7.11191    | 4.456    | 7.47306   | 4.460    |
| 0.1  | 6.13030   | 8.881    | 6.44069    | 8.893    | 6.76720   | 8.903    | 7.11062    | 8.912    | 7.47183   | 8.920    |
| 0.15 | 6.12785   | 13.326   | 6.43836    | 13.342   | 6.76499   | 13.357   | 7.10851    | 13.371   | 7.46982   | 13.383   |
| 0.2  | 6.12450   | 17.774   | 6.43518    | 17.796   | 6.76195   | 17.815   | 7.10563    | 17.833   | 7.46708   | 17.848   |
| 0.25 | 6.12034   | 22.228   | 6.43121    | 22.254   | 6.75818   | 22.278   | 7.10204    | 22.299   | 7.46366   | 22.318   |
| 0.3  | 6.11547   | 26.689   | 6.42658    | 26.718   | 6.75376   | 26.745   | 7.09784    | 26.770   | 7.45966   | 26.791   |
| 0.35 | 6.10999   | 31.157   | 6.42137    | 31.190   | 6.74881   | 31.219   | 7.09313    | 31.246   | 7.45518   | 31.270   |
| 0.4  | 6.10406   | 35.634   | 6.41572    | 35.668   | 6.74343   | 35.700   | 7.08802    | 35.728   | 7.45032   | 35.754   |
| 0.45 | 6.09781   | 40.119   | 6.40977    | 40.155   | 6.73778   | 40.188   | 7.08264    | 40.218   | 7.44519   | 40.245   |
| 0.5  | 6.09139   | 44.614   | 6.40367    | 44.651   | 6.73197   | 44.684   | 7.07711    | 44.713   | 7.43994   | 44.741   |
| 0.55 | 6.08496   | 49.118   | 6.39755    | 49.155   | 6.72616   | 49.187   | 7.07158    | 49.217   | 7.43468   | 49.244   |
| 0.6  | 6.07869   | 53.632   | 6.39160    | 53.667   | 6.72048   | 53.699   | 7.06618    | 53.727   | 7.42955   | 53.754   |
| 0.65 | 6.07273   | 58.155   | 6.38592    | 58.188   | 6.71509   | 58.218   | 7.06105    | 58.244   | 7.42467   | 58.269   |
| 0.7  | 6.06722   | 62.686   | 6.38068    | 62.716   | 6.71011   | 62.743   | 7.05631    | 62.768   | 7.42017   | 62.790   |
| 0.75 | 6.06229   | 67.226   | 6.37601    | 67.252   | 6.70565   | 67.275   | 7.05208    | 67.297   | 7.41614   | 67.316   |
| 0.8  | 6.05808   | 71.772   | 6.37201    | 71.794   | 6.70185   | 71.813   | 7.04847    | 71.831   | 7.41271   | 71.847   |
| 0.85 | 6.05471   | 76.324   | 6.36879    | 76.340   | 6.69880   | 76.356   | 7.04556    | 76.369   | 7.40994   | 76.382   |
| 0.9  | 6.05223   | 80.880   | 6.36644    | 80.891   | 6.69656   | 80.902   | 7.04343    | 80.911   | 7.40791   | 80.920   |
| 0.95 | 6.05071   | 85.439   | 6.36499    | 85.445   | 6.69518   | 85.450   | 7.04213    | 85.455   | 7.40668   | 85.460   |
| 1.0  | 6.05020   | 90.000   | 6.36451    | 90.000   | 6.69473   | 90.000   | 7.04169    | 90.000   | 7.40626   | 90.000   |
| 1.05 | 6.05071   | 94.561   | 6.36499    | 94.555   | 6.69518   | 94.550   | 7.04213    | 94.545   | 7.40668   | 94.540   |
| 1.1  | 6.05223   | 99.120   | 6.36644    | 99.109   | 6.69656   | 99.098   | 7.04343    | 99.089   | 7.40791   | 99.080   |
| 1.15 | 6.05471   | 103.676  | 6.36879    | 103.660  | 6.69880   | 103.644  | 7.04556    | 103.631  | 7.40994   | 103.618  |
| 1.2  | 6.05808   | 108.228  | 6.37201    | 108.206  | 6.70185   | 108.187  | 7.04847    | 108.169  | 7.41271   | 108.153  |
| 1.25 | 6.06229   | 112.774  | 6.37601    | 112.748  | 6.70565   | 112.725  | 7.05208    | 112.703  | 7.41614   | 112.684  |
| 1.3  | 6.06722   | 117.314  | 6.38068    | 117.284  | 6.71011   | 117.257  | 7.05631    | 117.232  | 7.42017   | 117.210  |
| 1.35 | 6.07273   | 121.845  | 6.38592    | 121.812  | 6.71509   | 121.782  | 7.06105    | 121.756  | 7.42467   | 121.731  |
| 1.4  | 6.07869   | 126.368  | 6.39160    | 126.333  | 6.72048   | 126.301  | 7.06618    | 126.273  | 7.42955   | 126.246  |
| 1.45 | 6.08496   | 130.882  | 6.39755    | 130.845  | 6.72616   | 130.813  | 7.07158    | 130.783  | 7.43468   | 130.756  |
| 1.5  | 6.09139   | 135.386  | 6.40367    | 135.350  | 6.73197   | 135.316  | 7.07711    | 135.286  | 7.43994   | 135.259  |
| 1.55 | 6.09781   | 139.881  | 6.40977    | 139.845  | 6.73778   | 139.812  | 7.08264    | 139.782  | 7.44519   | 139.755  |
| 1.6  | 6.10406   | 144.366  | 6.41572    | 144.332  | 6.74343   | 144.300  | 7.08802    | 144.272  | 7.45032   | 144.246  |
| 1.65 | 6.10999   | 148.843  | 6.42137    | 148.810  | 6.74881   | 148.781  | 7.09313    | 148.754  | 7.45518   | 148.730  |
| 1.7  | 6.11547   | 153.311  | 6.42658    | 153.282  | 6.75376   | 153.255  | 7.09784    | 153.230  | 7.45966   | 153.209  |
| 1.75 | 6.12034   | 157.772  | 6.43121    | 157.746  | 6.75818   | 157.722  | 7.10204    | 157.701  | 7.46366   | 157.682  |
| 1.8  | 6.12450   | 162.226  | 6.43518    | 162.204  | 6.76195   | 162.185  | 7.10563    | 162.167  | 7.46708   | 162.152  |
| 1.85 | 6.12785   | 166.674  | 6.43836    | 166.658  | 6.76499   | 166.643  | 7.10851    | 166.629  | 7.46982   | 166.617  |
| 1.9  | 6.13030   | 171.119  | 6.44069    | 171.107  | 6.76720   | 171.097  | 7.11062    | 171.088  | 7.47183   | 171.080  |
| 1.95 | 6.13179   | 175.560  | 6.44212    | 175.554  | 6.76855   | 175.549  | 7.11191    | 175.544  | 7.47306   | 175.540  |
| 2.0  | 6.13229   | 180.000  | 6.44259    | 180.000  | 6.76901   | 180.000  | 7.11234    | 180.000  | 7.47347   | 180.000  |

Example.  $\cosh(2.65 + i\,0.75) = 7.05208 / 67^{\circ}.297 = 7.05208 / 67^{\circ}.17'.49''$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 2.75$ |          | $x = 2.8$ |          | $x = 2.85$ |          | $x = 2.9$ |          | $x = 2.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 7.85328    | 0.000    | 8.25273   | 0.000    | 8.67281    | 0.000    | 9.11458   | 0.000    | 9.57915    | 0.000    |
| 0.05 | 7.85288    | 4.464    | 8.25235   | 4.467    | 8.67246    | 4.470    | 9.11424   | 4.473    | 9.57882    | 4.476    |
| 0.1  | 7.85172    | 8.928    | 8.25124   | 8.935    | 8.67140    | 8.941    | 9.11324   | 8.947    | 9.57787    | 8.952    |
| 0.15 | 7.84980    | 13.394   | 8.24942   | 13.404   | 8.66967    | 13.413   | 9.11160   | 13.421   | 9.57630    | 13.429   |
| 0.2  | 7.84720    | 17.863   | 8.24694   | 17.876   | 8.66731    | 17.888   | 9.10934   | 17.898   | 9.57416    | 17.908   |
| 0.25 | 7.84395    | 22.335   | 8.24385   | 22.351   | 8.66436    | 22.364   | 9.10655   | 22.378   | 9.57150    | 22.389   |
| 0.3  | 7.84015    | 26.811   | 8.24024   | 26.829   | 8.66092    | 26.845   | 9.10327   | 26.860   | 9.56838    | 26.873   |
| 0.35 | 7.83588    | 31.292   | 8.23617   | 31.312   | 8.65706    | 31.330   | 9.09959   | 31.346   | 9.56488    | 31.360   |
| 0.4  | 7.83125    | 35.778   | 8.23177   | 35.799   | 8.65287    | 35.818   | 9.09561   | 35.835   | 9.56109    | 35.851   |
| 0.45 | 7.82638    | 40.269   | 8.22713   | 40.291   | 8.64846    | 40.311   | 9.09142   | 40.329   | 9.55711    | 40.345   |
| 0.5  | 7.82138    | 44.766   | 8.22238   | 44.788   | 8.64395    | 44.808   | 9.08711   | 44.827   | 9.55301    | 44.843   |
| 0.55 | 7.81637    | 49.269   | 8.21762   | 49.290   | 8.63941    | 49.310   | 9.08281   | 49.329   | 9.54892    | 49.345   |
| 0.6  | 7.81149    | 53.777   | 8.21268   | 53.798   | 8.63500    | 53.817   | 9.07861   | 53.835   | 9.54492    | 53.850   |
| 0.65 | 7.80685    | 58.291   | 8.20857   | 58.311   | 8.63080    | 58.329   | 9.07461   | 58.345   | 9.54113    | 58.360   |
| 0.7  | 7.80257    | 62.810   | 8.20449   | 62.828   | 8.62691    | 62.845   | 9.07093   | 62.859   | 9.53762    | 62.873   |
| 0.75 | 7.79875    | 67.334   | 8.20085   | 67.350   | 8.62347    | 67.364   | 9.06764   | 67.377   | 9.53450    | 67.389   |
| 0.8  | 7.79547    | 71.862   | 8.19774   | 71.875   | 8.62051    | 71.887   | 9.06483   | 71.898   | 9.53182    | 71.907   |
| 0.85 | 7.79285    | 76.393   | 8.19524   | 76.404   | 8.61813    | 76.413   | 9.06257   | 76.421   | 9.52967    | 76.429   |
| 0.9  | 7.79092    | 80.927   | 8.19341   | 80.934   | 8.61639    | 80.940   | 9.06091   | 80.946   | 9.52809    | 80.951   |
| 0.95 | 7.78975    | 85.463   | 8.19230   | 85.467   | 8.61532    | 85.470   | 9.05990   | 85.473   | 9.52713    | 85.475   |
| 1.0  | 7.78935    | 90.000   | 8.19192   | 90.000   | 8.61497    | 90.000   | 9.05956   | 90.000   | 9.52681    | 90.000   |
| 1.05 | 7.78975    | 94.537   | 8.19230   | 94.533   | 8.61532    | 94.530   | 9.05990   | 94.527   | 9.52713    | 94.525   |
| 1.1  | 7.79092    | 99.073   | 8.19341   | 99.066   | 8.61639    | 99.060   | 9.06091   | 99.054   | 9.52809    | 99.049   |
| 1.15 | 7.79285    | 103.607  | 8.19524   | 103.596  | 8.61813    | 103.587  | 9.06257   | 103.579  | 9.52967    | 103.571  |
| 1.2  | 7.79547    | 108.138  | 8.19774   | 108.125  | 8.62051    | 108.113  | 9.06483   | 108.102  | 9.53182    | 108.093  |
| 1.25 | 7.79875    | 112.666  | 8.20085   | 112.650  | 8.62347    | 112.636  | 9.06764   | 112.623  | 9.53450    | 112.611  |
| 1.3  | 7.80257    | 117.190  | 8.20449   | 117.172  | 8.62691    | 117.155  | 9.07093   | 117.141  | 9.53762    | 117.127  |
| 1.35 | 7.80685    | 121.709  | 8.20857   | 121.689  | 8.63080    | 121.671  | 9.07461   | 121.655  | 9.54113    | 121.640  |
| 1.4  | 7.81149    | 126.223  | 8.21268   | 126.202  | 8.63500    | 126.183  | 9.07861   | 126.165  | 9.54492    | 126.150  |
| 1.45 | 7.81637    | 130.731  | 8.21762   | 130.710  | 8.63941    | 130.690  | 9.08281   | 130.671  | 9.54892    | 130.655  |
| 1.5  | 7.82138    | 135.234  | 8.22238   | 135.212  | 8.64395    | 135.192  | 9.08711   | 135.173  | 9.55301    | 135.157  |
| 1.55 | 7.82638    | 139.731  | 8.22713   | 139.709  | 8.64846    | 139.689  | 9.09142   | 139.671  | 9.55711    | 139.655  |
| 1.6  | 7.83125    | 144.222  | 8.23177   | 144.201  | 8.65287    | 144.182  | 9.09561   | 144.165  | 9.56109    | 144.149  |
| 1.65 | 7.83588    | 148.708  | 8.23617   | 148.688  | 8.65706    | 148.670  | 9.09959   | 148.654  | 9.56488    | 148.640  |
| 1.7  | 7.84015    | 153.189  | 8.24024   | 153.171  | 8.66092    | 153.155  | 9.10327   | 153.140  | 9.56838    | 153.127  |
| 1.75 | 7.84395    | 157.665  | 8.24385   | 157.649  | 8.66436    | 157.636  | 9.10655   | 157.622  | 9.57150    | 157.611  |
| 1.8  | 7.84720    | 162.137  | 8.24694   | 162.124  | 8.66731    | 162.112  | 9.10934   | 162.102  | 9.57416    | 162.092  |
| 1.85 | 7.84980    | 166.606  | 8.24942   | 166.596  | 8.66967    | 166.587  | 9.11160   | 166.579  | 9.57630    | 166.571  |
| 1.9  | 7.85172    | 171.072  | 8.25124   | 171.065  | 8.67140    | 171.059  | 9.11324   | 171.053  | 9.57787    | 171.048  |
| 1.95 | 7.85288    | 175.536  | 8.25235   | 175.533  | 8.67246    | 175.530  | 9.11424   | 175.527  | 9.57882    | 175.524  |
| 2.0  | 7.85328    | 180.000  | 8.25273   | 180.000  | 8.67281    | 180.000  | 9.11458   | 180.000  | 9.57915    | 180.000  |

Example.  $\cosh(2.90 + i0.9) = 9.06091 / 80^{\circ}.946 = 0.96091 / 80^{\circ}.56'.46''$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

| $x = 3.0$ |          |          | $x = 3.05$ |          | $x = 3.1$ |          | $x = 3.15$ |          | $x = 3.2$ |          |
|-----------|----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$       | $r$      | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0         | 10.06766 | 0.000    | 10.58135   | 0.000    | 11.12150  | 0.000    | 11.68946   | 0.000    | 12.28665  | 0.000    |
| 0.05      | 10.06737 | 4.478    | 10.58110   | 4.480    | 11.12120  | 4.482    | 11.68920   | 4.484    | 12.28640  | 4.485    |
| 0.1       | 10.06645 | 8.956    | 10.58020   | 8.960    | 11.12040  | 8.964    | 11.68840   | 8.968    | 12.28560  | 8.971    |
| 0.15      | 10.06500 | 13.436   | 10.57880   | 13.442   | 11.11900  | 13.447   | 11.68710   | 13.452   | 12.28440  | 13.457   |
| 0.2       | 10.06292 | 17.917   | 10.57680   | 17.925   | 11.11720  | 17.932   | 11.68540   | 17.938   | 12.28280  | 17.944   |
| 0.25      | 10.06040 | 22.400   | 10.57440   | 22.409   | 11.11490  | 22.418   | 11.68320   | 22.426   | 12.28070  | 22.433   |
| 0.3       | 10.05743 | 26.886   | 10.57160   | 26.896   | 11.11220  | 26.906   | 11.68060   | 26.915   | 12.27820  | 26.923   |
| 0.35      | 10.05410 | 31.374   | 10.56840   | 31.386   | 11.10920  | 31.396   | 11.67780   | 31.406   | 12.27550  | 31.415   |
| 0.4       | 10.05050 | 35.865   | 10.56500   | 35.878   | 11.10600  | 35.890   | 11.67470   | 35.900   | 12.27260  | 35.910   |
| 0.45      | 10.04670 | 40.360   | 10.56140   | 40.373   | 11.10250  | 40.385   | 11.67140   | 40.396   | 12.26950  | 40.406   |
| 0.5       | 10.04280 | 44.858   | 10.55770   | 44.871   | 11.09900  | 44.884   | 11.66810   | 44.895   | 12.26630  | 44.905   |
| 0.55      | 10.03890 | 49.360   | 10.55400   | 49.373   | 11.09540  | 49.385   | 11.66470   | 49.396   | 12.26310  | 49.406   |
| 0.6       | 10.03510 | 53.865   | 10.55040   | 53.878   | 11.09200  | 53.889   | 11.66140   | 53.900   | 12.26000  | 53.909   |
| 0.65      | 10.03150 | 58.373   | 10.54690   | 58.385   | 11.08880  | 58.396   | 11.65830   | 58.406   | 12.25700  | 58.415   |
| 0.7       | 10.02820 | 62.885   | 10.54380   | 62.896   | 11.08580  | 62.906   | 11.65540   | 62.915   | 12.25430  | 62.923   |
| 0.75      | 10.02520 | 67.399   | 10.54090   | 67.409   | 11.08310  | 67.417   | 11.65290   | 67.425   | 12.25180  | 67.432   |
| 0.8       | 10.02260 | 71.916   | 10.53850   | 71.924   | 11.08080  | 71.932   | 11.65070   | 71.938   | 12.24980  | 71.944   |
| 0.85      | 10.02060 | 76.435   | 10.53660   | 76.441   | 11.07890  | 76.447   | 11.64890   | 76.452   | 12.24810  | 76.457   |
| 0.9       | 10.01910 | 80.956   | 10.53520   | 80.960   | 11.07750  | 80.964   | 11.64770   | 80.967   | 12.24690  | 80.970   |
| 0.95      | 10.01820 | 85.478   | 10.53430   | 85.480   | 11.07670  | 85.482   | 11.64690   | 85.484   | 12.24610  | 85.485   |
| 1.0       | 10.01787 | 90.000   | 10.53399   | 90.000   | 11.07645  | 90.000   | 11.64661   | 90.000   | 12.24588  | 90.000   |
| 1.05      | 10.01820 | 94.522   | 10.53430   | 94.520   | 11.07670  | 94.518   | 11.64690   | 94.516   | 12.24610  | 94.515   |
| 1.1       | 10.01910 | 99.044   | 10.53520   | 99.040   | 11.07750  | 99.036   | 11.64770   | 99.033   | 12.24690  | 99.030   |
| 1.15      | 10.02060 | 103.565  | 10.53660   | 103.559  | 11.07890  | 103.553  | 11.64890   | 103.548  | 12.24810  | 103.543  |
| 1.2       | 10.02260 | 108.084  | 10.53850   | 108.076  | 11.08080  | 108.068  | 11.65070   | 108.062  | 12.24980  | 108.056  |
| 1.25      | 10.02520 | 112.601  | 10.54090   | 112.591  | 11.08310  | 112.583  | 11.65290   | 112.575  | 12.25180  | 112.568  |
| 1.3       | 10.02820 | 117.115  | 10.54380   | 117.104  | 11.08580  | 117.094  | 11.65540   | 117.085  | 12.25430  | 117.077  |
| 1.35      | 10.03150 | 121.627  | 10.54690   | 121.615  | 11.08880  | 121.604  | 11.65830   | 121.594  | 12.25700  | 121.585  |
| 1.4       | 10.03510 | 126.135  | 10.55040   | 126.122  | 11.09200  | 126.111  | 11.66140   | 126.100  | 12.26000  | 126.091  |
| 1.45      | 10.03890 | 130.640  | 10.55400   | 130.627  | 11.09540  | 130.615  | 11.66470   | 130.604  | 12.26310  | 130.594  |
| 1.5       | 10.04280 | 135.142  | 10.55770   | 135.129  | 11.09900  | 135.116  | 11.66810   | 135.105  | 12.26630  | 135.095  |
| 1.55      | 10.04670 | 139.640  | 10.56140   | 139.627  | 11.10250  | 139.615  | 11.67140   | 139.604  | 12.26950  | 139.594  |
| 1.6       | 10.05050 | 144.135  | 10.56500   | 144.122  | 11.10600  | 144.110  | 11.67470   | 144.100  | 12.27260  | 144.090  |
| 1.65      | 10.05410 | 148.626  | 10.56840   | 148.614  | 11.10920  | 148.604  | 11.67780   | 148.594  | 12.27550  | 148.585  |
| 1.7       | 10.05743 | 153.114  | 10.57160   | 153.104  | 11.11220  | 153.094  | 11.68060   | 153.085  | 12.27820  | 153.077  |
| 1.75      | 10.06040 | 157.600  | 10.57440   | 157.591  | 11.11490  | 157.582  | 11.68320   | 157.574  | 12.28070  | 157.567  |
| 1.8       | 10.06292 | 162.083  | 10.57680   | 162.075  | 11.11720  | 162.068  | 11.68540   | 162.062  | 12.28280  | 162.056  |
| 1.85      | 10.06500 | 166.564  | 10.57880   | 166.558  | 11.11900  | 166.553  | 11.68710   | 166.548  | 12.28440  | 166.543  |
| 1.9       | 10.06645 | 171.044  | 10.58020   | 171.040  | 11.12040  | 171.036  | 11.68840   | 171.032  | 12.28560  | 171.029  |
| 1.95      | 10.06737 | 175.522  | 10.58110   | 175.520  | 11.12120  | 175.518  | 11.68920   | 175.516  | 12.28640  | 175.515  |
| 2.0       | 10.06766 | 180.000  | 10.58135   | 180.000  | 11.12150  | 180.000  | 11.68946   | 180.000  | 12.28665  | 180.000  |

Example.  $\cosh(3.15 + i 0.15) = 11.68710 / 13^{\circ} 45' = 11.68710 / 13^{\circ} 27' .07''$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 3.25$ |          | $x = 3.3$ |          | $x = 3.35$ |          | $x = 3.4$ |          | $x = 3.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
|      | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 12.91456   | 0.000    | 13.57476  | 0.000    | 14.26891   | 0.000    | 14.99874  | 0.000    | 15.76607   | 0.000    |
| 0.05 | 12.91430   | 4.486    | 13.57455  | 4.488    | 14.26870   | 4.489    | 14.99853  | 4.490    | 15.76587   | 4.491    |
| 0.1  | 12.91360   | 8.973    | 13.57387  | 8.976    | 14.26805   | 8.978    | 14.99790  | 8.980    | 15.76530   | 8.982    |
| 0.15 | 12.91240   | 13.461   | 13.57275  | 13.465   | 14.26700   | 13.468   | 14.99692  | 13.471   | 15.76433   | 13.474   |
| 0.2  | 12.91085   | 17.949   | 13.57123  | 17.954   | 14.26556   | 17.959   | 14.99555  | 17.963   | 15.76303   | 17.966   |
| 0.25 | 12.90888   | 22.439   | 13.56936  | 22.445   | 14.26377   | 22.450   | 14.99385  | 22.455   | 15.76144   | 22.459   |
| 0.3  | 12.90658   | 26.930   | 13.56718  | 26.937   | 14.26167   | 26.943   | 14.99186  | 26.948   | 15.75950   | 26.953   |
| 0.35 | 12.90398   | 31.423   | 13.56470  | 31.431   | 14.25933   | 31.437   | 14.98960  | 31.443   | 15.75740   | 31.449   |
| 0.4  | 12.90117   | 35.918   | 13.56203  | 35.926   | 14.25614   | 35.933   | 14.98721  | 35.939   | 15.75510   | 35.945   |
| 0.45 | 12.89820   | 40.415   | 13.55918  | 40.423   | 14.25412   | 40.430   | 14.98466  | 40.437   | 15.75270   | 40.443   |
| 0.5  | 12.89518   | 44.914   | 13.55633  | 44.922   | 14.25137   | 44.929   | 14.98205  | 44.936   | 15.75020   | 44.942   |
| 0.55 | 12.89215   | 49.415   | 13.55346  | 49.423   | 14.24863   | 49.430   | 14.97945  | 49.437   | 15.74772   | 49.443   |
| 0.6  | 12.88910   | 53.918   | 13.55062  | 53.926   | 14.24595   | 53.933   | 14.97690  | 53.939   | 15.74529   | 53.945   |
| 0.65 | 12.88637   | 58.423   | 13.54796  | 58.431   | 14.24339   | 58.437   | 14.97448  | 58.443   | 15.74300   | 58.449   |
| 0.7  | 12.88380   | 62.930   | 13.54550  | 62.937   | 14.24106   | 62.943   | 14.97225  | 62.948   | 15.74088   | 62.953   |
| 0.75 | 12.88146   | 67.439   | 13.54322  | 67.445   | 14.23897   | 67.450   | 14.97023  | 67.455   | 15.73898   | 67.459   |
| 0.8  | 12.87949   | 71.949   | 13.54141  | 71.954   | 14.23718   | 71.959   | 14.96855  | 71.963   | 15.73736   | 71.966   |
| 0.85 | 12.87790   | 76.461   | 13.53991  | 76.465   | 14.23573   | 76.468   | 14.96716  | 76.471   | 15.73603   | 76.474   |
| 0.9  | 12.87670   | 80.973   | 13.53878  | 80.976   | 14.23470   | 80.978   | 14.96616  | 80.980   | 15.73510   | 80.982   |
| 0.95 | 12.87600   | 85.486   | 13.53810  | 85.488   | 14.23403   | 85.489   | 14.96556  | 85.490   | 15.73450   | 85.491   |
| 1.0  | 12.87578   | 90.000   | 13.53788  | 90.000   | 14.23382   | 90.000   | 14.96536  | 90.000   | 15.73432   | 90.000   |
| 1.05 | 12.87600   | 94.514   | 13.53810  | 94.512   | 14.23403   | 94.511   | 14.96556  | 94.510   | 15.73450   | 94.509   |
| 1.1  | 12.87670   | 99.027   | 13.53878  | 99.024   | 14.23470   | 99.022   | 14.96616  | 99.020   | 15.73510   | 99.018   |
| 1.15 | 12.87790   | 103.539  | 13.53991  | 103.535  | 14.23573   | 103.532  | 14.96716  | 103.529  | 15.73603   | 103.526  |
| 1.2  | 12.87949   | 108.051  | 13.54141  | 108.046  | 14.23718   | 108.041  | 14.96855  | 108.037  | 15.73736   | 108.034  |
| 1.25 | 12.88146   | 112.561  | 13.54322  | 112.555  | 14.23897   | 112.550  | 14.97023  | 112.545  | 15.73898   | 112.541  |
| 1.3  | 12.88380   | 117.070  | 13.54550  | 117.063  | 14.24106   | 117.057  | 14.97225  | 117.052  | 15.74088   | 117.047  |
| 1.35 | 12.88637   | 121.577  | 13.54796  | 121.569  | 14.24339   | 121.563  | 14.97448  | 121.557  | 15.74300   | 121.551  |
| 1.4  | 12.88910   | 126.082  | 13.55062  | 126.074  | 14.24595   | 126.067  | 14.97690  | 126.061  | 15.74529   | 126.055  |
| 1.45 | 12.89215   | 130.585  | 13.55346  | 130.577  | 14.24863   | 130.570  | 14.97945  | 130.563  | 15.74772   | 130.557  |
| 1.5  | 12.89518   | 135.086  | 13.55633  | 135.078  | 14.25137   | 135.071  | 14.98205  | 135.064  | 15.75020   | 135.058  |
| 1.55 | 12.89820   | 139.585  | 13.55918  | 139.577  | 14.25412   | 139.570  | 14.98466  | 139.563  | 15.75270   | 139.557  |
| 1.6  | 12.90117   | 144.082  | 13.56203  | 144.074  | 14.25614   | 144.067  | 14.98721  | 144.061  | 15.75510   | 144.055  |
| 1.65 | 12.90398   | 148.577  | 13.56470  | 148.569  | 14.25933   | 148.563  | 14.98960  | 148.557  | 15.75740   | 148.551  |
| 1.7  | 12.90658   | 153.070  | 13.56718  | 153.063  | 14.26167   | 153.057  | 14.99186  | 153.052  | 15.75950   | 153.047  |
| 1.75 | 12.90888   | 157.561  | 13.56936  | 157.555  | 14.26377   | 157.550  | 14.99385  | 157.545  | 15.76144   | 157.541  |
| 1.8  | 12.91085   | 162.051  | 13.57123  | 162.046  | 14.26556   | 162.041  | 14.99555  | 162.037  | 15.76303   | 162.034  |
| 1.85 | 12.91240   | 166.539  | 13.57275  | 166.535  | 14.26700   | 166.532  | 14.99692  | 166.529  | 15.76433   | 166.526  |
| 1.9  | 12.91360   | 171.027  | 13.57387  | 171.024  | 14.26805   | 171.022  | 14.99790  | 171.020  | 15.76530   | 171.018  |
| 1.95 | 12.91430   | 175.514  | 13.57455  | 175.512  | 14.26870   | 175.511  | 14.99853  | 175.510  | 15.76587   | 175.509  |
| 2.0  | 12.91456   | 180.000  | 13.57476  | 180.000  | 14.26891   | 180.000  | 14.99874  | 180.000  | 15.76607   | 180.000  |

Example.  $\cosh(3.4 + i0.75) = 14.97023 / 67^\circ.455 = 14.97023 / 67^\circ.27'.18''$ .



TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 3.5$ |          | $x = 3.55$ |          | $x = 3.6$ |          | $x = 3.65$ |          | $x = 3.7$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 16.57282  | 0.000    | 17.42102   | 0.000    | 18.31278  | 0.000    | 19.25033   | 0.000    | 20.23601  | 0.000    |
| 0.05 | 16.57260  | 4.402    | 17.42083   | 4.403    | 18.31260  | 4.493    | 19.25015   | 4.494    | 20.23585  | 4.495    |
| 0.1  | 16.57210  | 8.984    | 17.42030   | 8.985    | 18.31210  | 8.987    | 19.24970   | 8.988    | 20.23540  | 8.989    |
| 0.15 | 16.57120  | 13.476   | 17.41945   | 13.479   | 18.31130  | 13.481   | 19.24890   | 13.483   | 20.23460  | 13.484   |
| 0.2  | 16.56996  | 17.969   | 17.41830   | 17.972   | 18.31020  | 17.975   | 19.24790   | 17.977   | 20.23365  | 17.979   |
| 0.25 | 16.56840  | 22.463   | 17.41680   | 22.467   | 18.30880  | 22.470   | 19.24650   | 22.473   | 20.23240  | 22.475   |
| 0.3  | 16.56660  | 26.958   | 17.41510   | 26.962   | 18.30715  | 26.965   | 19.24496   | 26.969   | 20.23090  | 26.972   |
| 0.35 | 16.56460  | 31.454   | 17.41310   | 31.458   | 18.30530  | 31.462   | 19.24323   | 31.466   | 20.22930  | 31.469   |
| 0.4  | 16.56240  | 35.950   | 17.41110   | 35.955   | 18.30335  | 35.959   | 19.24134   | 35.963   | 20.22750  | 35.967   |
| 0.45 | 16.56010  | 40.448   | 17.40900   | 40.453   | 18.30126  | 40.458   | 19.23940   | 40.462   | 20.22560  | 40.465   |
| 0.5  | 16.55770  | 44.948   | 17.40665   | 44.953   | 18.29910  | 44.957   | 19.23732   | 44.961   | 20.22365  | 44.965   |
| 0.55 | 16.55538  | 49.448   | 17.40441   | 49.453   | 18.29699  | 49.458   | 19.23530   | 49.462   | 20.22170  | 49.465   |
| 0.6  | 16.55307  | 53.950   | 17.40222   | 53.955   | 18.29490  | 53.959   | 19.23331   | 53.963   | 20.21981  | 53.967   |
| 0.65 | 16.55087  | 58.454   | 17.40015   | 58.458   | 18.29292  | 58.462   | 19.23142   | 58.465   | 20.21804  | 58.469   |
| 0.7  | 16.54885  | 62.958   | 17.39822   | 62.962   | 18.29109  | 62.965   | 19.22968   | 62.969   | 20.21639  | 62.972   |
| 0.75 | 16.54702  | 67.463   | 17.39650   | 67.466   | 18.28942  | 67.470   | 19.22813   | 67.473   | 20.21490  | 67.475   |
| 0.8  | 16.54550  | 71.969   | 17.39504   | 71.972   | 18.28805  | 71.975   | 19.22681   | 71.977   | 20.21365  | 71.979   |
| 0.85 | 16.54428  | 76.476   | 17.39386   | 76.479   | 18.28694  | 76.481   | 19.22577   | 76.483   | 20.21268  | 76.484   |
| 0.9  | 16.54337  | 80.984   | 17.39300   | 80.985   | 18.28611  | 80.987   | 19.22496   | 80.988   | 20.21189  | 80.989   |
| 0.95 | 16.54281  | 85.492   | 17.39248   | 85.493   | 18.28560  | 85.493   | 19.22448   | 85.494   | 20.21140  | 85.495   |
| 1.0  | 16.54263  | 90.000   | 17.39230   | 90.000   | 18.28546  | 90.000   | 19.22434   | 90.000   | 20.21129  | 90.000   |
| 1.05 | 16.54281  | 94.508   | 17.39248   | 94.507   | 18.28560  | 94.507   | 19.22448   | 94.506   | 20.21140  | 94.506   |
| 1.1  | 16.54337  | 99.016   | 17.39300   | 99.015   | 18.28611  | 99.013   | 19.22496   | 99.012   | 20.21189  | 99.011   |
| 1.15 | 16.54428  | 103.524  | 17.39386   | 103.522  | 18.28694  | 103.519  | 19.22577   | 103.518  | 20.21268  | 103.516  |
| 1.2  | 16.54550  | 108.031  | 17.39504   | 108.028  | 18.28805  | 108.025  | 19.22681   | 108.023  | 20.21365  | 108.021  |
| 1.25 | 16.54702  | 112.537  | 17.39650   | 112.534  | 18.28942  | 112.530  | 19.22813   | 112.527  | 20.21490  | 112.525  |
| 1.3  | 16.54885  | 117.042  | 17.39822   | 117.038  | 18.29109  | 117.035  | 19.22968   | 117.031  | 20.21639  | 117.028  |
| 1.35 | 16.55087  | 121.547  | 17.40015   | 121.542  | 18.29292  | 121.538  | 19.23142   | 121.535  | 20.21804  | 121.531  |
| 1.4  | 16.55307  | 126.050  | 17.40222   | 126.045  | 18.29490  | 126.041  | 19.23331   | 126.037  | 20.21981  | 126.033  |
| 1.45 | 16.55538  | 130.552  | 17.40441   | 130.547  | 18.29699  | 130.542  | 19.23530   | 130.538  | 20.22170  | 130.535  |
| 1.5  | 16.55770  | 135.052  | 17.40665   | 135.047  | 18.29910  | 135.043  | 19.23732   | 135.039  | 20.22365  | 135.035  |
| 1.55 | 16.56010  | 139.552  | 17.40900   | 139.547  | 18.30126  | 139.542  | 19.23940   | 139.538  | 20.22560  | 139.535  |
| 1.6  | 16.56240  | 144.050  | 17.41110   | 144.045  | 18.30335  | 144.041  | 19.24134   | 144.037  | 20.22750  | 144.033  |
| 1.65 | 16.56460  | 148.546  | 17.41310   | 148.542  | 18.30530  | 148.538  | 19.24323   | 148.534  | 20.22930  | 148.531  |
| 1.7  | 16.56660  | 153.042  | 17.41510   | 153.038  | 18.30715  | 153.035  | 19.24496   | 153.031  | 20.23090  | 153.028  |
| 1.75 | 16.56840  | 157.537  | 17.41680   | 157.533  | 18.30880  | 157.530  | 19.24650   | 157.527  | 20.23240  | 157.525  |
| 1.8  | 16.56996  | 162.031  | 17.41830   | 162.028  | 18.31020  | 162.025  | 19.24790   | 162.023  | 20.23365  | 162.021  |
| 1.85 | 16.57120  | 166.524  | 17.41945   | 166.521  | 18.31130  | 166.519  | 19.24890   | 166.517  | 20.23460  | 166.516  |
| 1.9  | 16.57210  | 171.016  | 17.42030   | 171.015  | 18.31210  | 171.013  | 19.24970   | 171.012  | 20.23540  | 171.011  |
| 1.95 | 16.57260  | 175.508  | 17.42083   | 175.507  | 18.31260  | 175.507  | 19.25015   | 175.506  | 20.23585  | 175.505  |
| 2.0  | 16.57282  | 180.000  | 17.42102   | 180.000  | 18.31278  | 180.000  | 19.25033   | 180.000  | 20.23601  | 180.000  |

Example.  $\cosh(3.65 + i\ 0.05) = 19.25015 / 4^{\circ}.494 = 19.25015 / 4^{\circ}.29'.38''$ .

TABLE XI. HYPERBOLIC COSINES.  $\cosh(x + iq) = r/\gamma$ . CONTINUED

| $x = 3.75$ |          |          | $x = 3.8$ |          | $x = 3.85$ |          | $x = 3.9$ |          | $x = 3.95$ |          |
|------------|----------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$        | $r$      | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0          | 21.27230 | 0.000    | 22.36178  | 0.000    | 23.50717   | 0.000    | 24.71135  | 0.000    | 25.97731   | 0.000    |
| 0.05       | 21.27212 | 4.495    | 22.36163  | 4.495    | 23.50702   | 4.496    | 24.71120  | 4.496    | 25.97720   | 4.497    |
| 0.1        | 21.27170 | 8.990    | 22.36122  | 8.991    | 23.50664   | 8.992    | 24.71090  | 8.993    | 25.97680   | 8.993    |
| 0.15       | 21.27100 | 13.486   | 22.36055  | 13.487   | 23.50600   | 13.488   | 24.71024  | 13.489   | 25.97630   | 13.490   |
| 0.2        | 21.27000 | 17.981   | 22.35962  | 17.983   | 23.50512   | 17.985   | 24.70940  | 17.986   | 25.97550   | 17.988   |
| 0.25       | 21.26885 | 22.478   | 22.35850  | 22.480   | 23.50404   | 22.482   | 24.70840  | 22.483   | 25.97450   | 22.485   |
| 0.3        | 21.26745 | 26.974   | 22.35716  | 26.977   | 23.50277   | 26.979   | 24.70720  | 26.981   | 25.97337   | 26.983   |
| 0.35       | 21.26586 | 31.472   | 22.35555  | 31.474   | 23.50136   | 31.477   | 24.70580  | 31.479   | 25.97206   | 31.481   |
| 0.4        | 21.26415 | 35.970   | 22.35403  | 35.973   | 23.49980   | 35.975   | 24.70440  | 35.978   | 25.97066   | 35.980   |
| 0.45       | 21.26236 | 40.469   | 22.35230  | 40.472   | 23.49820   | 40.474   | 24.70280  | 40.477   | 25.96920   | 40.479   |
| 0.5        | 21.26052 | 44.968   | 22.35060  | 44.971   | 23.49650   | 44.974   | 24.70120  | 44.977   | 25.96770   | 44.979   |
| 0.55       | 21.25869 | 49.469   | 22.34883  | 49.472   | 23.49486   | 49.474   | 24.69964  | 49.477   | 25.96618   | 49.479   |
| 0.6        | 21.25685 | 53.970   | 22.34712  | 53.973   | 23.49322   | 53.975   | 24.69809  | 53.978   | 25.96471   | 53.980   |
| 0.65       | 21.25520 | 58.472   | 22.34550  | 58.474   | 23.49115   | 58.477   | 24.69662  | 58.479   | 25.96333   | 58.481   |
| 0.7        | 21.25362 | 62.974   | 22.34401  | 62.977   | 23.49028   | 62.979   | 24.69528  | 62.981   | 25.96205   | 62.983   |
| 0.75       | 21.25221 | 67.478   | 22.34270  | 67.480   | 23.48900   | 67.482   | 24.69406  | 67.483   | 25.96090   | 67.485   |
| 0.8        | 21.25102 | 71.981   | 22.34153  | 71.983   | 23.48791   | 71.985   | 24.69302  | 71.986   | 25.95991   | 71.988   |
| 0.85       | 21.25006 | 76.486   | 22.34061  | 76.487   | 23.48704   | 76.488   | 24.69223  | 76.489   | 25.95911   | 76.490   |
| 0.9        | 21.24935 | 80.990   | 22.33995  | 80.991   | 23.48640   | 80.992   | 24.69159  | 80.993   | 25.95854   | 80.993   |
| 0.95       | 21.24891 | 85.495   | 22.33932  | 85.496   | 23.48601   | 85.496   | 24.69120  | 85.496   | 25.95820   | 85.497   |
| 1.0        | 21.24878 | 90.000   | 22.33941  | 90.000   | 23.48589   | 90.000   | 24.69110  | 90.000   | 25.95806   | 90.000   |
| 1.05       | 21.24891 | 94.505   | 22.33952  | 94.504   | 23.48601   | 94.504   | 24.69120  | 94.504   | 25.95820   | 94.503   |
| 1.1        | 21.24935 | 99.010   | 22.33995  | 99.009   | 23.48640   | 99.008   | 24.69159  | 99.007   | 25.95854   | 99.007   |
| 1.15       | 21.25006 | 103.514  | 22.34061  | 103.513  | 23.48704   | 103.512  | 24.69223  | 103.511  | 25.95911   | 103.510  |
| 1.2        | 21.25102 | 108.019  | 22.34153  | 108.017  | 23.48791   | 108.015  | 24.69302  | 108.014  | 25.95991   | 108.013  |
| 1.25       | 21.25221 | 112.522  | 22.34270  | 112.520  | 23.48900   | 112.518  | 24.69406  | 112.517  | 25.96090   | 112.515  |
| 1.3        | 21.25362 | 117.026  | 22.34401  | 117.023  | 23.49028   | 117.021  | 24.69528  | 117.019  | 25.96205   | 117.017  |
| 1.35       | 21.25520 | 121.528  | 22.34550  | 121.526  | 23.49115   | 121.523  | 24.69662  | 121.521  | 25.96333   | 121.519  |
| 1.4        | 21.25685 | 126.030  | 22.34712  | 126.027  | 23.49322   | 126.025  | 24.69809  | 126.022  | 25.96471   | 126.020  |
| 1.45       | 21.25869 | 130.531  | 22.34883  | 130.528  | 23.49486   | 130.525  | 24.69964  | 130.523  | 25.96618   | 130.521  |
| 1.5        | 21.26052 | 135.032  | 22.35060  | 135.029  | 23.49650   | 135.026  | 24.70120  | 135.023  | 25.96770   | 135.021  |
| 1.55       | 21.26236 | 139.531  | 22.35230  | 139.528  | 23.49820   | 139.526  | 24.70280  | 139.523  | 25.96920   | 139.521  |
| 1.6        | 21.26415 | 144.030  | 22.35403  | 144.027  | 23.49980   | 144.025  | 24.70440  | 144.022  | 25.97066   | 144.020  |
| 1.65       | 21.26586 | 148.528  | 22.35555  | 148.526  | 23.50136   | 148.523  | 24.70580  | 148.521  | 25.97206   | 148.519  |
| 1.7        | 21.26745 | 153.026  | 22.35716  | 153.023  | 23.50277   | 153.021  | 24.70720  | 153.019  | 25.97337   | 153.017  |
| 1.75       | 21.26885 | 157.522  | 22.35850  | 157.520  | 23.50404   | 157.518  | 24.70840  | 157.517  | 25.97450   | 157.515  |
| 1.8        | 21.27000 | 162.019  | 22.35962  | 162.017  | 23.50512   | 162.015  | 24.70940  | 162.014  | 25.97550   | 162.012  |
| 1.85       | 21.27100 | 166.514  | 22.36055  | 166.513  | 23.50600   | 166.512  | 24.71024  | 166.511  | 25.97630   | 166.510  |
| 1.9        | 21.27170 | 171.010  | 22.36122  | 171.009  | 23.50664   | 171.008  | 24.71090  | 171.007  | 25.97680   | 171.007  |
| 1.95       | 21.27212 | 175.505  | 22.36163  | 175.505  | 23.50702   | 175.504  | 24.71120  | 175.504  | 25.97720   | 175.503  |
| 2.0        | 21.27230 | 180.000  | 22.36178  | 180.000  | 23.50717   | 180.000  | 24.71135  | 180.000  | 25.97731   | 180.000  |

Example.  $\cosh(3.85 + i \underline{1.05}) = 23.48601 / \underline{94^\circ.504} = 23.48601 / \underline{94^\circ.30'.14''}$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ 

| $q$  | $x = 0$  |          | $x = 0.05$ |          | $x = 0.1$ |          | $x = 0.15$ |          | $x = 0.2$ |          |
|------|----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
|      | $r$      | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 0.00     | 90       | 0.04996    | 0.00     | 0.09967   | 0.00     | 0.14889    | 0.00     | 0.19738   | 0.00     |
| 0.05 | 0.07870  | 90       | 0.09322    | 57.368   | 0.12699   | 37.846   | 0.16840    | 27.190   | 0.21246   | 20.849   |
| 0.1  | 0.15838  | 90       | 0.16607    | 72.040   | 0.18711   | 56.914   | 0.21732    | 45.420   | 0.25294   | 36.955   |
| 0.15 | 0.24008  | 90       | 0.24520    | 77.558   | 0.25987   | 66.084   | 0.28232    | 56.148   | 0.31045   | 47.863   |
| 0.2  | 0.32492  | 90       | 0.32870    | 80.329   | 0.33969   | 71.092   | 0.35699    | 62.612   | 0.37939   | 55.054   |
| 0.25 | 0.41421  | 90       | 0.41727    | 81.937   | 0.42568   | 74.107   | 0.43932    | 66.700   | 0.45731   | 59.848   |
| 0.3  | 0.50953  | 90       | 0.51180    | 82.942   | 0.51851   | 76.025   | 0.52931    | 69.373   | 0.54368   | 63.082   |
| 0.35 | 0.61280  | 90       | 0.61455    | 83.585   | 0.61970   | 77.267   | 0.62802    | 71.131   | 0.63915   | 65.250   |
| 0.4  | 0.72654  | 90       | 0.72778    | 83.987   | 0.73143   | 78.047   | 0.73734    | 72.245   | 0.74525   | 66.641   |
| 0.45 | 0.85408  | 90       | 0.85476    | 84.209   | 0.85678   | 78.478   | 0.86004    | 72.864   | 0.86439   | 67.419   |
| 0.5  | 1.00000  | 90       | 1.00000    | 84.279   | 1.00000   | 78.616   | 1.00000    | 73.064   | 1.00000   | 67.670   |
| 0.55 | 1.17085  | 90       | 1.16991    | 84.209   | 1.16717   | 78.478   | 1.16274    | 72.864   | 1.15688   | 67.419   |
| 0.6  | 1.37638  | 90       | 1.37404    | 83.987   | 1.36718   | 78.047   | 1.35623    | 72.245   | 1.34183   | 66.641   |
| 0.65 | 1.63185  | 90       | 1.62722    | 83.585   | 1.61369   | 77.267   | 1.59231    | 71.131   | 1.56459   | 65.250   |
| 0.7  | 1.96261  | 90       | 1.95388    | 82.942   | 1.92859   | 76.025   | 1.88925    | 69.373   | 1.83933   | 63.082   |
| 0.75 | 2.41421  | 90       | 2.39735    | 81.937   | 2.34919   | 74.107   | 2.27623    | 66.700   | 2.18669   | 59.848   |
| 0.8  | 3.07768  | 90       | 3.04234    | 80.329   | 2.94391   | 71.092   | 2.80120    | 62.612   | 2.63581   | 55.054   |
| 0.85 | 4.16530  | 90       | 4.07824    | 77.558   | 3.84810   | 66.084   | 3.54212    | 56.148   | 3.22115   | 47.863   |
| 0.9  | 6.31375  | 90       | 6.02149    | 72.040   | 5.34442   | 56.914   | 4.60155    | 45.420   | 3.95347   | 36.955   |
| 0.95 | 12.70620 | 90       | 10.72750   | 57.368   | 7.87464   | 37.846   | 5.93842    | 27.190   | 4.70673   | 20.849   |
| 1.0  | $\infty$ | 90       | 20.01667   | 0.00     | 10.03331  | 0.00     | 6.71659    | 0.00     | 5.06649   | 0.00     |
| 1.05 | 12.70620 | 90       | 10.72750   | 57.368   | 7.87464   | 37.846   | 5.93842    | 27.190   | 4.70673   | 20.849   |
| 1.1  | 6.31375  | 90       | 6.02149    | 72.040   | 5.34442   | 56.914   | 4.60155    | 45.420   | 3.95347   | 36.955   |
| 1.15 | 4.16530  | 90       | 4.07824    | 77.558   | 3.84810   | 66.084   | 3.54212    | 56.148   | 3.22115   | 47.863   |
| 1.2  | 3.07768  | 90       | 3.04234    | 80.329   | 2.94391   | 71.092   | 2.80120    | 62.612   | 2.63581   | 55.054   |
| 1.25 | 2.41421  | 90       | 2.39735    | 81.937   | 2.34919   | 74.107   | 2.27623    | 66.700   | 2.18669   | 59.848   |
| 1.3  | 1.96261  | 90       | 1.95388    | 82.942   | 1.92859   | 76.025   | 1.88925    | 69.373   | 1.83933   | 63.082   |
| 1.35 | 1.63185  | 90       | 1.62722    | 83.585   | 1.61369   | 77.267   | 1.59231    | 71.131   | 1.56459   | 65.250   |
| 1.4  | 1.37638  | 90       | 1.37404    | 83.987   | 1.36718   | 78.047   | 1.35623    | 72.245   | 1.34183   | 66.641   |
| 1.45 | 1.17085  | 90       | 1.16991    | 84.209   | 1.16717   | 78.478   | 1.16274    | 72.864   | 1.15688   | 67.419   |
| 1.5  | 1.00000  | 90       | 1.00000    | 84.279   | 1.00000   | 78.616   | 1.00000    | 73.064   | 1.00000   | 67.670   |
| 1.55 | 0.85408  | 90       | 0.85476    | 84.209   | 0.85678   | 78.478   | 0.86004    | 72.864   | 0.86439   | 67.419   |
| 1.6  | 0.72654  | 90       | 0.72778    | 83.987   | 0.73143   | 78.047   | 0.73734    | 72.245   | 0.74525   | 66.641   |
| 1.65 | 0.61280  | 90       | 0.61455    | 83.585   | 0.61970   | 77.267   | 0.62802    | 71.131   | 0.63915   | 65.250   |
| 1.7  | 0.50953  | 90       | 0.51180    | 82.942   | 0.51851   | 76.025   | 0.52931    | 69.373   | 0.54368   | 63.082   |
| 1.75 | 0.41421  | 90       | 0.41727    | 81.937   | 0.42568   | 74.107   | 0.43932    | 66.700   | 0.45731   | 59.848   |
| 1.8  | 0.32492  | 90       | 0.32870    | 80.329   | 0.33969   | 71.092   | 0.35699    | 62.612   | 0.37939   | 55.054   |
| 1.85 | 0.24008  | 90       | 0.24520    | 77.558   | 0.25987   | 66.084   | 0.28232    | 56.148   | 0.31045   | 47.863   |
| 1.9  | 0.15838  | 90       | 0.16607    | 72.040   | 0.18711   | 56.914   | 0.21732    | 45.420   | 0.25294   | 36.955   |
| 1.95 | 0.07870  | 90       | 0.09322    | 57.368   | 0.12699   | 37.846   | 0.16840    | 27.190   | 0.21246   | 20.849   |
| 2.0  | 0.00     | 90       | 0.04996    | 0.00     | 0.09967   | 0.00     | 0.14889    | 0.00     | 0.19738   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(0.1 + i 0.25) = 0.42568 / 74^\circ.107 = 0.42568 / 74^\circ.06'.25''$ .  
 $\tanh(0.1 + i 1.2) = 2.94391 / 71^\circ.092 = 2.94391 / 71^\circ.05'.31''$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 0.25$ |          | $x = 0.3$ |          | $x = 0.35$ |          | $x = 0.4$ |          | $x = 0.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
|      | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 0.24492    | 0.00     | 0.29131   | 0.00     | 0.33638    | 0.00     | 0.37995   | 0.00     | 0.42190    | 0.00     |
| 0.05 | 0.25721    | 16.710   | 0.30168   | 13.805   | 0.34534    | 11.652   | 0.38784   | 9.990    | 0.42894    | 8.665    |
| 0.1  | 0.29145    | 30.669   | 0.33123   | 25.891   | 0.37127    | 22.164   | 0.41090   | 19.185   | 0.44965    | 16.753   |
| 0.15 | 0.34237    | 41.063   | 0.37657   | 35.492   | 0.41192    | 30.900   | 0.44758   | 27.076   | 0.48295    | 23.858   |
| 0.2  | 0.40561    | 48.442   | 0.43445   | 42.715   | 0.46491    | 37.770   | 0.49617   | 33.498   | 0.52758    | 29.796   |
| 0.25 | 0.47875    | 53.612   | 0.50275   | 48.001   | 0.52849    | 42.989   | 0.55525   | 38.527   | 0.58242    | 34.560   |
| 0.3  | 0.56098    | 57.214   | 0.58056   | 51.799   | 0.60177    | 46.843   | 0.62401   | 42.331   | 0.64675    | 38.242   |
| 0.35 | 0.65262    | 59.679   | 0.66796   | 54.453   | 0.68466    | 49.590   | 0.70225   | 45.093   | 0.72031    | 40.958   |
| 0.4  | 0.75486    | 61.281   | 0.76580   | 56.200   | 0.77774    | 51.423   | 0.79033   | 46.961   | 0.80327    | 42.815   |
| 0.45 | 0.86968    | 62.184   | 0.87570   | 57.194   | 0.88225    | 52.474   | 0.88914   | 48.039   | 0.89620    | 43.896   |
| 0.5  | 1.00000    | 62.476   | 1.00000   | 57.518   | 1.00000    | 52.817   | 1.00000   | 48.392   | 1.00000    | 44.250   |
| 0.55 | 1.14985    | 62.184   | 1.14195   | 57.194   | 1.13347    | 52.474   | 1.12469   | 48.039   | 1.11583    | 43.896   |
| 0.6  | 1.32476    | 61.281   | 1.30582   | 56.200   | 1.28577    | 51.423   | 1.26529   | 46.961   | 1.24492    | 42.815   |
| 0.65 | 1.53228    | 59.679   | 1.49710   | 54.453   | 1.46059    | 49.590   | 1.42400   | 45.093   | 1.38830    | 40.958   |
| 0.7  | 1.78259    | 57.214   | 1.72246   | 51.799   | 1.66176    | 46.843   | 1.60255   | 42.331   | 1.54620    | 38.242   |
| 0.75 | 2.08878    | 53.612   | 1.98907   | 48.001   | 1.89219    | 42.989   | 1.80100   | 38.527   | 1.71608    | 34.560   |
| 0.8  | 2.46545    | 48.442   | 2.30177   | 42.715   | 2.15096    | 37.770   | 2.01545   | 33.498   | 1.89545    | 29.796   |
| 0.85 | 2.92081    | 41.063   | 2.65553   | 35.492   | 2.42764    | 30.900   | 2.23422   | 27.076   | 2.07060    | 23.858   |
| 0.9  | 3.43113    | 30.669   | 3.01903   | 25.891   | 2.69344    | 22.164   | 2.43370   | 19.185   | 2.22397    | 16.753   |
| 0.95 | 3.88795    | 16.710   | 3.31480   | 13.805   | 2.89571    | 11.652   | 2.57838   | 9.990    | 2.33132    | 8.665    |
| 1.0  | 4.08299    | 0.00     | 3.43274   | 0.00     | 2.97287    | 0.00     | 2.63193   | 0.00     | 2.37024    | 0.00     |
| 1.05 | 3.88795    | 16.710   | 3.31480   | 13.805   | 2.89571    | 11.652   | 2.57838   | 9.990    | 2.33132    | 8.665    |
| 1.1  | 3.43113    | 30.669   | 3.01903   | 25.891   | 2.69344    | 22.164   | 2.43370   | 19.185   | 2.22397    | 16.753   |
| 1.15 | 2.92081    | 41.063   | 2.65553   | 35.492   | 2.42764    | 30.900   | 2.23422   | 27.076   | 2.07060    | 23.858   |
| 1.2  | 2.46545    | 48.442   | 2.30177   | 42.715   | 2.15096    | 37.770   | 2.01545   | 33.498   | 1.89545    | 29.796   |
| 1.25 | 2.08878    | 53.612   | 1.98907   | 48.001   | 1.89219    | 42.989   | 1.80100   | 38.527   | 1.71608    | 34.560   |
| 1.3  | 1.78259    | 57.214   | 1.72246   | 51.799   | 1.66176    | 46.843   | 1.60255   | 42.331   | 1.54620    | 38.242   |
| 1.35 | 1.53228    | 59.679   | 1.49710   | 54.453   | 1.46059    | 49.590   | 1.42400   | 45.093   | 1.38830    | 40.958   |
| 1.4  | 1.32476    | 61.281   | 1.30582   | 56.200   | 1.28577    | 51.423   | 1.26529   | 46.961   | 1.24492    | 42.815   |
| 1.45 | 1.14985    | 62.184   | 1.14195   | 57.194   | 1.13347    | 52.474   | 1.12469   | 48.039   | 1.11583    | 43.896   |
| 1.5  | 1.00000    | 62.476   | 1.00000   | 57.518   | 1.00000    | 52.817   | 1.00000   | 48.392   | 1.00000    | 44.250   |
| 1.55 | 0.86968    | 62.184   | 0.87570   | 57.194   | 0.88225    | 52.474   | 0.88914   | 48.039   | 0.89620    | 43.896   |
| 1.6  | 0.75486    | 61.281   | 0.76580   | 56.200   | 0.77774    | 51.423   | 0.79033   | 46.961   | 0.80327    | 42.815   |
| 1.65 | 0.65262    | 59.679   | 0.66796   | 54.453   | 0.68466    | 49.590   | 0.70225   | 45.093   | 0.72031    | 40.958   |
| 1.7  | 0.56098    | 57.214   | 0.58056   | 51.799   | 0.60177    | 46.843   | 0.62401   | 42.331   | 0.64675    | 38.242   |
| 1.75 | 0.47875    | 53.612   | 0.50275   | 48.001   | 0.52849    | 42.989   | 0.55525   | 38.527   | 0.58242    | 34.560   |
| 1.8  | 0.40561    | 48.442   | 0.43445   | 42.715   | 0.46491    | 37.770   | 0.49617   | 33.498   | 0.52758    | 29.796   |
| 1.85 | 0.34237    | 41.063   | 0.37657   | 35.492   | 0.41192    | 30.900   | 0.44758   | 27.076   | 0.48295    | 23.858   |
| 1.9  | 0.29145    | 30.669   | 0.33123   | 25.891   | 0.37127    | 22.164   | 0.41090   | 19.185   | 0.44965    | 16.753   |
| 1.95 | 0.25721    | 16.710   | 0.30168   | 13.805   | 0.34534    | 11.652   | 0.38784   | 9.990    | 0.42894    | 8.665    |
| 2.0  | 0.24492    | 0.00     | 0.29131   | 0.00     | 0.33638    | 0.00     | 0.37995   | 0.00     | 0.42190    | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(0.4 + i0) = 0.37995/\angle 0^\circ$ .

$\tanh(0.45 + i1.1) = 2.22397/\angle 16^\circ.753 = 2.22397/\angle 16^\circ.45'.11''$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 0.5$ |          | $x = 0.55$ |          | $x = 0.6$ |          | $x = 0.65$ |          | $x = 0.7$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
|      | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 0.46211   | 0.00     | 0.50052    | 0.00     | 0.53704   | 0.00     | 0.57167    | 0.00     | 0.60437   | 0.00     |
| 0.05 | 0.46846   | 7.582    | 0.50628    | 6.680    | 0.54230   | 5.917    | 0.57648    | 5.263    | 0.60878   | 4.696    |
| 0.1  | 0.48720   | 14.732   | 0.52334    | 13.027   | 0.55790   | 11.570   | 0.59079    | 10.312   | 0.62194   | 9.217    |
| 0.15 | 0.51758   | 21.122   | 0.55115    | 18.773   | 0.58344   | 16.739   | 0.61428    | 14.966   | 0.64357   | 13.409   |
| 0.2  | 0.55865   | 26.572   | 0.58900    | 23.753   | 0.61835   | 21.276   | 0.64650    | 19.090   | 0.67331   | 17.153   |
| 0.25 | 0.60952   | 31.035   | 0.63616    | 27.897   | 0.66205   | 25.101   | 0.68606    | 22.604   | 0.71076   | 20.372   |
| 0.3  | 0.66956   | 34.544   | 0.69209    | 31.204   | 0.71405   | 28.190   | 0.73523    | 25.471   | 0.75548   | 23.017   |
| 0.35 | 0.73847   | 37.169   | 0.75645    | 33.707   | 0.77399   | 30.553   | 0.79092    | 27.683   | 0.80711   | 25.074   |
| 0.4  | 0.81628   | 38.983   | 0.82914    | 35.453   | 0.84168   | 32.214   | 0.85377    | 29.248   | 0.86531   | 26.539   |
| 0.45 | 0.90328   | 40.046   | 0.91025    | 36.482   | 0.91703   | 33.198   | 0.92354    | 30.180   | 0.92973   | 27.414   |
| 0.5  | 1.00000   | 40.395   | 1.00000    | 36.822   | 1.00000   | 33.524   | 1.00000    | 30.489   | 1.00000   | 27.705   |
| 0.55 | 1.10708   | 40.046   | 1.09860    | 36.482   | 1.09048   | 33.198   | 1.08279    | 30.180   | 1.07558   | 27.414   |
| 0.6  | 1.22508   | 38.983   | 1.20607    | 35.453   | 1.18810   | 32.214   | 1.17128    | 29.248   | 1.15566   | 26.539   |
| 0.65 | 1.35414   | 37.169   | 1.32197    | 33.707   | 1.29201   | 30.553   | 1.26434    | 27.683   | 1.23898   | 25.074   |
| 0.7  | 1.49352   | 34.544   | 1.44490    | 31.204   | 1.40047   | 28.190   | 1.36012    | 25.471   | 1.32366   | 23.017   |
| 0.75 | 1.64064   | 31.035   | 1.57193    | 27.897   | 1.51047   | 25.101   | 1.45568    | 22.604   | 1.40695   | 20.372   |
| 0.8  | 1.79004   | 26.572   | 1.69780    | 23.753   | 1.61722   | 21.276   | 1.54680    | 19.090   | 1.48519   | 17.153   |
| 0.85 | 1.93206   | 21.122   | 1.81438    | 18.773   | 1.71398   | 16.739   | 1.62793    | 14.966   | 1.55384   | 13.409   |
| 0.9  | 2.05254   | 14.732   | 1.91081    | 13.027   | 1.79243   | 11.570   | 1.69266    | 10.312   | 1.60788   | 9.217    |
| 0.95 | 2.13405   | 7.582    | 1.97520    | 6.680    | 1.84400   | 5.917    | 1.73467    | 5.263    | 1.64262   | 4.696    |
| 1.0  | 2.16395   | 0.00     | 1.99792    | 0.00     | 1.86202   | 0.00     | 1.74926    | 0.00     | 1.65462   | 0.00     |
| 1.05 | 2.13405   | 7.582    | 1.97520    | 6.680    | 1.84400   | 5.917    | 1.73467    | 5.263    | 1.64262   | 4.696    |
| 1.1  | 2.05254   | 14.732   | 1.91081    | 13.027   | 1.79243   | 11.570   | 1.69266    | 10.312   | 1.60788   | 9.217    |
| 1.15 | 1.93206   | 21.122   | 1.81438    | 18.773   | 1.71398   | 16.739   | 1.62793    | 14.966   | 1.55384   | 13.409   |
| 1.2  | 1.79004   | 26.572   | 1.69780    | 23.753   | 1.61722   | 21.276   | 1.54680    | 19.090   | 1.48519   | 17.153   |
| 1.25 | 1.64064   | 31.035   | 1.57193    | 27.897   | 1.51047   | 25.101   | 1.45568    | 22.604   | 1.40695   | 20.372   |
| 1.3  | 1.49352   | 34.544   | 1.44490    | 31.204   | 1.40047   | 28.190   | 1.36012    | 25.471   | 1.32366   | 23.017   |
| 1.35 | 1.35414   | 37.169   | 1.32197    | 33.707   | 1.29201   | 30.553   | 1.26434    | 27.683   | 1.23898   | 25.074   |
| 1.4  | 1.22508   | 38.983   | 1.20607    | 35.453   | 1.18810   | 32.214   | 1.17128    | 29.248   | 1.15566   | 26.539   |
| 1.45 | 1.10708   | 40.046   | 1.09860    | 36.482   | 1.09048   | 33.198   | 1.08279    | 30.180   | 1.07558   | 27.414   |
| 1.5  | 1.00000   | 40.395   | 1.00000    | 36.822   | 1.00000   | 33.524   | 1.00000    | 30.489   | 1.00000   | 27.705   |
| 1.55 | 0.90328   | 40.046   | 0.91025    | 36.482   | 0.91703   | 33.198   | 0.92354    | 30.180   | 0.92973   | 27.414   |
| 1.6  | 0.81628   | 38.983   | 0.82914    | 35.453   | 0.84168   | 32.214   | 0.85377    | 29.248   | 0.86531   | 26.539   |
| 1.65 | 0.73847   | 37.169   | 0.75645    | 33.707   | 0.77399   | 30.553   | 0.79092    | 27.683   | 0.80711   | 25.074   |
| 1.7  | 0.66956   | 34.544   | 0.69209    | 31.204   | 0.71405   | 28.190   | 0.73523    | 25.471   | 0.75548   | 23.017   |
| 1.75 | 0.60952   | 31.035   | 0.63616    | 27.897   | 0.66205   | 25.101   | 0.68606    | 22.604   | 0.71076   | 20.372   |
| 1.8  | 0.55865   | 26.572   | 0.58900    | 23.753   | 0.61835   | 21.276   | 0.64650    | 19.090   | 0.67331   | 17.153   |
| 1.85 | 0.51758   | 21.122   | 0.55115    | 18.773   | 0.58344   | 16.739   | 0.61428    | 14.966   | 0.64357   | 13.409   |
| 1.9  | 0.48720   | 14.732   | 0.52334    | 13.027   | 0.55790   | 11.570   | 0.59079    | 10.312   | 0.62194   | 9.217    |
| 1.95 | 0.46846   | 7.582    | 0.50628    | 6.680    | 0.54230   | 5.917    | 0.57648    | 5.263    | 0.60878   | 4.696    |
| 2.0  | 0.46211   | 0.00     | 0.50052    | 0.00     | 0.53704   | 0.00     | 0.57167    | 0.00     | 0.60437   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(0.7 + i\mathbf{0.7}) = 1.32366 / \underline{23.017} = 1.32366 / \underline{23.017.017.}$

$\tanh(0.6 + i\mathbf{1.5}) = 1.0000 / \underline{33.524} = 1.0000 / \underline{33.317.267.}$

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 0.75$ |          | $x = 0.8$ |          | $x = 0.85$ |          | $x = 0.9$ |          | $x = 0.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
|      | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 0.63515    | 0.00     | 0.66403   | 0.00     | 0.69107    | 0.00     | 0.71629   | 0.00     | 0.73978    | 0.00     |
| 0.05 | 0.63921    | 4.202    | 0.66777   | 3.767    | 0.69451    | 3.384    | 0.71947   | 3.043    | 0.74269    | 2.741    |
| 0.1  | 0.65131    | 8.257    | 0.67892   | 7.411    | 0.70478    | 6.662    | 0.72892   | 5.995    | 0.75141    | 5.401    |
| 0.15 | 0.67125    | 12.036   | 0.69730   | 10.819   | 0.72172    | 9.737    | 0.74453   | 8.771    | 0.76578    | 7.909    |
| 0.2  | 0.69871    | 15.432   | 0.72264   | 13.898   | 0.74509    | 12.526   | 0.76607   | 11.297   | 0.78561    | 10.196   |
| 0.25 | 0.73333    | 18.371   | 0.75461   | 16.576   | 0.77459    | 14.964   | 0.79326   | 13.513   | 0.81065    | 12.208   |
| 0.3  | 0.77471    | 20.804   | 0.79285   | 18.807   | 0.80986    | 17.004   | 0.82576   | 15.375   | 0.84054    | 13.904   |
| 0.35 | 0.82247    | 22.707   | 0.83695   | 20.559   | 0.85051    | 18.613   | 0.86316   | 16.848   | 0.87492    | 15.250   |
| 0.4  | 0.87623    | 24.068   | 0.88650   | 21.819   | 0.89611    | 19.773   | 0.90504   | 17.914   | 0.91333    | 16.226   |
| 0.45 | 0.93557    | 24.885   | 0.94104   | 22.576   | 0.94614    | 20.472   | 0.95086   | 18.557   | 0.95523    | 16.816   |
| 0.5  | 1.00000    | 25.157   | 1.00000   | 22.828   | 1.00000    | 20.706   | 1.00000   | 18.772   | 1.00000    | 17.013   |
| 0.55 | 1.06887    | 24.885   | 1.06265   | 22.576   | 1.05693    | 20.472   | 1.05168   | 18.557   | 1.04687    | 16.816   |
| 0.6  | 1.14125    | 24.068   | 1.12803   | 21.819   | 1.11594    | 19.773   | 1.10492   | 17.914   | 1.09490    | 16.226   |
| 0.65 | 1.21585    | 22.707   | 1.19482   | 20.559   | 1.17576    | 18.613   | 1.15853   | 16.848   | 1.14297    | 15.250   |
| 0.7  | 1.29081    | 20.804   | 1.26128   | 18.807   | 1.23478    | 17.004   | 1.21101   | 15.375   | 1.18971    | 13.904   |
| 0.75 | 1.36365    | 18.371   | 1.32519   | 16.576   | 1.29101    | 14.964   | 1.26062   | 13.513   | 1.23358    | 12.208   |
| 0.8  | 1.43121    | 15.432   | 1.38382   | 13.898   | 1.34212    | 12.526   | 1.30536   | 11.297   | 1.27289    | 10.196   |
| 0.85 | 1.48976    | 12.036   | 1.43411   | 10.819   | 1.38559    | 9.737    | 1.34313   | 8.771    | 1.30586    | 7.909    |
| 0.9  | 1.53537    | 8.257    | 1.47293   | 7.411    | 1.41889    | 6.662    | 1.37189   | 5.995    | 1.33083    | 5.401    |
| 0.95 | 1.56444    | 4.202    | 1.49751   | 3.767    | 1.43986    | 3.384    | 1.38992   | 3.043    | 1.34645    | 2.741    |
| 1.0  | 1.57443    | 0.00     | 1.50594   | 0.00     | 1.44703    | 0.00     | 1.39606   | 0.00     | 1.35175    | 0.00     |
| 1.05 | 1.56444    | 4.202    | 1.49751   | 3.767    | 1.43986    | 3.384    | 1.38992   | 3.043    | 1.34645    | 2.741    |
| 1.1  | 1.53537    | 8.257    | 1.47293   | 7.411    | 1.41889    | 6.662    | 1.37189   | 5.995    | 1.33083    | 5.401    |
| 1.15 | 1.48976    | 12.036   | 1.43411   | 10.819   | 1.38559    | 9.737    | 1.34313   | 8.771    | 1.30586    | 7.909    |
| 1.2  | 1.43121    | 15.432   | 1.38382   | 13.898   | 1.34212    | 12.526   | 1.30536   | 11.297   | 1.27289    | 10.196   |
| 1.25 | 1.36365    | 18.371   | 1.32519   | 16.576   | 1.29101    | 14.964   | 1.26062   | 13.513   | 1.23358    | 12.208   |
| 1.3  | 1.29081    | 20.804   | 1.26128   | 18.807   | 1.23478    | 17.004   | 1.21101   | 15.375   | 1.18971    | 13.904   |
| 1.35 | 1.21585    | 22.707   | 1.19482   | 20.559   | 1.17576    | 18.613   | 1.15853   | 16.848   | 1.14297    | 15.250   |
| 1.4  | 1.14125    | 24.068   | 1.12803   | 21.819   | 1.11594    | 19.773   | 1.10492   | 17.914   | 1.09490    | 16.226   |
| 1.45 | 1.06887    | 24.885   | 1.06265   | 22.576   | 1.05693    | 20.472   | 1.05168   | 18.557   | 1.04687    | 16.816   |
| 1.5  | 1.00000    | 25.157   | 1.00000   | 22.828   | 1.00000    | 20.706   | 1.00000   | 18.772   | 1.00000    | 17.013   |
| 1.55 | 0.93557    | 24.885   | 0.94104   | 22.576   | 0.94614    | 20.472   | 0.95086   | 18.557   | 0.95523    | 16.816   |
| 1.6  | 0.87623    | 24.068   | 0.88650   | 21.819   | 0.89611    | 19.773   | 0.90504   | 17.914   | 0.91333    | 16.226   |
| 1.65 | 0.82247    | 22.707   | 0.83695   | 20.559   | 0.85051    | 18.613   | 0.86316   | 16.848   | 0.87492    | 15.250   |
| 1.7  | 0.77471    | 20.804   | 0.79285   | 18.807   | 0.80986    | 17.004   | 0.82576   | 15.375   | 0.84054    | 13.904   |
| 1.75 | 0.73333    | 18.371   | 0.75461   | 16.576   | 0.77459    | 14.964   | 0.79326   | 13.513   | 0.81065    | 12.208   |
| 1.8  | 0.69871    | 15.432   | 0.72264   | 13.898   | 0.74509    | 12.526   | 0.76607   | 11.297   | 0.78561    | 10.196   |
| 1.85 | 0.67125    | 12.036   | 0.69730   | 10.819   | 0.72172    | 9.737    | 0.74453   | 8.771    | 0.76578    | 7.909    |
| 1.9  | 0.65131    | 8.257    | 0.67892   | 7.411    | 0.70478    | 6.662    | 0.72892   | 5.995    | 0.75141    | 5.401    |
| 1.95 | 0.63921    | 4.202    | 0.66777   | 3.767    | 0.69451    | 3.384    | 0.71947   | 3.043    | 0.74269    | 2.741    |
| 2.0  | 0.63515    | 0.00     | 0.66403   | 0.00     | 0.69107    | 0.00     | 0.71629   | 0.00     | 0.73978    | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(0.9 + i1.0) = 1.39606/\underline{0^\circ}$ .

$\tanh(0.95 + i1.55) = 0.95523/\sqrt{16^\circ.816} = 0.95523/\sqrt{16^\circ.48'.58''}$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 1.0$ |          | $x = 1.05$ |          | $x = 1.1$ |          | $x = 1.15$ |          | $x = 1.2$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 0.76159   | 0.00     | 0.78181    | 0.00     | 0.80050   | 0.00     | 0.81775    | 0.00     | 0.83365   | 0.00     |
| 0.05 | 0.76428   | 2.470    | 0.78427    | 2.228    | 0.80277   | 2.010    | 0.81084    | 1.815    | 0.83557   | 1.639    |
| 0.1  | 0.77229   | 4.870    | 0.79164    | 4.394    | 0.80954   | 3.966    | 0.82605    | 3.582    | 0.84127   | 3.235    |
| 0.15 | 0.78552   | 7.134    | 0.80380    | 6.440    | 0.82071   | 5.815    | 0.83630    | 5.254    | 0.85067   | 4.747    |
| 0.2  | 0.80376   | 9.206    | 0.82058    | 8.315    | 0.83611   | 7.513    | 0.85043    | 6.790    | 0.86361   | 6.138    |
| 0.25 | 0.82678   | 11.032   | 0.84172    | 9.971    | 0.85551   | 9.015    | 0.86822    | 8.151    | 0.87991   | 7.371    |
| 0.3  | 0.85425   | 12.575   | 0.86693    | 11.373   | 0.87862   | 10.288   | 0.88939    | 9.306    | 0.89927   | 8.419    |
| 0.35 | 0.88580   | 13.802   | 0.89585    | 12.492   | 0.90509   | 11.305   | 0.91359    | 10.231   | 0.92138   | 9.258    |
| 0.4  | 0.92098   | 14.694   | 0.92802    | 13.305   | 0.93449   | 12.045   | 0.94042    | 10.904   | 0.94585   | 9.870    |
| 0.45 | 0.95925   | 15.233   | 0.96294    | 13.798   | 0.96632   | 12.495   | 0.96941    | 11.313   | 0.97223   | 10.242   |
| 0.5  | 1.00000   | 15.414   | 1.00000    | 13.963   | 1.00000   | 12.646   | 1.00000    | 11.451   | 1.00000   | 10.368   |
| 0.55 | 1.04248   | 15.233   | 1.03849    | 13.798   | 1.03486   | 12.495   | 1.03155    | 11.313   | 1.02854   | 10.242   |
| 0.6  | 1.08581   | 14.694   | 1.07756    | 13.305   | 1.07010   | 12.045   | 1.06335    | 10.904   | 1.05725   | 9.870    |
| 0.65 | 1.12892   | 13.802   | 1.11626    | 12.492   | 1.10486   | 11.305   | 1.09458    | 10.231   | 1.08533   | 9.258    |
| 0.7  | 1.17061   | 12.575   | 1.15349    | 11.373   | 1.13815   | 10.288   | 1.12437    | 9.306    | 1.11201   | 8.419    |
| 0.75 | 1.20951   | 11.032   | 1.18804    | 9.971    | 1.16889   | 9.015    | 1.15179    | 8.151    | 1.13649   | 7.371    |
| 0.8  | 1.24415   | 9.206    | 1.21866    | 8.315    | 1.19602   | 7.513    | 1.17587    | 6.790    | 1.15793   | 6.138    |
| 0.85 | 1.27305   | 7.134    | 1.24409    | 6.440    | 1.21846   | 5.815    | 1.19574    | 5.254    | 1.17555   | 4.747    |
| 0.9  | 1.29485   | 4.870    | 1.26320    | 4.394    | 1.23527   | 3.966    | 1.21058    | 3.582    | 1.18868   | 3.235    |
| 0.95 | 1.30843   | 2.470    | 1.27506    | 2.228    | 1.24569   | 2.010    | 1.21976    | 1.815    | 1.19680   | 1.639    |
| 1.0  | 1.31304   | 0.00     | 1.27908    | 0.00     | 1.24922   | 0.00     | 1.22286    | 0.00     | 1.19954   | 0.00     |
| 1.05 | 1.30843   | 2.470    | 1.27506    | 2.228    | 1.24569   | 2.010    | 1.21976    | 1.815    | 1.19680   | 1.639    |
| 1.1  | 1.29485   | 4.870    | 1.26320    | 4.394    | 1.23527   | 3.966    | 1.21058    | 3.582    | 1.18868   | 3.235    |
| 1.15 | 1.27305   | 7.134    | 1.24409    | 6.440    | 1.21846   | 5.815    | 1.19574    | 5.254    | 1.17555   | 4.747    |
| 1.2  | 1.24415   | 9.206    | 1.21866    | 8.315    | 1.19602   | 7.513    | 1.17587    | 6.790    | 1.15793   | 6.138    |
| 1.25 | 1.20951   | 11.032   | 1.18804    | 9.971    | 1.16889   | 9.015    | 1.15179    | 8.151    | 1.13649   | 7.371    |
| 1.3  | 1.17061   | 12.575   | 1.15349    | 11.373   | 1.13815   | 10.288   | 1.12437    | 9.306    | 1.11201   | 8.419    |
| 1.35 | 1.12892   | 13.802   | 1.11626    | 12.492   | 1.10486   | 11.305   | 1.09458    | 10.231   | 1.08533   | 9.258    |
| 1.4  | 1.08581   | 14.694   | 1.07756    | 13.305   | 1.07010   | 12.045   | 1.06335    | 10.904   | 1.05725   | 9.870    |
| 1.45 | 1.04248   | 15.233   | 1.03849    | 13.798   | 1.03486   | 12.495   | 1.03155    | 11.313   | 1.02854   | 10.242   |
| 1.5  | 1.00000   | 15.414   | 1.00000    | 13.963   | 1.00000   | 12.646   | 1.00000    | 11.451   | 1.00000   | 10.368   |
| 1.55 | 0.95925   | 15.233   | 0.96294    | 13.798   | 0.96632   | 12.495   | 0.96941    | 11.313   | 0.97223   | 10.242   |
| 1.6  | 0.92098   | 14.694   | 0.92802    | 13.305   | 0.93449   | 12.045   | 0.94042    | 10.904   | 0.94585   | 9.870    |
| 1.65 | 0.88580   | 13.802   | 0.89585    | 12.492   | 0.90509   | 11.305   | 0.91359    | 10.231   | 0.92138   | 9.258    |
| 1.7  | 0.85425   | 12.575   | 0.86693    | 11.373   | 0.87862   | 10.288   | 0.88939    | 9.306    | 0.89927   | 8.419    |
| 1.75 | 0.82678   | 11.032   | 0.84172    | 9.971    | 0.85551   | 9.015    | 0.86822    | 8.151    | 0.87991   | 7.371    |
| 1.8  | 0.80376   | 9.206    | 0.82058    | 8.315    | 0.83611   | 7.513    | 0.85043    | 6.790    | 0.86361   | 6.138    |
| 1.85 | 0.78552   | 7.134    | 0.80380    | 6.440    | 0.82071   | 5.815    | 0.83630    | 5.254    | 0.85067   | 4.747    |
| 1.9  | 0.77229   | 4.870    | 0.79164    | 4.394    | 0.80954   | 3.966    | 0.82605    | 3.582    | 0.84127   | 3.235    |
| 1.95 | 0.76428   | 2.470    | 0.78427    | 2.228    | 0.80277   | 2.010    | 0.81084    | 1.815    | 0.83557   | 1.639    |
| 2.0  | 0.76159   | 0.00     | 0.78181    | 0.00     | 0.80050   | 0.00     | 0.81775    | 0.00     | 0.83365   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(1.1 + i 0.7) = 1.13815 / 10^{\circ}.288 = 1.13815 / 10^{\circ}.17'.17''$ .  
 $\tanh(1.2 + i 1.7) = 0.89927 / 8^{\circ}.419 = 0.89927 / 8^{\circ}.25'.08''$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 1.25$ |          | $x = 1.3$ |          | $x = 1.35$ |          | $x = 1.4$ |          | $x = 1.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 0.84828    | 0.00     | 0.86172   | 0.00     | 0.87405    | 0.00     | 0.88535   | 0.00     | 0.89569    | 0.00     |
| 0.05 | 0.85004    | 1.481    | 0.86333   | 1.338    | 0.87552    | 1.210    | 0.88669   | 1.094    | 0.89692    | 0.989    |
| 0.1  | 0.85526    | 2.924    | 0.86811   | 2.643    | 0.87990    | 2.389    | 0.89069   | 2.161    | 0.90057    | 1.953    |
| 0.15 | 0.86387    | 4.291    | 0.87599   | 3.879    | 0.88711    | 3.508    | 0.89728   | 3.172    | 0.90659    | 2.869    |
| 0.2  | 0.87573    | 5.549    | 0.88684   | 5.017    | 0.89702    | 4.538    | 0.90634   | 4.104    | 0.91485    | 3.712    |
| 0.25 | 0.89063    | 6.666    | 0.90047   | 6.029    | 0.90947    | 5.454    | 0.91769   | 4.934    | 0.92521    | 4.463    |
| 0.3  | 0.90833    | 7.616    | 0.91663   | 6.890    | 0.92421    | 6.234    | 0.93114   | 5.640    | 0.93746    | 5.103    |
| 0.35 | 0.92852    | 8.378    | 0.93504   | 7.581    | 0.94099    | 6.860    | 0.94642   | 6.207    | 0.95137    | 5.617    |
| 0.4  | 0.95082    | 8.933    | 0.95534   | 8.085    | 0.95947    | 7.317    | 0.96323   | 6.622    | 0.96665    | 5.993    |
| 0.45 | 0.97481    | 9.272    | 0.97715   | 8.393    | 0.97928    | 7.596    | 0.98122   | 6.875    | 0.98298    | 6.222    |
| 0.5  | 1.00000    | 9.385    | 1.00000   | 8.496    | 1.00000    | 7.689    | 1.00000   | 6.960    | 1.00000    | 6.299    |
| 0.55 | 1.02584    | 9.272    | 1.02338   | 8.393    | 1.02116    | 7.596    | 1.01914   | 6.875    | 1.01733    | 6.222    |
| 0.6  | 1.05173    | 8.933    | 1.04674   | 8.085    | 1.04224    | 7.317    | 1.03817   | 6.622    | 1.03450    | 5.993    |
| 0.65 | 1.07699    | 8.378    | 1.06948   | 7.581    | 1.06271    | 6.860    | 1.05661   | 6.207    | 1.05112    | 5.617    |
| 0.7  | 1.10092    | 7.616    | 1.09096   | 6.890    | 1.08200    | 6.234    | 1.07395   | 5.640    | 1.06671    | 5.103    |
| 0.75 | 1.12280    | 6.666    | 1.11054   | 6.029    | 1.09955    | 5.454    | 1.08963   | 4.934    | 1.08084    | 4.463    |
| 0.8  | 1.14192    | 5.549    | 1.12760   | 5.017    | 1.11483    | 4.538    | 1.10335   | 4.104    | 1.09308    | 3.712    |
| 0.85 | 1.15758    | 4.291    | 1.14156   | 3.879    | 1.12726    | 3.508    | 1.11448   | 3.172    | 1.10304    | 2.869    |
| 0.9  | 1.16924    | 2.924    | 1.15193   | 2.643    | 1.13650    | 2.389    | 1.12272   | 2.161    | 1.11041    | 1.953    |
| 0.95 | 1.17642    | 1.481    | 1.15831   | 1.338    | 1.14218    | 1.210    | 1.12779   | 1.094    | 1.11493    | 0.989    |
| 1.0  | 1.17885    | 0.00     | 1.16047   | 0.00     | 1.14410    | 0.00     | 1.12950   | 0.00     | 1.11646    | 0.00     |
| 1.05 | 1.17642    | 1.481    | 1.15831   | 1.338    | 1.14218    | 1.210    | 1.12779   | 1.094    | 1.11493    | 0.989    |
| 1.1  | 1.16924    | 2.924    | 1.15193   | 2.643    | 1.13650    | 2.389    | 1.12272   | 2.161    | 1.11041    | 1.953    |
| 1.15 | 1.15758    | 4.291    | 1.14156   | 3.879    | 1.12726    | 3.508    | 1.11448   | 3.172    | 1.10304    | 2.869    |
| 1.2  | 1.14192    | 5.549    | 1.12760   | 5.017    | 1.11483    | 4.538    | 1.10335   | 4.104    | 1.09308    | 3.712    |
| 1.25 | 1.12280    | 6.666    | 1.11054   | 6.029    | 1.09955    | 5.454    | 1.08963   | 4.934    | 1.08084    | 4.463    |
| 1.3  | 1.10092    | 7.616    | 1.09096   | 6.890    | 1.08200    | 6.234    | 1.07395   | 5.640    | 1.06671    | 5.103    |
| 1.35 | 1.07699    | 8.378    | 1.06948   | 7.581    | 1.06271    | 6.860    | 1.05661   | 6.207    | 1.05112    | 5.617    |
| 1.4  | 1.05173    | 8.933    | 1.04674   | 8.085    | 1.04224    | 7.317    | 1.03817   | 6.622    | 1.03450    | 5.993    |
| 1.45 | 1.02584    | 9.272    | 1.02338   | 8.393    | 1.02116    | 7.596    | 1.01914   | 6.875    | 1.01733    | 6.222    |
| 1.5  | 1.00000    | 9.385    | 1.00000   | 8.496    | 1.00000    | 7.689    | 1.00000   | 6.960    | 1.00000    | 6.299    |
| 1.55 | 0.97481    | 9.272    | 0.97715   | 8.393    | 0.97928    | 7.596    | 0.98122   | 6.875    | 0.98298    | 6.222    |
| 1.6  | 0.95082    | 8.933    | 0.95534   | 8.085    | 0.95947    | 7.317    | 0.96323   | 6.622    | 0.96665    | 5.993    |
| 1.65 | 0.92852    | 8.378    | 0.93504   | 7.581    | 0.94099    | 6.860    | 0.94642   | 6.207    | 0.95137    | 5.617    |
| 1.7  | 0.90833    | 7.616    | 0.91663   | 6.890    | 0.92421    | 6.234    | 0.93114   | 5.640    | 0.93746    | 5.103    |
| 1.75 | 0.89063    | 6.666    | 0.90047   | 6.029    | 0.90947    | 5.454    | 0.91769   | 4.934    | 0.92521    | 4.463    |
| 1.8  | 0.87573    | 5.549    | 0.88684   | 5.017    | 0.89702    | 4.538    | 0.90634   | 4.104    | 0.91485    | 3.712    |
| 1.85 | 0.86387    | 4.291    | 0.87599   | 3.879    | 0.88711    | 3.508    | 0.89728   | 3.172    | 0.90659    | 2.869    |
| 1.9  | 0.85526    | 2.924    | 0.86811   | 2.643    | 0.87990    | 2.389    | 0.89069   | 2.161    | 0.90057    | 1.953    |
| 1.95 | 0.85004    | 1.481    | 0.86333   | 1.338    | 0.87552    | 1.210    | 0.88669   | 1.094    | 0.89692    | 0.989    |
| 2.0  | 0.84828    | 0.00     | 0.86172   | 0.00     | 0.87405    | 0.00     | 0.88535   | 0.00     | 0.89569    | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(1.25 + i\ 0.25) = 0.89063 / \sqrt{6^{\circ}.666} = 0.89063 / \sqrt{6^{\circ}.39'.58''}$   
 $\tanh(1.25 + i\ 1.25) = 1.12280 / \sqrt{6^{\circ}.666} = 1.12280 / \sqrt{6^{\circ}.39'.58''}$



TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 1.5$ |          | $x = 1.55$ |          | $x = 1.6$ |          | $x = 1.65$ |          | $x = 1.7$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 0.90515   | 0.00     | 0.91379    | 0.00     | 0.92167   | 0.00     | 0.92886    | 0.00     | 0.93541   | 0.00     |
| 0.05 | 0.90627   | 0.894    | 0.91481    | 0.809    | 0.92260   | 0.732    | 0.92970    | 0.662    | 0.93618   | 0.599    |
| 0.1  | 0.90960   | 1.767    | 0.91785    | 1.598    | 0.92537   | 1.446    | 0.93223    | 1.308    | 0.93848   | 1.183    |
| 0.15 | 0.91509   | 2.595    | 0.92285    | 2.347    | 0.92993   | 2.123    | 0.93638    | 1.921    | 0.94226   | 1.738    |
| 0.2  | 0.92263   | 3.357    | 0.92972    | 3.038    | 0.93619   | 2.748    | 0.94207    | 2.486    | 0.94744   | 2.249    |
| 0.25 | 0.93207   | 4.038    | 0.93832    | 3.653    | 0.94401   | 3.305    | 0.94920    | 2.990    | 0.95392   | 2.705    |
| 0.3  | 0.94323   | 4.617    | 0.94847    | 4.177    | 0.95325   | 3.780    | 0.95760    | 3.420    | 0.96155   | 3.095    |
| 0.35 | 0.95588   | 5.083    | 0.95998    | 4.599    | 0.96371   | 4.161    | 0.96710    | 3.766    | 0.97018   | 3.407    |
| 0.4  | 0.96976   | 5.423    | 0.97259    | 4.908    | 0.97516   | 4.440    | 0.97749    | 4.019    | 0.97961   | 3.636    |
| 0.45 | 0.98458   | 5.631    | 0.98603    | 5.096    | 0.98735   | 4.611    | 0.98854    | 4.173    | 0.98962   | 3.776    |
| 0.5  | 1.00000   | 5.700    | 1.00000    | 5.159    | 1.00000   | 4.668    | 1.00000    | 4.225    | 1.00000   | 3.822    |
| 0.55 | 1.01566   | 5.631    | 1.01417    | 5.096    | 1.01281   | 4.611    | 1.01159    | 4.173    | 1.01049   | 3.776    |
| 0.6  | 1.03118   | 5.423    | 1.02818    | 4.908    | 1.02548   | 4.440    | 1.02303    | 4.019    | 1.02082   | 3.636    |
| 0.65 | 1.04616   | 5.083    | 1.04169    | 4.599    | 1.03766   | 4.161    | 1.03402    | 3.766    | 1.03074   | 3.407    |
| 0.7  | 1.06019   | 4.617    | 1.05433    | 4.177    | 1.04904   | 3.780    | 1.04428    | 3.420    | 1.03999   | 3.095    |
| 0.75 | 1.07289   | 4.038    | 1.06574    | 3.653    | 1.05931   | 3.305    | 1.05353    | 2.990    | 1.04831   | 2.705    |
| 0.8  | 1.08386   | 3.357    | 1.07559    | 3.038    | 1.06817   | 2.748    | 1.06149    | 2.486    | 1.05548   | 2.249    |
| 0.85 | 1.09279   | 2.595    | 1.08360    | 2.347    | 1.07535   | 2.123    | 1.06795    | 1.921    | 1.06128   | 1.738    |
| 0.9  | 1.09938   | 1.767    | 1.08951    | 1.598    | 1.08065   | 1.446    | 1.07270    | 1.308    | 1.06556   | 1.183    |
| 0.95 | 1.10343   | 0.894    | 1.09313    | 0.809    | 1.08390   | 0.732    | 1.07562    | 0.662    | 1.06817   | 0.599    |
| 1.0  | 1.10479   | 0.00     | 1.09436    | 0.00     | 1.08500   | 0.00     | 1.07659    | 0.00     | 1.06906   | 0.00     |
| 1.05 | 1.10343   | 0.894    | 1.09313    | 0.809    | 1.08390   | 0.732    | 1.07562    | 0.662    | 1.06817   | 0.599    |
| 1.1  | 1.09938   | 1.767    | 1.08951    | 1.598    | 1.08065   | 1.446    | 1.07270    | 1.308    | 1.06556   | 1.183    |
| 1.15 | 1.09279   | 2.595    | 1.08360    | 2.347    | 1.07535   | 2.123    | 1.06795    | 1.921    | 1.06128   | 1.738    |
| 1.2  | 1.08386   | 3.357    | 1.07559    | 3.038    | 1.06817   | 2.748    | 1.06149    | 2.486    | 1.05548   | 2.249    |
| 1.25 | 1.07289   | 4.038    | 1.06574    | 3.653    | 1.05931   | 3.305    | 1.05353    | 2.990    | 1.04831   | 2.705    |
| 1.3  | 1.06019   | 4.617    | 1.05433    | 4.177    | 1.04904   | 3.780    | 1.04428    | 3.420    | 1.03999   | 3.095    |
| 1.35 | 1.04616   | 5.083    | 1.04169    | 4.599    | 1.03766   | 4.161    | 1.03402    | 3.766    | 1.03074   | 3.407    |
| 1.4  | 1.03118   | 5.423    | 1.02818    | 4.908    | 1.02548   | 4.440    | 1.02303    | 4.019    | 1.02082   | 3.636    |
| 1.45 | 1.01566   | 5.631    | 1.01417    | 5.096    | 1.01281   | 4.611    | 1.01159    | 4.173    | 1.01049   | 3.776    |
| 1.5  | 1.00000   | 5.700    | 1.00000    | 5.159    | 1.00000   | 4.668    | 1.00000    | 4.225    | 1.00000   | 3.822    |
| 1.55 | 0.98458   | 5.631    | 0.98603    | 5.096    | 0.98735   | 4.611    | 0.98854    | 4.173    | 0.98962   | 3.776    |
| 1.6  | 0.96976   | 5.423    | 0.97259    | 4.908    | 0.97516   | 4.440    | 0.97749    | 4.019    | 0.97961   | 3.636    |
| 1.65 | 0.95588   | 5.083    | 0.95998    | 4.599    | 0.96371   | 4.161    | 0.96710    | 3.766    | 0.97018   | 3.407    |
| 1.7  | 0.94323   | 4.617    | 0.94847    | 4.177    | 0.95325   | 3.780    | 0.95760    | 3.420    | 0.96155   | 3.095    |
| 1.75 | 0.93207   | 4.038    | 0.93832    | 3.653    | 0.94401   | 3.305    | 0.94920    | 2.990    | 0.95392   | 2.705    |
| 1.8  | 0.92263   | 3.357    | 0.92972    | 3.038    | 0.93619   | 2.748    | 0.94207    | 2.486    | 0.94744   | 2.249    |
| 1.85 | 0.91509   | 2.595    | 0.92285    | 2.347    | 0.92993   | 2.123    | 0.93638    | 1.921    | 0.94226   | 1.738    |
| 1.9  | 0.90960   | 1.767    | 0.91785    | 1.598    | 0.92537   | 1.446    | 0.93223    | 1.308    | 0.93848   | 1.183    |
| 1.95 | 0.90627   | 0.894    | 0.91481    | 0.809    | 0.92260   | 0.732    | 0.92970    | 0.662    | 0.93618   | 0.599    |
| 2.0  | 0.90515   | 0.00     | 0.91379    | 0.00     | 0.92167   | 0.00     | 0.92886    | 0.00     | 0.93541   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(1.7 + i \underline{0.7}) = 1.03999 \sqrt[3]{3^{\circ}.095} = 1.03999 \sqrt[3]{3^{\circ}.05'.42''}$ .  
 $\tanh(1.6 + i \underline{1.7}) = 0.95325 \sqrt[3]{3^{\circ}.780} = 0.95325 \sqrt[3]{3^{\circ}.46'.48''}$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 1.75$ |          | $x = 1.8$ |          | $x = 1.85$ |          | $x = 1.9$ |          | $x = 1.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
|      | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 0.94138    | 0.00     | 0.94681   | 0.00     | 0.95175    | 0.00     | 0.95624   | 0.00     | 0.96032    | 0.00     |
| 0.05 | 0.94208    | 0.542    | 0.94745   | 0.490    | 0.95232    | 0.443    | 0.95677   | 0.401    | 0.96080    | 0.363    |
| 0.1  | 0.94417    | 1.070    | 0.94935   | 0.968    | 0.95406    | 0.876    | 0.95834   | 0.793    | 0.96222    | 0.717    |
| 0.15 | 0.94761    | 1.572    | 0.95247   | 1.422    | 0.95690    | 1.287    | 0.96092   | 1.165    | 0.96457    | 1.053    |
| 0.2  | 0.95232    | 2.035    | 0.95676   | 1.841    | 0.96079    | 1.666    | 0.96445   | 1.507    | 0.96778    | 1.364    |
| 0.25 | 0.95821    | 2.448    | 0.96211   | 2.215    | 0.96565    | 2.004    | 0.96886   | 1.813    | 0.97178    | 1.640    |
| 0.3  | 0.96514    | 2.800    | 0.96840   | 2.533    | 0.97136    | 2.292    | 0.97405   | 2.074    | 0.97649    | 1.877    |
| 0.35 | 0.97297    | 3.083    | 0.97551   | 2.789    | 0.97781    | 2.524    | 0.97990   | 2.285    | 0.98179    | 2.067    |
| 0.4  | 0.98153    | 3.291    | 0.98327   | 2.977    | 0.98485    | 2.694    | 0.98628   | 2.438    | 0.98757    | 2.206    |
| 0.45 | 0.99061    | 3.417    | 0.99149   | 3.092    | 0.99230    | 2.798    | 0.99303   | 2.531    | 0.99369    | 2.291    |
| 0.5  | 1.00000    | 3.459    | 1.00000   | 3.130    | 1.00000    | 2.833    | 1.00000   | 2.562    | 1.00000    | 2.319    |
| 0.55 | 1.00948    | 3.417    | 1.00858   | 3.092    | 1.00776    | 2.798    | 1.00702   | 2.531    | 1.00635    | 2.291    |
| 0.6  | 1.01882    | 3.291    | 1.01702   | 2.977    | 1.01539    | 2.694    | 1.01392   | 2.438    | 1.01258    | 2.206    |
| 0.65 | 1.02778    | 3.083    | 1.02511   | 2.789    | 1.02269    | 2.524    | 1.02051   | 2.285    | 1.01854    | 2.067    |
| 0.7  | 1.03612    | 2.800    | 1.03263   | 2.533    | 1.02948    | 2.292    | 1.02664   | 2.074    | 1.02408    | 1.877    |
| 0.75 | 1.04362    | 2.448    | 1.03939   | 2.215    | 1.03558    | 2.004    | 1.03214   | 1.813    | 1.02904    | 1.640    |
| 0.8  | 1.05007    | 2.035    | 1.04520   | 1.841    | 1.04081    | 1.666    | 1.03686   | 1.507    | 1.03329    | 1.364    |
| 0.85 | 1.05529    | 1.572    | 1.04990   | 1.422    | 1.04504    | 1.287    | 1.04067   | 1.165    | 1.03673    | 1.053    |
| 0.9  | 1.05913    | 1.070    | 1.05336   | 0.968    | 1.04816    | 0.876    | 1.04347   | 0.793    | 1.03926    | 0.717    |
| 0.95 | 1.06148    | 0.542    | 1.05547   | 0.490    | 1.05006    | 0.443    | 1.04519   | 0.401    | 1.04080    | 0.363    |
| 1.0  | 1.06228    | 0.00     | 1.05619   | 0.00     | 1.05070    | 0.00     | 1.04576   | 0.00     | 1.04131    | 0.00     |
| 1.05 | 1.06148    | 0.542    | 1.05547   | 0.490    | 1.05006    | 0.443    | 1.04519   | 0.401    | 1.04080    | 0.363    |
| 1.1  | 1.05913    | 1.070    | 1.05336   | 0.968    | 1.04816    | 0.876    | 1.04347   | 0.793    | 1.03926    | 0.717    |
| 1.15 | 1.05529    | 1.572    | 1.04990   | 1.422    | 1.04504    | 1.287    | 1.04067   | 1.165    | 1.03673    | 1.053    |
| 1.2  | 1.05007    | 2.035    | 1.04520   | 1.841    | 1.04081    | 1.666    | 1.03686   | 1.507    | 1.03329    | 1.364    |
| 1.25 | 1.04362    | 2.448    | 1.03939   | 2.215    | 1.03558    | 2.004    | 1.03214   | 1.813    | 1.02904    | 1.640    |
| 1.3  | 1.03612    | 2.800    | 1.03263   | 2.533    | 1.02948    | 2.292    | 1.02664   | 2.074    | 1.02408    | 1.877    |
| 1.35 | 1.02778    | 3.083    | 1.02511   | 2.789    | 1.02269    | 2.524    | 1.02051   | 2.285    | 1.01854    | 2.067    |
| 1.4  | 1.01882    | 3.291    | 1.01702   | 2.977    | 1.01539    | 2.694    | 1.01392   | 2.438    | 1.01258    | 2.206    |
| 1.45 | 1.00948    | 3.417    | 1.00858   | 3.092    | 1.00776    | 2.798    | 1.00702   | 2.531    | 1.00635    | 2.291    |
| 1.5  | 1.00000    | 3.459    | 1.00000   | 3.130    | 1.00000    | 2.833    | 1.00000   | 2.562    | 1.00000    | 2.319    |
| 1.55 | 0.99061    | 3.417    | 0.99149   | 3.092    | 0.99230    | 2.798    | 0.99303   | 2.531    | 0.99369    | 2.291    |
| 1.6  | 0.98153    | 3.291    | 0.98327   | 2.977    | 0.98485    | 2.694    | 0.98628   | 2.438    | 0.98757    | 2.206    |
| 1.65 | 0.97297    | 3.083    | 0.97551   | 2.789    | 0.97781    | 2.524    | 0.97990   | 2.285    | 0.98179    | 2.067    |
| 1.7  | 0.96514    | 2.800    | 0.96840   | 2.533    | 0.97136    | 2.292    | 0.97405   | 2.074    | 0.97649    | 1.877    |
| 1.75 | 0.95821    | 2.448    | 0.96211   | 2.215    | 0.96565    | 2.004    | 0.96886   | 1.813    | 0.97178    | 1.640    |
| 1.8  | 0.95232    | 2.035    | 0.95676   | 1.841    | 0.96079    | 1.666    | 0.96445   | 1.507    | 0.96778    | 1.364    |
| 1.85 | 0.94761    | 1.572    | 0.95247   | 1.422    | 0.95690    | 1.287    | 0.96092   | 1.165    | 0.96457    | 1.053    |
| 1.9  | 0.94417    | 1.070    | 0.94935   | 0.968    | 0.95406    | 0.876    | 0.95834   | 0.793    | 0.96222    | 0.717    |
| 1.95 | 0.94208    | 0.542    | 0.94745   | 0.490    | 0.95232    | 0.443    | 0.95677   | 0.401    | 0.96080    | 0.363    |
| 2.0  | 0.94138    | 0.00     | 0.94681   | 0.00     | 0.95175    | 0.00     | 0.95624   | 0.00     | 0.96032    | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(1.9 + i\,0.05) = 0.95677 / 0^{\circ}.401 = 0.95677 / 0^{\circ}.24'.04''$ .

$\tanh(1.95 + i\,1.5) = 1.000 / 2^{\circ}.319 = 1.000 / 2^{\circ}.19'.08''$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

| $q$  | $x = 2.0$ |          | $x = 2.05$ |          | $x = 2.1$ |          | $x = 2.15$ |          | $x = 2.2$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
|      | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 0.96403   | 0.00     | 0.96740    | 0.00     | 0.97045   | 0.00     | 0.97323    | 0.00     | 0.97574   | 0.00     |
| 0.05 | 0.96446   | 0.328    | 0.96779    | 0.297    | 0.97081   | 0.268    | 0.97355    | 0.243    | 0.97604   | 0.220    |
| 0.1  | 0.96576   | 0.649    | 0.96897    | 0.587    | 0.97188   | 0.531    | 0.97452    | 0.481    | 0.97692   | 0.435    |
| 0.15 | 0.96789   | 0.953    | 0.97090    | 0.862    | 0.97363   | 0.780    | 0.97611    | 0.706    | 0.97842   | 0.639    |
| 0.2  | 0.97080   | 1.234    | 0.97354    | 1.117    | 0.97603   | 1.010    | 0.97829    | 0.914    | 0.98033   | 0.827    |
| 0.25 | 0.97443   | 1.484    | 0.97684    | 1.343    | 0.97902   | 1.215    | 0.98100    | 1.099    | 0.98279   | 0.995    |
| 0.3  | 0.97870   | 1.698    | 0.98071    | 1.537    | 0.98253   | 1.390    | 0.98418    | 1.258    | 0.98567   | 1.138    |
| 0.35 | 0.98351   | 1.870    | 0.98507    | 1.692    | 0.98648   | 1.531    | 0.98776    | 1.385    | 0.98892   | 1.253    |
| 0.4  | 0.98875   | 1.996    | 0.98981    | 1.806    | 0.99078   | 1.634    | 0.99165    | 1.479    | 0.99244   | 1.337    |
| 0.45 | 0.99429   | 2.072    | 0.99483    | 1.876    | 0.99532   | 1.698    | 0.99577    | 1.536    | 0.99617   | 1.389    |
| 0.5  | 1.00000   | 2.098    | 1.00000    | 1.899    | 1.00000   | 1.718    | 1.00000    | 1.555    | 1.00000   | 1.406    |
| 0.55 | 1.00574   | 2.072    | 1.00520    | 1.876    | 1.00470   | 1.698    | 1.00425    | 1.536    | 1.00385   | 1.389    |
| 0.6  | 1.01138   | 1.996    | 1.01029    | 1.806    | 1.00931   | 1.634    | 1.00842    | 1.479    | 1.00762   | 1.337    |
| 0.65 | 1.01676   | 1.870    | 1.01516    | 1.692    | 1.01371   | 1.531    | 1.01240    | 1.385    | 1.01121   | 1.253    |
| 0.7  | 1.02176   | 1.698    | 1.01967    | 1.537    | 1.01778   | 1.390    | 1.01608    | 1.258    | 1.01454   | 1.138    |
| 0.75 | 1.02624   | 1.484    | 1.02371    | 1.343    | 1.02143   | 1.215    | 1.01937    | 1.099    | 1.01751   | 0.995    |
| 0.8  | 1.03008   | 1.234    | 1.02718    | 1.117    | 1.02456   | 1.010    | 1.02220    | 0.914    | 1.02006   | 0.827    |
| 0.85 | 1.03318   | 0.953    | 1.02998    | 0.862    | 1.02708   | 0.780    | 1.02447    | 0.706    | 1.02206   | 0.639    |
| 0.9  | 1.03545   | 0.649    | 1.03203    | 0.587    | 1.02894   | 0.531    | 1.02614    | 0.481    | 1.02363   | 0.435    |
| 0.95 | 1.03685   | 0.328    | 1.03328    | 0.297    | 1.03007   | 0.268    | 1.02717    | 0.243    | 1.02455   | 0.220    |
| 1.0  | 1.03731   | 0.00     | 1.03370    | 0.00     | 1.03045   | 0.00     | 1.02751    | 0.00     | 1.02486   | 0.00     |
| 1.05 | 1.03685   | 0.328    | 1.03328    | 0.297    | 1.03007   | 0.268    | 1.02717    | 0.243    | 1.02455   | 0.220    |
| 1.1  | 1.03545   | 0.649    | 1.03202    | 0.587    | 1.02894   | 0.531    | 1.02614    | 0.481    | 1.02363   | 0.435    |
| 1.15 | 1.03318   | 0.953    | 1.02998    | 0.862    | 1.02708   | 0.780    | 1.02447    | 0.706    | 1.02206   | 0.639    |
| 1.2  | 1.03008   | 1.234    | 1.02718    | 1.117    | 1.02456   | 1.010    | 1.02220    | 0.914    | 1.02006   | 0.827    |
| 1.25 | 1.02624   | 1.484    | 1.02371    | 1.343    | 1.02143   | 1.215    | 1.01937    | 1.099    | 1.01751   | 0.995    |
| 1.3  | 1.02176   | 1.698    | 1.01967    | 1.537    | 1.01778   | 1.390    | 1.01608    | 1.258    | 1.01454   | 1.138    |
| 1.35 | 1.01676   | 1.870    | 1.01516    | 1.692    | 1.01371   | 1.531    | 1.01240    | 1.385    | 1.01121   | 1.253    |
| 1.4  | 1.01138   | 1.996    | 1.01029    | 1.806    | 1.00931   | 1.634    | 1.00842    | 1.479    | 1.00762   | 1.337    |
| 1.45 | 1.00574   | 2.072    | 1.00520    | 1.876    | 1.00470   | 1.698    | 1.00425    | 1.536    | 1.00385   | 1.389    |
| 1.5  | 1.00000   | 2.098    | 1.00000    | 1.899    | 1.00000   | 1.718    | 1.00000    | 1.555    | 1.00000   | 1.406    |
| 1.55 | 0.99429   | 2.072    | 0.99483    | 1.876    | 0.99532   | 1.698    | 0.99577    | 1.536    | 0.99617   | 1.389    |
| 1.6  | 0.98875   | 1.996    | 0.98981    | 1.806    | 0.99078   | 1.634    | 0.99165    | 1.479    | 0.99244   | 1.337    |
| 1.65 | 0.98351   | 1.870    | 0.98507    | 1.692    | 0.98648   | 1.531    | 0.98776    | 1.385    | 0.98892   | 1.253    |
| 1.7  | 0.97870   | 1.698    | 0.98071    | 1.537    | 0.98253   | 1.390    | 0.98418    | 1.258    | 0.98567   | 1.138    |
| 1.75 | 0.97443   | 1.484    | 0.97684    | 1.343    | 0.97902   | 1.215    | 0.98100    | 1.099    | 0.98279   | 0.995    |
| 1.8  | 0.97080   | 1.234    | 0.97354    | 1.117    | 0.97603   | 1.010    | 0.97829    | 0.914    | 0.98033   | 0.827    |
| 1.85 | 0.96789   | 0.953    | 0.97090    | 0.862    | 0.97363   | 0.780    | 0.97611    | 0.706    | 0.97842   | 0.639    |
| 1.9  | 0.96576   | 0.649    | 0.96897    | 0.587    | 0.97188   | 0.531    | 0.97452    | 0.481    | 0.97692   | 0.435    |
| 1.95 | 0.96446   | 0.328    | 0.96779    | 0.297    | 0.97081   | 0.268    | 0.97355    | 0.243    | 0.97604   | 0.220    |
| 2.0  | 0.96403   | 0.00     | 0.96740    | 0.00     | 0.97045   | 0.00     | 0.97323    | 0.00     | 0.97574   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(2.2 + i0) = 0.97574 / 0^\circ$ . $\tanh(2.2 + i1.95) = 0.97604 / 0^\circ.220 = 0.97604 / 0^\circ.13'.12''$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 2.25$ |          | $x = 2.3$ |          | $x = 2.35$ |          | $x = 2.4$ |          | $x = 2.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 0.97803    | 0.00     | 0.98010   | 0.00     | 0.98197    | 0.00     | 0.98367   | 0.00     | 0.98522    | 0.00     |
| 0.05 | 0.97829    | 0.199    | 0.98034   | 0.180    | 0.98219    | 0.163    | 0.98388   | 0.147    | 0.98540    | 0.134    |
| 0.1  | 0.97909    | 0.394    | 0.98106   | 0.356    | 0.98285    | 0.322    | 0.98447   | 0.291    | 0.98594    | 0.264    |
| 0.15 | 0.98040    | 0.578    | 0.98225   | 0.523    | 0.98393    | 0.473    | 0.98544   | 0.428    | 0.98682    | 0.388    |
| 0.2  | 0.98219    | 0.748    | 0.98387   | 0.677    | 0.98539    | 0.613    | 0.98677   | 0.554    | 0.98802    | 0.501    |
| 0.25 | 0.98441    | 0.900    | 0.98589   | 0.814    | 0.98722    | 0.737    | 0.98843   | 0.666    | 0.98953    | 0.603    |
| 0.3  | 0.98703    | 1.030    | 0.98825   | 0.932    | 0.98937    | 0.843    | 0.99037   | 0.763    | 0.99129    | 0.690    |
| 0.35 | 0.98997    | 1.134    | 0.99092   | 1.027    | 0.99178    | 0.929    | 0.99256   | 0.841    | 0.99326    | 0.760    |
| 0.4  | 0.99316    | 1.211    | 0.99381   | 1.095    | 0.99440    | 0.991    | 0.99493   | 0.896    | 0.99541    | 0.812    |
| 0.45 | 0.99653    | 1.257    | 0.99686   | 1.138    | 0.99716    | 1.030    | 0.99743   | 0.931    | 0.99767    | 0.843    |
| 0.5  | 1.00000    | 1.273    | 1.00000   | 1.152    | 1.00000    | 1.042    | 1.00000   | 0.942    | 1.00000    | 0.853    |
| 0.55 | 1.00348    | 1.257    | 1.00315   | 1.138    | 1.00285    | 1.030    | 1.00258   | 0.931    | 1.00233    | 0.843    |
| 0.6  | 1.00689    | 1.211    | 1.00623   | 1.095    | 1.00564    | 0.991    | 1.00510   | 0.896    | 1.00461    | 0.812    |
| 0.65 | 1.01014    | 1.134    | 1.00917   | 1.027    | 1.00829    | 0.929    | 1.00750   | 0.841    | 1.00678    | 0.760    |
| 0.7  | 1.01314    | 1.030    | 1.01189   | 0.932    | 1.01075    | 0.843    | 1.00972   | 0.763    | 1.00879    | 0.690    |
| 0.75 | 1.01583    | 0.900    | 1.01431   | 0.814    | 1.01295    | 0.737    | 1.01171   | 0.666    | 1.01058    | 0.603    |
| 0.8  | 1.01814    | 0.748    | 1.01640   | 0.677    | 1.01482    | 0.613    | 1.01340   | 0.554    | 1.01212    | 0.501    |
| 0.85 | 1.01999    | 0.578    | 1.01807   | 0.523    | 1.01634    | 0.473    | 1.01477   | 0.428    | 1.01336    | 0.388    |
| 0.9  | 1.02136    | 0.394    | 1.01930   | 0.356    | 1.01745    | 0.322    | 1.01578   | 0.291    | 1.01426    | 0.264    |
| 0.95 | 1.02219    | 0.199    | 1.02005   | 0.180    | 1.01813    | 0.163    | 1.01639   | 0.147    | 1.01482    | 0.134    |
| 1.0  | 1.02247    | 0.00     | 1.02031   | 0.00     | 1.01836    | 0.00     | 1.01659   | 0.00     | 1.01500    | 0.00     |
| 1.05 | 1.02219    | 0.199    | 1.02005   | 0.180    | 1.01813    | 0.163    | 1.01639   | 0.147    | 1.01482    | 0.134    |
| 1.1  | 1.02136    | 0.394    | 1.01930   | 0.356    | 1.01745    | 0.322    | 1.01578   | 0.291    | 1.01426    | 0.264    |
| 1.15 | 1.01999    | 0.578    | 1.01807   | 0.523    | 1.01634    | 0.473    | 1.01477   | 0.428    | 1.01336    | 0.388    |
| 1.2  | 1.01814    | 0.748    | 1.01640   | 0.677    | 1.01482    | 0.613    | 1.01340   | 0.554    | 1.01212    | 0.501    |
| 1.25 | 1.01583    | 0.900    | 1.01431   | 0.814    | 1.01295    | 0.737    | 1.01171   | 0.666    | 1.01058    | 0.603    |
| 1.3  | 1.01314    | 1.030    | 1.01189   | 0.932    | 1.01075    | 0.843    | 1.00972   | 0.763    | 1.00879    | 0.690    |
| 1.35 | 1.01014    | 1.134    | 1.00917   | 1.027    | 1.00829    | 0.929    | 1.00750   | 0.841    | 1.00678    | 0.760    |
| 1.4  | 1.00689    | 1.211    | 1.00623   | 1.095    | 1.00564    | 0.991    | 1.00510   | 0.896    | 1.00461    | 0.812    |
| 1.45 | 1.00348    | 1.257    | 1.00315   | 1.138    | 1.00285    | 1.030    | 1.00258   | 0.931    | 1.00233    | 0.843    |
| 1.5  | 1.00000    | 1.273    | 1.00000   | 1.152    | 1.00000    | 1.042    | 1.00000   | 0.942    | 1.00000    | 0.853    |
| 1.55 | 0.99653    | 1.257    | 0.99686   | 1.138    | 0.99716    | 1.030    | 0.99743   | 0.931    | 0.99767    | 0.843    |
| 1.6  | 0.99316    | 1.211    | 0.99381   | 1.095    | 0.99440    | 0.991    | 0.99493   | 0.896    | 0.99541    | 0.812    |
| 1.65 | 0.98997    | 1.134    | 0.99092   | 1.027    | 0.99178    | 0.929    | 0.99256   | 0.841    | 0.99326    | 0.760    |
| 1.7  | 0.98703    | 1.030    | 0.98825   | 0.932    | 0.98937    | 0.843    | 0.99037   | 0.763    | 0.99129    | 0.690    |
| 1.75 | 0.98441    | 0.900    | 0.98589   | 0.814    | 0.98722    | 0.737    | 0.98843   | 0.666    | 0.98953    | 0.603    |
| 1.8  | 0.98219    | 0.748    | 0.98387   | 0.677    | 0.98539    | 0.613    | 0.98677   | 0.554    | 0.98802    | 0.501    |
| 1.85 | 0.98040    | 0.578    | 0.98225   | 0.523    | 0.98393    | 0.473    | 0.98544   | 0.428    | 0.98682    | 0.388    |
| 1.9  | 0.97909    | 0.394    | 0.98106   | 0.356    | 0.98285    | 0.322    | 0.98447   | 0.291    | 0.98594    | 0.264    |
| 1.95 | 0.97829    | 0.199    | 0.98034   | 0.180    | 0.98219    | 0.163    | 0.98388   | 0.147    | 0.98540    | 0.134    |
| 2.0  | 0.97803    | 0.00     | 0.98010   | 0.00     | 0.98197    | 0.00     | 0.98367   | 0.00     | 0.98522    | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(2.45 + i \underline{0.7}) = 1.00879 / \underline{0^{\circ}.690} = 1.00879 / \underline{0^{\circ}.41'.24''}$ .  
 $\tanh(2.45 + i \underline{1.7}) = 0.99129 / \underline{0^{\circ}.690} = 0.99129 / \underline{0^{\circ}.41'.24''}$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 2.5$ |          | $x = 2.55$ |          | $x = 2.6$ |          | $x = 2.65$ |          | $x = 2.7$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 0.98661   | 0.00     | 0.98788    | 0.00     | 0.98903   | 0.00     | 0.99007    | 0.00     | 0.99101   | 0.00     |
| 0.05 | 0.98678   | 0.121    | 0.98803    | 0.109    | 0.98916   | 0.099    | 0.99019    | 0.089    | 0.99112   | 0.081    |
| 0.1  | 0.98727   | 0.239    | 0.98847    | 0.216    | 0.98956   | 0.195    | 0.99055    | 0.177    | 0.99145   | 0.160    |
| 0.15 | 0.98806   | 0.350    | 0.98919    | 0.317    | 0.99022   | 0.287    | 0.99114    | 0.260    | 0.99198   | 0.235    |
| 0.2  | 0.98916   | 0.454    | 0.99018    | 0.411    | 0.99111   | 0.372    | 0.99196    | 0.336    | 0.99272   | 0.304    |
| 0.25 | 0.99052   | 0.546    | 0.99142    | 0.494    | 0.99223   | 0.447    | 0.99297    | 0.405    | 0.99363   | 0.366    |
| 0.3  | 0.99211   | 0.625    | 0.99286    | 0.565    | 0.99354   | 0.512    | 0.99415    | 0.463    | 0.99471   | 0.419    |
| 0.35 | 0.99390   | 0.688    | 0.99448    | 0.623    | 0.99500   | 0.563    | 0.99548    | 0.510    | 0.99591   | 0.461    |
| 0.4  | 0.99584   | 0.734    | 0.99624    | 0.665    | 0.99660   | 0.601    | 0.99692    | 0.544    | 0.99721   | 0.492    |
| 0.45 | 0.99789   | 0.763    | 0.99809    | 0.690    | 0.99828   | 0.624    | 0.99844    | 0.565    | 0.99859   | 0.511    |
| 0.5  | 1.00000   | 0.772    | 1.00000    | 0.699    | 1.00000   | 0.632    | 1.00000    | 0.573    | 1.00000   | 0.518    |
| 0.55 | 1.00211   | 0.763    | 1.00191    | 0.690    | 1.00173   | 0.624    | 1.00156    | 0.565    | 1.00141   | 0.511    |
| 0.6  | 1.00417   | 0.734    | 1.00377    | 0.665    | 1.00342   | 0.601    | 1.00309    | 0.544    | 1.00280   | 0.492    |
| 0.65 | 1.00614   | 0.688    | 1.00555    | 0.623    | 1.00502   | 0.563    | 1.00454    | 0.510    | 1.00411   | 0.461    |
| 0.7  | 1.00795   | 0.625    | 1.00719    | 0.565    | 1.00651   | 0.512    | 1.00589    | 0.463    | 1.00532   | 0.419    |
| 0.75 | 1.00958   | 0.546    | 1.00866    | 0.494    | 1.00783   | 0.447    | 1.00708    | 0.405    | 1.00641   | 0.366    |
| 0.8  | 1.01096   | 0.454    | 1.00991    | 0.411    | 1.00897   | 0.372    | 1.00811    | 0.336    | 1.00733   | 0.304    |
| 0.85 | 1.01208   | 0.350    | 1.01092    | 0.317    | 1.00988   | 0.287    | 1.00894    | 0.260    | 1.00808   | 0.235    |
| 0.9  | 1.01290   | 0.239    | 1.01166    | 0.216    | 1.01055   | 0.195    | 1.00954    | 0.177    | 1.00863   | 0.160    |
| 0.95 | 1.01340   | 0.121    | 1.01212    | 0.109    | 1.01096   | 0.099    | 1.00991    | 0.089    | 1.00896   | 0.081    |
| 1.0  | 1.01357   | 0.00     | 1.01227    | 0.00     | 1.01110   | 0.00     | 1.01003    | 0.00     | 1.00907   | 0.00     |
| 1.05 | 1.01340   | 0.121    | 1.01212    | 0.109    | 1.01096   | 0.099    | 1.00991    | 0.089    | 1.00896   | 0.081    |
| 1.1  | 1.01290   | 0.239    | 1.01166    | 0.216    | 1.01055   | 0.195    | 1.00954    | 0.177    | 1.00863   | 0.160    |
| 1.15 | 1.01208   | 0.350    | 1.01092    | 0.317    | 1.00988   | 0.287    | 1.00894    | 0.260    | 1.00808   | 0.235    |
| 1.2  | 1.01096   | 0.454    | 1.00991    | 0.411    | 1.00897   | 0.372    | 1.00811    | 0.336    | 1.00733   | 0.304    |
| 1.25 | 1.00958   | 0.546    | 1.00866    | 0.494    | 1.00783   | 0.447    | 1.00708    | 0.405    | 1.00641   | 0.366    |
| 1.3  | 1.00795   | 0.625    | 1.00719    | 0.565    | 1.00651   | 0.512    | 1.00589    | 0.463    | 1.00532   | 0.419    |
| 1.35 | 1.00614   | 0.688    | 1.00555    | 0.623    | 1.00502   | 0.563    | 1.00454    | 0.510    | 1.00411   | 0.461    |
| 1.4  | 1.00417   | 0.734    | 1.00377    | 0.665    | 1.00342   | 0.601    | 1.00309    | 0.544    | 1.00280   | 0.492    |
| 1.45 | 1.00211   | 0.763    | 1.00191    | 0.690    | 1.00173   | 0.624    | 1.00156    | 0.565    | 1.00141   | 0.511    |
| 1.5  | 1.00000   | 0.772    | 1.00000    | 0.699    | 1.00000   | 0.632    | 1.00000    | 0.573    | 1.00000   | 0.518    |
| 1.55 | 0.99789   | 0.763    | 0.99809    | 0.690    | 0.99828   | 0.624    | 0.99844    | 0.565    | 0.99859   | 0.511    |
| 1.6  | 0.99584   | 0.734    | 0.99624    | 0.665    | 0.99660   | 0.601    | 0.99692    | 0.544    | 0.99721   | 0.492    |
| 1.65 | 0.99390   | 0.688    | 0.99448    | 0.623    | 0.99500   | 0.563    | 0.99548    | 0.510    | 0.99591   | 0.461    |
| 1.7  | 0.99211   | 0.625    | 0.99286    | 0.565    | 0.99354   | 0.512    | 0.99415    | 0.463    | 0.99471   | 0.419    |
| 1.75 | 0.99052   | 0.546    | 0.99142    | 0.494    | 0.99223   | 0.447    | 0.99297    | 0.405    | 0.99363   | 0.366    |
| 1.8  | 0.98916   | 0.454    | 0.99018    | 0.411    | 0.99111   | 0.372    | 0.99196    | 0.336    | 0.99272   | 0.304    |
| 1.85 | 0.98806   | 0.350    | 0.98919    | 0.317    | 0.99022   | 0.287    | 0.99114    | 0.260    | 0.99198   | 0.235    |
| 1.9  | 0.98727   | 0.239    | 0.98847    | 0.216    | 0.98956   | 0.195    | 0.99055    | 0.177    | 0.99145   | 0.160    |
| 1.95 | 0.98678   | 0.121    | 0.98803    | 0.109    | 0.98916   | 0.099    | 0.99019    | 0.089    | 0.99112   | 0.081    |
| 2.0  | 0.98661   | 0.00     | 0.98788    | 0.00     | 0.98903   | 0.00     | 0.99007    | 0.00     | 0.99101   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(2.5 + i \underline{0.25}) = 0.99052 / \underline{0^\circ.546} = 0.99052 / \underline{0^\circ.32'.46''}$ .  
 $\tanh(2.5 + i \underline{1.75}) = 0.99052 / \underline{0^\circ.546} = 0.99052 / \underline{0^\circ.32'.46''}$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 2.75$ |          | $x = 2.8$ |          | $x = 2.85$ |          | $x = 2.9$ |          | $x = 2.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 0.99186    | 0.00     | 0.99263   | 0.00     | 0.99333    | 0.00     | 0.99396   | 0.00     | 0.99454    | 0.00     |
| 0.05 | 0.99196    | 0.073    | 0.99272   | 0.066    | 0.99341    | 0.060    | 0.99404   | 0.054    | 0.99460    | 0.049    |
| 0.1  | 0.99226    | 0.145    | 0.99299   | 0.131    | 0.99366    | 0.119    | 0.99426   | 0.107    | 0.99480    | 0.097    |
| 0.15 | 0.99275    | 0.213    | 0.99343   | 0.192    | 0.99406    | 0.174    | 0.99462   | 0.158    | 0.99513    | 0.143    |
| 0.2  | 0.99341    | 0.275    | 0.99404   | 0.249    | 0.99460    | 0.226    | 0.99511   | 0.204    | 0.99558    | 0.184    |
| 0.25 | 0.99424    | 0.331    | 0.99478   | 0.300    | 0.99528    | 0.271    | 0.99573   | 0.245    | 0.99613    | 0.222    |
| 0.3  | 0.99521    | 0.379    | 0.99566   | 0.343    | 0.99607    | 0.310    | 0.99645   | 0.281    | 0.99679    | 0.254    |
| 0.35 | 0.99630    | 0.417    | 0.99665   | 0.378    | 0.99697    | 0.342    | 0.99726   | 0.309    | 0.99752    | 0.280    |
| 0.4  | 0.99748    | 0.446    | 0.99772   | 0.403    | 0.99793    | 0.365    | 0.99813   | 0.330    | 0.99831    | 0.299    |
| 0.45 | 0.99872    | 0.463    | 0.99884   | 0.419    | 0.99895    | 0.379    | 0.99905   | 0.343    | 0.99914    | 0.310    |
| 0.5  | 1.00000    | 0.468    | 1.00000   | 0.424    | 1.00000    | 0.383    | 1.00000   | 0.347    | 1.00000    | 0.313    |
| 0.55 | 1.00128    | 0.463    | 1.00116   | 0.419    | 1.00105    | 0.379    | 1.00095   | 0.343    | 1.00086    | 0.310    |
| 0.6  | 1.00253    | 0.446    | 1.00229   | 0.403    | 1.00207    | 0.365    | 1.00187   | 0.330    | 1.00169    | 0.299    |
| 0.65 | 1.00372    | 0.417    | 1.00336   | 0.378    | 1.00304    | 0.342    | 1.00275   | 0.309    | 1.00249    | 0.280    |
| 0.7  | 1.00482    | 0.379    | 1.00436   | 0.343    | 1.00394    | 0.310    | 1.00356   | 0.281    | 1.00323    | 0.254    |
| 0.75 | 1.00580    | 0.331    | 1.00524   | 0.300    | 1.00474    | 0.271    | 1.00429   | 0.245    | 1.00388    | 0.222    |
| 0.8  | 1.00664    | 0.275    | 1.00600   | 0.249    | 1.00543    | 0.226    | 1.00491   | 0.204    | 1.00444    | 0.184    |
| 0.85 | 1.00731    | 0.213    | 1.00661   | 0.192    | 1.00598    | 0.174    | 1.00541   | 0.158    | 1.00489    | 0.143    |
| 0.9  | 1.00780    | 0.145    | 1.00706   | 0.131    | 1.00638    | 0.119    | 1.00578   | 0.107    | 1.00522    | 0.097    |
| 0.95 | 1.00810    | 0.073    | 1.00733   | 0.066    | 1.00663    | 0.060    | 1.00600   | 0.054    | 1.00543    | 0.049    |
| 1.0  | 1.00821    | 0.00     | 1.00742   | 0.00     | 1.00671    | 0.00     | 1.00607   | 0.00     | 1.00549    | 0.00     |
| 1.05 | 1.00810    | 0.073    | 1.00733   | 0.066    | 1.00663    | 0.060    | 1.00600   | 0.054    | 1.00543    | 0.049    |
| 1.1  | 1.00780    | 0.145    | 1.00706   | 0.131    | 1.00638    | 0.119    | 1.00578   | 0.107    | 1.00522    | 0.097    |
| 1.15 | 1.00731    | 0.213    | 1.00661   | 0.192    | 1.00598    | 0.174    | 1.00541   | 0.158    | 1.00489    | 0.143    |
| 1.2  | 1.00664    | 0.275    | 1.00600   | 0.249    | 1.00543    | 0.226    | 1.00491   | 0.204    | 1.00444    | 0.184    |
| 1.25 | 1.00580    | 0.331    | 1.00524   | 0.300    | 1.00474    | 0.271    | 1.00429   | 0.245    | 1.00388    | 0.222    |
| 1.3  | 1.00482    | 0.379    | 1.00436   | 0.343    | 1.00394    | 0.310    | 1.00356   | 0.281    | 1.00323    | 0.254    |
| 1.35 | 1.00372    | 0.417    | 1.00336   | 0.378    | 1.00304    | 0.342    | 1.00275   | 0.309    | 1.00249    | 0.280    |
| 1.4  | 1.00253    | 0.446    | 1.00229   | 0.403    | 1.00207    | 0.365    | 1.00187   | 0.330    | 1.00169    | 0.299    |
| 1.45 | 1.00128    | 0.463    | 1.00116   | 0.419    | 1.00105    | 0.379    | 1.00095   | 0.343    | 1.00086    | 0.310    |
| 1.5  | 1.00000    | 0.468    | 1.00000   | 0.424    | 1.00000    | 0.383    | 1.00000   | 0.347    | 1.00000    | 0.313    |
| 1.55 | 0.99872    | 0.463    | 0.99884   | 0.419    | 0.99895    | 0.379    | 0.99905   | 0.343    | 0.99914    | 0.310    |
| 1.6  | 0.99748    | 0.446    | 0.99772   | 0.403    | 0.99793    | 0.365    | 0.99813   | 0.330    | 0.99831    | 0.299    |
| 1.65 | 0.99630    | 0.417    | 0.99665   | 0.378    | 0.99697    | 0.342    | 0.99726   | 0.309    | 0.99752    | 0.280    |
| 1.7  | 0.99521    | 0.379    | 0.99566   | 0.343    | 0.99607    | 0.310    | 0.99645   | 0.281    | 0.99679    | 0.254    |
| 1.75 | 0.99424    | 0.331    | 0.99478   | 0.300    | 0.99528    | 0.271    | 0.99573   | 0.245    | 0.99613    | 0.222    |
| 1.8  | 0.99341    | 0.275    | 0.99404   | 0.249    | 0.99460    | 0.226    | 0.99511   | 0.204    | 0.99558    | 0.184    |
| 1.85 | 0.99275    | 0.213    | 0.99343   | 0.192    | 0.99406    | 0.174    | 0.99462   | 0.158    | 0.99513    | 0.143    |
| 1.9  | 0.99226    | 0.145    | 0.99299   | 0.131    | 0.99366    | 0.119    | 0.99426   | 0.107    | 0.99480    | 0.097    |
| 1.95 | 0.99196    | 0.073    | 0.99272   | 0.066    | 0.99341    | 0.060    | 0.99404   | 0.054    | 0.99460    | 0.049    |
| 2.0  | 0.99186    | 0.00     | 0.99263   | 0.00     | 0.99333    | 0.00     | 0.99396   | 0.00     | 0.99454    | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(2.9 + i\,0.5) = 1.0000 / 0^{\circ}.347 = 1.0000 / 0^{\circ}.26'.49''$ .  
 $\tanh(2.95 + i\,1.75) = 0.99613 / 0^{\circ}.222 = 0.99613 / 0^{\circ}.13'.19''$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 3.0$ |          | $x = 3.05$ |          | $x = 3.1$ |          | $x = 3.15$ |          | $x = 3.2$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 0.99505   | 0.00     | 0.99552    | 0.00     | 0.99595   | 0.00     | 0.99633    | 0.00     | 0.99668   | 0.00     |
| 0.05 | 0.99511   | 0.044    | 0.99558    | 0.040    | 0.99600   | 0.037    | 0.99638    | 0.033    | 0.99672   | 0.030    |
| 0.1  | 0.99530   | 0.088    | 0.99575    | 0.079    | 0.99615   | 0.072    | 0.99651    | 0.065    | 0.99685   | 0.059    |
| 0.15 | 0.99559   | 0.129    | 0.99601    | 0.117    | 0.99639   | 0.106    | 0.99673    | 0.096    | 0.99704   | 0.086    |
| 0.2  | 0.99600   | 0.167    | 0.99638    | 0.151    | 0.99672   | 0.137    | 0.99703    | 0.124    | 0.99732   | 0.112    |
| 0.25 | 0.99650   | 0.201    | 0.99683    | 0.182    | 0.99714   | 0.165    | 0.99741    | 0.149    | 0.99765   | 0.135    |
| 0.3  | 0.99709   | 0.229    | 0.99737    | 0.208    | 0.99762   | 0.188    | 0.99784    | 0.170    | 0.99805   | 0.154    |
| 0.35 | 0.99775   | 0.253    | 0.99797    | 0.229    | 0.99816   | 0.207    | 0.99833    | 0.188    | 0.99849   | 0.170    |
| 0.4  | 0.99847   | 0.270    | 0.99862    | 0.244    | 0.99875   | 0.221    | 0.99887    | 0.200    | 0.99897   | 0.181    |
| 0.45 | 0.99923   | 0.281    | 0.99930    | 0.254    | 0.99937   | 0.230    | 0.99943    | 0.208    | 0.99948   | 0.188    |
| 0.5  | 1.00000   | 0.284    | 1.00000    | 0.257    | 1.00000   | 0.233    | 1.00000    | 0.211    | 1.00000   | 0.191    |
| 0.55 | 1.00078   | 0.281    | 1.00070    | 0.254    | 1.00064   | 0.230    | 1.00058    | 0.208    | 1.00052   | 0.188    |
| 0.6  | 1.00153   | 0.270    | 1.00139    | 0.244    | 1.00125   | 0.221    | 1.00114    | 0.200    | 1.00103   | 0.181    |
| 0.65 | 1.00225   | 0.253    | 1.00204    | 0.229    | 1.00185   | 0.207    | 1.00167    | 0.188    | 1.00151   | 0.170    |
| 0.7  | 1.00292   | 0.229    | 1.00264    | 0.208    | 1.00239   | 0.188    | 1.00216    | 0.170    | 1.00196   | 0.154    |
| 0.75 | 1.00351   | 0.201    | 1.00318    | 0.182    | 1.00287   | 0.165    | 1.00260    | 0.149    | 1.00235   | 0.135    |
| 0.8  | 1.00402   | 0.167    | 1.00363    | 0.151    | 1.00329   | 0.137    | 1.00297    | 0.124    | 1.00269   | 0.112    |
| 0.85 | 1.00443   | 0.129    | 1.00400    | 0.117    | 1.00360   | 0.106    | 1.00328    | 0.096    | 1.00297   | 0.086    |
| 0.9  | 1.00473   | 0.088    | 1.00427    | 0.079    | 1.00387   | 0.072    | 1.00350    | 0.065    | 1.00316   | 0.059    |
| 0.95 | 1.00491   | 0.044    | 1.00444    | 0.040    | 1.00399   | 0.037    | 1.00363    | 0.033    | 1.00329   | 0.030    |
| 1.0  | 1.00497   | 0.00     | 1.00450    | 0.00     | 1.00407   | 0.00     | 1.00368    | 0.00     | 1.00333   | 0.00     |
| 1.05 | 1.00491   | 0.044    | 1.00444    | 0.040    | 1.00399   | 0.037    | 1.00363    | 0.033    | 1.00329   | 0.030    |
| 1.1  | 1.00473   | 0.088    | 1.00427    | 0.079    | 1.00387   | 0.072    | 1.00350    | 0.065    | 1.00316   | 0.059    |
| 1.15 | 1.00443   | 0.129    | 1.00400    | 0.117    | 1.00360   | 0.106    | 1.00328    | 0.096    | 1.00297   | 0.086    |
| 1.2  | 1.00402   | 0.167    | 1.00363    | 0.151    | 1.00329   | 0.137    | 1.00297    | 0.124    | 1.00269   | 0.112    |
| 1.25 | 1.00351   | 0.201    | 1.00318    | 0.182    | 1.00287   | 0.165    | 1.00260    | 0.149    | 1.00235   | 0.135    |
| 1.3  | 1.00292   | 0.229    | 1.00264    | 0.208    | 1.00239   | 0.188    | 1.00216    | 0.170    | 1.00196   | 0.154    |
| 1.35 | 1.00225   | 0.253    | 1.00204    | 0.229    | 1.00185   | 0.207    | 1.00167    | 0.188    | 1.00151   | 0.170    |
| 1.4  | 1.00153   | 0.270    | 1.00139    | 0.244    | 1.00125   | 0.221    | 1.00114    | 0.200    | 1.00103   | 0.181    |
| 1.45 | 1.00078   | 0.281    | 1.00070    | 0.254    | 1.00064   | 0.230    | 1.00058    | 0.208    | 1.00052   | 0.188    |
| 1.5  | 1.00000   | 0.284    | 1.00000    | 0.257    | 1.00000   | 0.233    | 1.00000    | 0.211    | 1.00000   | 0.191    |
| 1.55 | 0.99923   | 0.281    | 0.99930    | 0.254    | 0.99937   | 0.230    | 0.99943    | 0.208    | 0.99948   | 0.188    |
| 1.6  | 0.99847   | 0.270    | 0.99862    | 0.244    | 0.99875   | 0.221    | 0.99887    | 0.200    | 0.99897   | 0.181    |
| 1.65 | 0.99775   | 0.253    | 0.99797    | 0.229    | 0.99816   | 0.207    | 0.99833    | 0.188    | 0.99849   | 0.170    |
| 1.7  | 0.99709   | 0.229    | 0.99737    | 0.208    | 0.99762   | 0.188    | 0.99784    | 0.170    | 0.99805   | 0.154    |
| 1.75 | 0.99650   | 0.201    | 0.99683    | 0.182    | 0.99714   | 0.165    | 0.99741    | 0.149    | 0.99765   | 0.135    |
| 1.8  | 0.99600   | 0.167    | 0.99638    | 0.151    | 0.99672   | 0.137    | 0.99703    | 0.124    | 0.99732   | 0.112    |
| 1.85 | 0.99559   | 0.129    | 0.99601    | 0.117    | 0.99639   | 0.106    | 0.99673    | 0.096    | 0.99704   | 0.086    |
| 1.9  | 0.99530   | 0.088    | 0.99575    | 0.079    | 0.99615   | 0.072    | 0.99651    | 0.065    | 0.99685   | 0.059    |
| 1.95 | 0.99511   | 0.044    | 0.99558    | 0.040    | 0.99600   | 0.037    | 0.99638    | 0.033    | 0.99672   | 0.030    |
| 2.0  | 0.99505   | 0.00     | 0.99552    | 0.00     | 0.99595   | 0.00     | 0.99633    | 0.00     | 0.99668   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(3.2 + i0) = 0.99668 / 0^\circ$ .

$\tanh(3.2 + i1.05) = 1.00329 / 0^\circ.030 = 1.00329 / 0^\circ.1'.48''$ .

TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 3.25$ |          | $x = 3.3$ |          | $x = 3.35$ |          | $x = 3.4$ |          | $x = 3.45$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 0.99700    | 0.00     | 0.99728   | 0.00     | 0.99754    | 0.00     | 0.99777   | 0.00     | 0.99799    | 0.00     |
| 0.05 | 0.99703    | 0.027    | 0.99732   | 0.024    | 0.99757    | 0.022    | 0.99780   | 0.020    | 0.99801    | 0.018    |
| 0.1  | 0.99714    | 0.053    | 0.99741   | 0.048    | 0.99766    | 0.044    | 0.99788   | 0.039    | 0.99809    | 0.036    |
| 0.15 | 0.99732    | 0.078    | 0.99758   | 0.071    | 0.99781    | 0.064    | 0.99802   | 0.058    | 0.99821    | 0.052    |
| 0.2  | 0.99757    | 0.101    | 0.99780   | 0.092    | 0.99801    | 0.083    | 0.99820   | 0.075    | 0.99837    | 0.068    |
| 0.25 | 0.99788    | 0.122    | 0.99807   | 0.110    | 0.99826    | 0.100    | 0.99843   | 0.090    | 0.99858    | 0.082    |
| 0.3  | 0.99823    | 0.139    | 0.99840   | 0.126    | 0.99855    | 0.114    | 0.99869   | 0.103    | 0.99882    | 0.094    |
| 0.35 | 0.99864    | 0.153    | 0.98877   | 0.139    | 0.99888    | 0.126    | 0.99899   | 0.114    | 0.99909    | 0.103    |
| 0.4  | 0.99907    | 0.164    | 0.99916   | 0.148    | 0.99924    | 0.134    | 0.99931   | 0.121    | 0.99938    | 0.110    |
| 0.45 | 0.99953    | 0.170    | 0.99958   | 0.154    | 0.99962    | 0.139    | 0.99965   | 0.126    | 0.99968    | 0.114    |
| 0.5  | 1.00000    | 0.172    | 1.00000   | 0.156    | 1.00000    | 0.141    | 1.00000   | 0.128    | 1.00000    | 0.116    |
| 0.55 | 1.00047    | 0.170    | 1.00042   | 0.154    | 1.00039    | 0.139    | 1.00035   | 0.126    | 1.00032    | 0.114    |
| 0.6  | 1.00093    | 0.164    | 1.00084   | 0.148    | 1.00076    | 0.134    | 1.00069   | 0.121    | 1.00062    | 0.110    |
| 0.65 | 1.00137    | 0.153    | 1.00124   | 0.139    | 1.00112    | 0.126    | 1.00101   | 0.114    | 1.00092    | 0.103    |
| 0.7  | 1.00177    | 0.139    | 1.00160   | 0.126    | 1.00145    | 0.114    | 1.00131   | 0.103    | 1.00118    | 0.094    |
| 0.75 | 1.00213    | 0.122    | 1.00193   | 0.110    | 1.00174    | 0.100    | 1.00158   | 0.090    | 1.00143    | 0.082    |
| 0.8  | 1.00244    | 0.101    | 1.00220   | 0.092    | 1.00199    | 0.083    | 1.00180   | 0.075    | 1.00163    | 0.068    |
| 0.85 | 1.00268    | 0.078    | 1.00243   | 0.071    | 1.00220    | 0.064    | 1.00199   | 0.058    | 1.00180    | 0.052    |
| 0.9  | 1.00286    | 0.053    | 1.00259   | 0.048    | 1.00234    | 0.044    | 1.00212   | 0.039    | 1.00192    | 0.036    |
| 0.95 | 1.00297    | 0.027    | 1.00269   | 0.024    | 1.00243    | 0.022    | 1.00220   | 0.020    | 1.00199    | 0.018    |
| 1.0  | 1.00301    | 0.00     | 1.00273   | 0.00     | 1.00246    | 0.00     | 1.00223   | 0.00     | 1.00202    | 0.00     |
| 1.05 | 1.00297    | 0.027    | 1.00269   | 0.024    | 1.00243    | 0.022    | 1.00220   | 0.020    | 1.00199    | 0.018    |
| 1.1  | 1.00286    | 0.053    | 1.00259   | 0.048    | 1.00234    | 0.044    | 1.00212   | 0.039    | 1.00192    | 0.036    |
| 1.15 | 1.00268    | 0.078    | 1.00243   | 0.071    | 1.00220    | 0.064    | 1.00199   | 0.058    | 1.00180    | 0.052    |
| 1.2  | 1.00244    | 0.101    | 1.00220   | 0.092    | 1.00199    | 0.083    | 1.00180   | 0.075    | 1.00163    | 0.068    |
| 1.25 | 1.00213    | 0.122    | 1.00193   | 0.110    | 1.00174    | 0.100    | 1.00158   | 0.090    | 1.00143    | 0.082    |
| 1.3  | 1.00177    | 0.139    | 1.00160   | 0.126    | 1.00145    | 0.114    | 1.00131   | 0.103    | 1.00118    | 0.094    |
| 1.35 | 1.00137    | 0.153    | 1.00124   | 0.139    | 1.00112    | 0.126    | 1.00101   | 0.114    | 1.00092    | 0.103    |
| 1.4  | 1.00093    | 0.164    | 1.00084   | 0.148    | 1.00076    | 0.134    | 1.00069   | 0.121    | 1.00062    | 0.110    |
| 1.45 | 1.00047    | 0.170    | 1.00042   | 0.154    | 1.00039    | 0.139    | 1.00035   | 0.126    | 1.00032    | 0.114    |
| 1.5  | 1.00000    | 0.172    | 1.00000   | 0.156    | 1.00000    | 0.141    | 1.00000   | 0.128    | 1.00000    | 0.116    |
| 1.55 | 0.99953    | 0.170    | 0.99958   | 0.154    | 0.99962    | 0.139    | 0.99965   | 0.126    | 0.99968    | 0.114    |
| 1.6  | 0.99907    | 0.164    | 0.99916   | 0.148    | 0.99924    | 0.134    | 0.99931   | 0.121    | 0.99938    | 0.110    |
| 1.65 | 0.99864    | 0.153    | 0.99877   | 0.139    | 0.99888    | 0.126    | 0.99899   | 0.114    | 0.99909    | 0.103    |
| 1.7  | 0.99823    | 0.139    | 0.99840   | 0.126    | 0.99855    | 0.114    | 0.99869   | 0.103    | 0.99882    | 0.094    |
| 1.75 | 0.99788    | 0.122    | 0.99807   | 0.110    | 0.99826    | 0.100    | 0.99843   | 0.090    | 0.99858    | 0.082    |
| 1.8  | 0.99757    | 0.101    | 0.99780   | 0.092    | 0.99801    | 0.083    | 0.99820   | 0.075    | 0.99837    | 0.068    |
| 1.85 | 0.99732    | 0.078    | 0.99758   | 0.071    | 0.99781    | 0.064    | 0.99802   | 0.058    | 0.99821    | 0.052    |
| 1.9  | 0.99714    | 0.053    | 0.99741   | 0.048    | 0.99766    | 0.044    | 0.99788   | 0.039    | 0.99809    | 0.036    |
| 1.95 | 0.99703    | 0.027    | 0.99732   | 0.024    | 0.99757    | 0.022    | 0.99780   | 0.020    | 0.99801    | 0.018    |
| 2.0  | 0.99700    | 0.00     | 0.99728   | 0.00     | 0.99754    | 0.00     | 0.99777   | 0.00     | 0.99799    | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(3.4 + i\,0.7) = 1.00131 / \underline{0^{\circ}.103} = 1.00131 / \underline{0^{\circ}.06'.11''}$ .  
 $\tanh(3.45 + i\,1.4) = 1.00062 / \underline{0^{\circ}.110} = 1.00062 / \underline{0^{\circ}.06'.36''}$ .



TABLE XII. HYPERBOLIC TANGENTS.  $\tanh(x + i\gamma) = r/\gamma$ . CONTINUED

|      | $x = 3.5$ |          | $x = 3.55$ |          | $x = 3.6$ |          | $x = 3.65$ |          | $x = 3.7$ |          |
|------|-----------|----------|------------|----------|-----------|----------|------------|----------|-----------|----------|
| $q$  | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ |
| 0    | 0.99818   | 0.00     | 0.99835    | 0.00     | 0.99851   | 0.00     | 0.99865    | 0.00     | 0.99878   | 0.00     |
| 0.05 | 0.99820   | 0.016    | 0.99837    | 0.015    | 0.99853   | 0.013    | 0.99867    | 0.012    | 0.99879   | 0.011    |
| 0.1  | 0.99827   | 0.032    | 0.99843    | 0.029    | 0.99858   | 0.026    | 0.99872    | 0.024    | 0.99884   | 0.022    |
| 0.15 | 0.99838   | 0.047    | 0.99853    | 0.043    | 0.99867   | 0.039    | 0.99880    | 0.035    | 0.99891   | 0.032    |
| 0.2  | 0.99853   | 0.061    | 0.99867    | 0.055    | 0.99879   | 0.050    | 0.99891    | 0.046    | 0.99901   | 0.041    |
| 0.25 | 0.99871   | 0.074    | 0.99883    | 0.067    | 0.99894   | 0.061    | 0.99905    | 0.055    | 0.99914   | 0.050    |
| 0.3  | 0.99893   | 0.084    | 0.99903    | 0.076    | 0.99912   | 0.069    | 0.99921    | 0.063    | 0.99928   | 0.057    |
| 0.35 | 0.99917   | 0.093    | 0.99925    | 0.084    | 0.99932   | 0.076    | 0.99939    | 0.069    | 0.99945   | 0.063    |
| 0.4  | 0.99944   | 0.099    | 0.99949    | 0.090    | 0.99954   | 0.081    | 0.99958    | 0.074    | 0.99962   | 0.067    |
| 0.45 | 0.99972   | 0.103    | 0.99974    | 0.093    | 0.99977   | 0.085    | 0.99979    | 0.076    | 0.99981   | 0.069    |
| 0.5  | 1.00000   | 0.104    | 1.00000    | 0.095    | 1.00000   | 0.086    | 1.00000    | 0.077    | 1.00000   | 0.070    |
| 0.55 | 1.00028   | 0.103    | 1.00026    | 0.093    | 1.00023   | 0.085    | 1.00021    | 0.076    | 1.00019   | 0.069    |
| 0.6  | 1.00056   | 0.099    | 1.00051    | 0.090    | 1.00046   | 0.081    | 1.00042    | 0.074    | 1.00038   | 0.067    |
| 0.65 | 1.00083   | 0.093    | 1.00075    | 0.084    | 1.00068   | 0.076    | 1.00061    | 0.069    | 1.00056   | 0.063    |
| 0.7  | 1.00107   | 0.084    | 1.00097    | 0.076    | 1.00088   | 0.069    | 1.00079    | 0.063    | 1.00072   | 0.057    |
| 0.75 | 1.00129   | 0.074    | 1.00117    | 0.067    | 1.00106   | 0.061    | 1.00096    | 0.055    | 1.00087   | 0.050    |
| 0.8  | 1.00148   | 0.061    | 1.00134    | 0.056    | 1.00121   | 0.050    | 1.00109    | 0.046    | 1.00099   | 0.041    |
| 0.85 | 1.00163   | 0.048    | 1.00147    | 0.043    | 1.00133   | 0.039    | 1.00120    | 0.035    | 1.00109   | 0.032    |
| 0.9  | 1.00174   | 0.032    | 1.00157    | 0.029    | 1.00142   | 0.026    | 1.00129    | 0.024    | 1.00117   | 0.022    |
| 0.95 | 1.00180   | 0.016    | 1.00163    | 0.015    | 1.00148   | 0.013    | 1.00134    | 0.012    | 1.00121   | 0.011    |
| 1.0  | 1.00183   | 0.00     | 1.00165    | 0.00     | 1.00149   | 0.00     | 1.00135    | 0.00     | 1.00122   | 0.00     |
| 1.05 | 1.00180   | 0.016    | 1.00163    | 0.015    | 1.00148   | 0.013    | 1.00134    | 0.012    | 1.00121   | 0.011    |
| 1.1  | 1.00174   | 0.032    | 1.00157    | 0.029    | 1.00142   | 0.026    | 1.00129    | 0.024    | 1.00117   | 0.022    |
| 1.15 | 1.00163   | 0.048    | 1.00147    | 0.043    | 1.00133   | 0.039    | 1.00120    | 0.035    | 1.00109   | 0.032    |
| 1.2  | 1.00148   | 0.061    | 1.00134    | 0.056    | 1.00121   | 0.050    | 1.00109    | 0.046    | 1.00099   | 0.041    |
| 1.25 | 1.00129   | 0.074    | 1.00117    | 0.067    | 1.00106   | 0.061    | 1.00096    | 0.055    | 1.00087   | 0.050    |
| 1.3  | 1.00107   | 0.084    | 1.00097    | 0.076    | 1.00088   | 0.069    | 1.00079    | 0.063    | 1.00072   | 0.057    |
| 1.35 | 1.00083   | 0.093    | 1.00075    | 0.084    | 1.00068   | 0.076    | 1.00061    | 0.069    | 1.00056   | 0.063    |
| 1.4  | 1.00056   | 0.099    | 1.00051    | 0.090    | 1.00046   | 0.081    | 1.00042    | 0.074    | 1.00038   | 0.067    |
| 1.45 | 1.00028   | 0.103    | 1.00026    | 0.093    | 1.00023   | 0.085    | 1.00021    | 0.076    | 1.00019   | 0.069    |
| 1.5  | 1.00000   | 0.104    | 1.00000    | 0.095    | 1.00000   | 0.086    | 1.00000    | 0.077    | 1.00000   | 0.070    |
| 1.55 | 0.99972   | 0.103    | 0.99974    | 0.093    | 0.99977   | 0.085    | 0.99979    | 0.076    | 0.99981   | 0.069    |
| 1.6  | 0.99944   | 0.099    | 0.99949    | 0.090    | 0.99954   | 0.081    | 0.99958    | 0.074    | 0.99962   | 0.067    |
| 1.65 | 0.99917   | 0.093    | 0.99925    | 0.084    | 0.99932   | 0.076    | 0.99939    | 0.069    | 0.99945   | 0.063    |
| 1.7  | 0.99893   | 0.084    | 0.99903    | 0.076    | 0.99912   | 0.069    | 0.99921    | 0.063    | 0.99928   | 0.057    |
| 1.75 | 0.99871   | 0.074    | 0.99883    | 0.067    | 0.99894   | 0.061    | 0.99905    | 0.055    | 0.99914   | 0.050    |
| 1.8  | 0.99853   | 0.061    | 0.99867    | 0.055    | 0.99879   | 0.050    | 0.99891    | 0.046    | 0.99901   | 0.041    |
| 1.85 | 0.99838   | 0.047    | 0.99853    | 0.043    | 0.99867   | 0.039    | 0.99880    | 0.035    | 0.99891   | 0.032    |
| 1.9  | 0.99827   | 0.032    | 0.99843    | 0.029    | 0.99858   | 0.026    | 0.99872    | 0.024    | 0.99884   | 0.022    |
| 1.95 | 0.99820   | 0.016    | 0.99837    | 0.015    | 0.99853   | 0.013    | 0.99867    | 0.012    | 0.99879   | 0.011    |
| 2.0  | 0.99818   | 0.00     | 0.99835    | 0.00     | 0.99851   | 0.00     | 0.99865    | 0.00     | 0.99878   | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(3.6 + i0) = 0.99851/\underline{0^\circ}$ .

$\tanh(3.7 + i1.7) = 0.99928/\sqrt{0^\circ.057} = 0.99928/\sqrt{0^\circ.03'.25''}$ .

TABLE XII. HYPERBOLIC TANGENTS:  $\tanh(x + iq) = r/\gamma$ . CONTINUED

|      | $x = 3.75$ |          | $x = 3.8$ |          | $x = 3.85$ |          | $x = 3.9$ |          | $x = 3.95$ |          |
|------|------------|----------|-----------|----------|------------|----------|-----------|----------|------------|----------|
| $q$  | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ | $r$       | $\gamma$ | $r$        | $\gamma$ |
| 0    | 0.99889    | 0.00     | 0.99900   | 0.00     | 0.99909    | 0.00     | 0.99918   | 0.00     | 0.99926    | 0.00     |
| 0.05 | 0.99891    | 0.010    | 0.99901   | 0.009    | 0.99911    | 0.008    | 0.99919   | 0.007    | 0.99927    | 0.007    |
| 0.1  | 0.99895    | 0.020    | 0.99905   | 0.018    | 0.99914    | 0.016    | 0.99922   | 0.015    | 0.99930    | 0.013    |
| 0.15 | 0.99902    | 0.029    | 0.99911   | 0.026    | 0.99919    | 0.024    | 0.99927   | 0.021    | 0.99934    | 0.019    |
| 0.2  | 0.99911    | 0.037    | 0.99919   | 0.034    | 0.99927    | 0.031    | 0.99934   | 0.028    | 0.99940    | 0.025    |
| 0.25 | 0.99922    | 0.045    | 0.99929   | 0.041    | 0.99936    | 0.037    | 0.99942   | 0.033    | 0.99948    | 0.030    |
| 0.3  | 0.99935    | 0.051    | 0.99941   | 0.046    | 0.99947    | 0.042    | 0.99952   | 0.038    | 0.99956    | 0.034    |
| 0.35 | 0.99950    | 0.056    | 0.99955   | 0.051    | 0.99959    | 0.046    | 0.99963   | 0.042    | 0.99966    | 0.038    |
| 0.4  | 0.99966    | 0.060    | 0.99969   | 0.055    | 0.99972    | 0.049    | 0.99975   | 0.045    | 0.99977    | 0.040    |
| 0.45 | 0.99983    | 0.063    | 0.99984   | 0.057    | 0.99986    | 0.051    | 0.99987   | 0.046    | 0.99988    | 0.042    |
| 0.5  | 1.00000    | 0.063    | 1.00000   | 0.057    | 1.00000    | 0.052    | 1.00000   | 0.047    | 1.00000    | 0.043    |
| 0.55 | 1.00017    | 0.063    | 1.00016   | 0.057    | 1.00014    | 0.051    | 1.00013   | 0.046    | 1.00012    | 0.042    |
| 0.6  | 1.00034    | 0.060    | 1.00031   | 0.055    | 1.00028    | 0.049    | 1.00025   | 0.045    | 1.00023    | 0.040    |
| 0.65 | 1.00050    | 0.056    | 1.00045   | 0.051    | 1.00041    | 0.046    | 1.00037   | 0.042    | 1.00034    | 0.038    |
| 0.7  | 1.00065    | 0.051    | 1.00059   | 0.046    | 1.00053    | 0.042    | 1.00048   | 0.038    | 1.00044    | 0.034    |
| 0.75 | 1.00078    | 0.045    | 1.00071   | 0.041    | 1.00064    | 0.037    | 1.00058   | 0.033    | 1.00052    | 0.030    |
| 0.8  | 1.00089    | 0.037    | 1.00081   | 0.034    | 1.00073    | 0.031    | 1.00066   | 0.028    | 1.00060    | 0.025    |
| 0.85 | 1.00099    | 0.029    | 1.00089   | 0.026    | 1.00081    | 0.024    | 1.00073   | 0.021    | 1.00066    | 0.019    |
| 0.9  | 1.00105    | 0.020    | 1.00095   | 0.018    | 1.00086    | 0.016    | 1.00078   | 0.015    | 1.00071    | 0.013    |
| 0.95 | 1.00109    | 0.010    | 1.00099   | 0.009    | 1.00089    | 0.008    | 1.00081   | 0.007    | 1.00073    | 0.007    |
| 1.0  | 1.00111    | 0.00     | 1.00100   | 0.00     | 1.00090    | 0.00     | 1.00082   | 0.00     | 1.00074    | 0.00     |
| 1.05 | 1.00109    | 0.010    | 1.00099   | 0.009    | 1.00089    | 0.008    | 1.00081   | 0.007    | 1.00073    | 0.007    |
| 1.1  | 1.00105    | 0.020    | 1.00095   | 0.018    | 1.00086    | 0.016    | 1.00078   | 0.015    | 1.00071    | 0.013    |
| 1.15 | 1.00099    | 0.029    | 1.00089   | 0.026    | 1.00081    | 0.024    | 1.00073   | 0.021    | 1.00066    | 0.019    |
| 1.2  | 1.00089    | 0.037    | 1.00081   | 0.034    | 1.00073    | 0.031    | 1.00066   | 0.028    | 1.00060    | 0.025    |
| 1.25 | 1.00078    | 0.045    | 1.00071   | 0.041    | 1.00064    | 0.037    | 1.00058   | 0.033    | 1.00052    | 0.030    |
| 1.3  | 1.00065    | 0.051    | 1.00059   | 0.046    | 1.00053    | 0.042    | 1.00048   | 0.038    | 1.00044    | 0.034    |
| 1.35 | 1.00050    | 0.056    | 1.00045   | 0.051    | 1.00041    | 0.046    | 1.00037   | 0.042    | 1.00034    | 0.038    |
| 1.4  | 1.00034    | 0.060    | 1.00031   | 0.055    | 1.00028    | 0.049    | 1.00025   | 0.045    | 1.00023    | 0.040    |
| 1.45 | 1.00017    | 0.063    | 1.00016   | 0.057    | 1.00014    | 0.051    | 1.00013   | 0.046    | 1.00012    | 0.042    |
| 1.5  | 1.00000    | 0.063    | 1.00000   | 0.057    | 1.00000    | 0.052    | 1.00000   | 0.047    | 1.00000    | 0.043    |
| 1.55 | 0.99983    | 0.063    | 0.99984   | 0.057    | 0.99986    | 0.051    | 0.99987   | 0.046    | 0.99988    | 0.042    |
| 1.6  | 0.99966    | 0.060    | 0.99969   | 0.055    | 0.99972    | 0.049    | 0.99975   | 0.045    | 0.99977    | 0.040    |
| 1.65 | 0.99950    | 0.056    | 0.99955   | 0.051    | 0.99959    | 0.046    | 0.99963   | 0.042    | 0.99966    | 0.038    |
| 1.7  | 0.99935    | 0.051    | 0.99941   | 0.046    | 0.99947    | 0.042    | 0.99952   | 0.038    | 0.99956    | 0.034    |
| 1.75 | 0.99922    | 0.045    | 0.99929   | 0.041    | 0.99936    | 0.037    | 0.99942   | 0.033    | 0.99948    | 0.030    |
| 1.8  | 0.99911    | 0.037    | 0.99919   | 0.034    | 0.99927    | 0.031    | 0.99934   | 0.028    | 0.99940    | 0.025    |
| 1.85 | 0.99902    | 0.029    | 0.99911   | 0.026    | 0.99919    | 0.024    | 0.99927   | 0.021    | 0.99934    | 0.019    |
| 1.9  | 0.99895    | 0.020    | 0.99905   | 0.018    | 0.99914    | 0.016    | 0.99922   | 0.015    | 0.99930    | 0.013    |
| 1.95 | 0.99891    | 0.010    | 0.99901   | 0.009    | 0.99911    | 0.008    | 0.99919   | 0.007    | 0.99927    | 0.007    |
| 2.0  | 0.99889    | 0.00     | 0.99900   | 0.00     | 0.99909    | 0.00     | 0.99918   | 0.00     | 0.99926    | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\tanh(3.95 + i0.9) = 1.00071 / \overline{0.013} = 1.00071 / \overline{0.0'47''}$ .  
 $\tanh(3.95 + i1.9) = 0.99930 / \overline{0.013} = 0.99930 / \overline{0.0'47''}$ .

TABLE XIII. FUNCTIONS OF  $4 + iq$ .  $f(4 + iq) = u + iv$

| $q$  | sinh     |          | cosh     |          | tanh    |         |
|------|----------|----------|----------|----------|---------|---------|
|      | $u$      | $v$      | $u$      | $v$      | $u$     | $v$     |
| 0    | 27.28992 | 0.00     | 27.30823 | 0.00     | 0.99933 | 0.00    |
| 0.05 | 27.20579 | 2.14258  | 27.22405 | 2.14114  | 0.99934 | 0.00010 |
| 0.1  | 26.95392 | 4.27195  | 26.97202 | 4.26908  | 0.99936 | 0.00021 |
| 0.15 | 26.53588 | 6.37498  | 26.55370 | 6.37071  | 0.99940 | 0.00030 |
| 0.2  | 25.95425 | 8.43871  | 25.97166 | 8.43305  | 0.99946 | 0.00039 |
| 0.25 | 25.21260 | 10.45041 | 25.22951 | 10.44340 | 0.99953 | 0.00047 |
| 0.3  | 24.31551 | 12.39768 | 24.33181 | 12.38935 | 0.99961 | 0.00054 |
| 0.35 | 23.26848 | 14.26851 | 23.28410 | 14.25895 | 0.99970 | 0.00060 |
| 0.4  | 22.07800 | 16.05138 | 22.09282 | 16.04061 | 0.99979 | 0.00064 |
| 0.45 | 20.75141 | 17.73528 | 20.76534 | 17.72339 | 0.99989 | 0.00066 |
| 0.5  | 19.29688 | 19.30983 | 19.30983 | 19.29688 | 1.00000 | 0.00067 |
| 0.55 | 17.72339 | 20.76534 | 17.73528 | 20.75141 | 1.00011 | 0.00066 |
| 0.6  | 16.04061 | 22.09282 | 16.05138 | 22.07800 | 1.00021 | 0.00064 |
| 0.65 | 14.25895 | 23.28410 | 14.26851 | 23.26848 | 1.00030 | 0.00060 |
| 0.7  | 12.39835 | 24.33181 | 12.39768 | 24.31551 | 1.00039 | 0.00054 |
| 0.75 | 10.44340 | 25.22951 | 10.45041 | 25.21260 | 1.00047 | 0.00047 |
| 0.8  | 8.43305  | 25.97166 | 8.43871  | 25.95425 | 1.00054 | 0.00039 |
| 0.85 | 6.37071  | 26.55370 | 6.37498  | 26.53588 | 1.00060 | 0.00030 |
| 0.9  | 4.26908  | 26.97202 | 4.27195  | 26.95392 | 1.00064 | 0.00021 |
| 0.95 | 2.14114  | 27.22405 | 2.14258  | 27.20579 | 1.00066 | 0.00011 |
| 1.0  | 0.00     | 27.30823 | 0.00     | 27.28992 | 1.00067 | 0.00    |
| 1.05 | 2.14114  | 27.22405 | 2.14258  | 27.20579 | 1.00066 | 0.00011 |
| 1.1  | 4.26908  | 26.97202 | 4.27195  | 26.95392 | 1.00064 | 0.00021 |
| 1.15 | 6.37071  | 26.55370 | 6.37498  | 26.53588 | 1.00060 | 0.00030 |
| 1.2  | 8.43305  | 25.97166 | 8.43871  | 25.95425 | 1.00054 | 0.00039 |
| 1.25 | 10.44340 | 25.22951 | 10.45041 | 25.21260 | 1.00047 | 0.00047 |
| 1.3  | 12.39835 | 24.33181 | 12.39768 | 24.31551 | 1.00039 | 0.00054 |
| 1.35 | 14.25895 | 23.28410 | 14.26851 | 23.26848 | 1.00030 | 0.00060 |
| 1.4  | 16.04061 | 22.09282 | 16.05138 | 22.07800 | 1.00021 | 0.00064 |
| 1.45 | 17.72339 | 20.76534 | 17.73528 | 20.75141 | 1.00011 | 0.00066 |
| 1.5  | 19.29688 | 19.30983 | 19.30983 | 19.29688 | 1.00000 | 0.00067 |
| 1.55 | 20.75141 | 17.73528 | 20.76534 | 17.72339 | 0.99989 | 0.00066 |
| 1.6  | 22.07800 | 16.05138 | 22.09282 | 16.04061 | 0.99979 | 0.00064 |
| 1.65 | 23.26848 | 14.26851 | 23.28410 | 14.25895 | 0.99970 | 0.00060 |
| 1.7  | 24.31551 | 12.39768 | 24.33181 | 12.38935 | 0.99961 | 0.00054 |
| 1.75 | 25.21260 | 10.45041 | 25.22951 | 10.44340 | 0.99953 | 0.00047 |
| 1.8  | 25.95425 | 8.43871  | 25.97166 | 8.43305  | 0.99946 | 0.00039 |
| 1.85 | 26.53588 | 6.37498  | 26.55370 | 6.37071  | 0.99940 | 0.00030 |
| 1.9  | 26.95392 | 4.27195  | 26.97202 | 4.26908  | 0.99936 | 0.00021 |
| 1.95 | 27.20579 | 2.14258  | 27.22405 | 2.14114  | 0.99934 | 0.00010 |
| 2.0  | 27.28992 | 0.00     | 27.30823 | 0.00     | 0.99933 | 0.00    |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(4 + i \underline{0.7}) = 12.38935 + i 24.33181$ .  
 $\cosh(4 + i \underline{1.25}) = -10.45041 + i 25.21260$ .

TABLE XIII. FUNCTIONS OF  $4 + iq$ .  $f(4 + iq) = r/\gamma$

| $q$  | sinh     |          | cosh     |          | tanh    |          |
|------|----------|----------|----------|----------|---------|----------|
|      | $r$      | $\gamma$ | $r$      | $\gamma$ | $r$     | $\gamma$ |
| 0    | 27.28992 | 0.00     | 27.30823 | 0.00     | 0.99933 | 0.00     |
| 0.05 | 27.29002 | 4.503    | 27.30810 | 4.497    | 0.99934 | 0.006    |
| 0.1  | 27.29036 | 9.006    | 27.30780 | 8.994    | 0.99936 | 0.012    |
| 0.15 | 27.29090 | 13.509   | 27.30723 | 13.491   | 0.99940 | 0.018    |
| 0.2  | 27.29166 | 18.011   | 27.30650 | 17.989   | 0.99946 | 0.023    |
| 0.25 | 27.29260 | 22.514   | 27.30550 | 22.486   | 0.99953 | 0.027    |
| 0.3  | 27.29370 | 27.016   | 27.30445 | 26.984   | 0.99961 | 0.031    |
| 0.35 | 27.29492 | 31.517   | 27.30324 | 31.483   | 0.99970 | 0.034    |
| 0.4  | 27.29624 | 36.018   | 27.30190 | 35.982   | 0.99979 | 0.037    |
| 0.45 | 27.29764 | 40.519   | 27.30050 | 40.481   | 0.99990 | 0.038    |
| 0.5  | 27.29908 | 45.019   | 27.29908 | 44.981   | 1.00000 | 0.038    |
| 0.55 | 27.30050 | 49.519   | 27.29764 | 49.481   | 1.00010 | 0.038    |
| 0.6  | 27.30190 | 54.018   | 27.29624 | 53.982   | 1.00021 | 0.037    |
| 0.65 | 27.30324 | 58.517   | 27.29492 | 58.483   | 1.00030 | 0.034    |
| 0.7  | 27.30445 | 63.016   | 27.29370 | 62.984   | 1.00039 | 0.031    |
| 0.75 | 27.30550 | 67.514   | 27.29260 | 67.486   | 1.00047 | 0.027    |
| 0.8  | 27.30650 | 72.011   | 27.29166 | 71.989   | 1.00054 | 0.023    |
| 0.85 | 27.30723 | 76.509   | 27.29090 | 76.491   | 1.00060 | 0.018    |
| 0.9  | 27.30780 | 81.006   | 27.29036 | 80.994   | 1.00064 | 0.012    |
| 0.95 | 27.30810 | 85.503   | 27.29002 | 85.497   | 1.00066 | 0.006    |
| 1.0  | 27.30823 | 90       | 27.28992 | 90       | 1.00067 | 0.00     |
| 1.05 | 27.30810 | 94.497   | 27.29002 | 94.503   | 1.00066 | 0.006    |
| 1.1  | 27.30780 | 98.994   | 27.29036 | 99.006   | 1.00064 | 0.012    |
| 1.15 | 27.30723 | 103.491  | 27.29090 | 103.509  | 1.00060 | 0.018    |
| 1.2  | 27.30650 | 107.989  | 27.29166 | 108.011  | 1.00054 | 0.023    |
| 1.25 | 27.30550 | 112.486  | 27.29260 | 112.514  | 1.00047 | 0.027    |
| 1.3  | 27.30445 | 116.984  | 27.29370 | 117.016  | 1.00039 | 0.031    |
| 1.35 | 27.30324 | 121.483  | 27.29492 | 121.517  | 1.00030 | 0.034    |
| 1.4  | 27.30190 | 125.982  | 27.29624 | 126.018  | 1.00021 | 0.037    |
| 1.45 | 27.30050 | 130.481  | 27.29764 | 130.519  | 1.00010 | 0.038    |
| 1.5  | 27.29908 | 134.981  | 27.29908 | 135.019  | 1.00000 | 0.038    |
| 1.55 | 27.29764 | 139.481  | 27.30050 | 139.519  | 0.99990 | 0.038    |
| 1.6  | 27.29624 | 143.982  | 27.30190 | 144.018  | 0.99979 | 0.037    |
| 1.65 | 27.29492 | 148.483  | 27.30324 | 148.517  | 0.99970 | 0.034    |
| 1.7  | 27.29370 | 152.984  | 27.30445 | 153.016  | 0.99961 | 0.031    |
| 1.75 | 27.29260 | 157.486  | 27.30550 | 157.514  | 0.99953 | 0.027    |
| 1.8  | 27.29166 | 161.989  | 27.30650 | 162.011  | 0.99946 | 0.023    |
| 1.85 | 27.29090 | 166.491  | 27.30723 | 166.509  | 0.99940 | 0.018    |
| 1.9  | 27.29036 | 170.994  | 27.30780 | 171.006  | 0.99936 | 0.012    |
| 1.95 | 27.29002 | 175.497  | 27.30810 | 175.503  | 0.99934 | 0.006    |
| 2.0  | 27.28992 | 180      | 27.30823 | 180      | 0.99933 | 0.00     |

Note. Negative quantities are in heavy type.

Examples.  $\sinh(4 + i \underline{1.0}) = 27.30823 / 90^\circ$ .

$\tanh(4 + i \underline{1.5}) = 1.0000 \sqrt{0^\circ.038} = 1.0000 \sqrt{0^\circ.02'17''}$ .

TABLE XIV. SEMI-EXPONENTIALS.

 $\frac{e^x}{2}$  and  $\log_{10} \left( \frac{e^x}{2} \right)$ 

| $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ | $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ | $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ |
|------|-----------------|---------------------------|------|-----------------|---------------------------|------|-----------------|---------------------------|
| 4.00 | 27.299          | 1.4361479                 | 4.50 | 45.009          | 1.6532952                 | 5.00 | 74.207          | 1.8704424                 |
| 4.01 | 27.573          | 1.4404909                 | 4.51 | 45.461          | 1.6576381                 | 5.01 | 74.952          | 1.8747854                 |
| 4.02 | 27.851          | 1.4448338                 | 4.52 | 45.918          | 1.6619811                 | 5.02 | 75.706          | 1.8791283                 |
| 4.03 | 28.130          | 1.4491768                 | 4.53 | 46.379          | 1.6663240                 | 5.03 | 76.467          | 1.8834712                 |
| 4.04 | 28.413          | 1.4535197                 | 4.54 | 46.845          | 1.6706669                 | 5.04 | 77.235          | 1.8878142                 |
| 4.05 | 28.699          | 1.4578627                 | 4.55 | 47.316          | 1.6750099                 | 5.05 | 78.011          | 1.8921571                 |
| 4.06 | 28.987          | 1.4622056                 | 4.56 | 47.792          | 1.6793528                 | 5.06 | 78.795          | 1.8965001                 |
| 4.07 | 29.278          | 1.4665485                 | 4.57 | 48.272          | 1.6836958                 | 5.07 | 79.587          | 1.9008430                 |
| 4.08 | 29.573          | 1.4708915                 | 4.58 | 48.757          | 1.6880387                 | 5.08 | 80.387          | 1.9051860                 |
| 4.09 | 29.870          | 1.4752344                 | 4.59 | 49.247          | 1.6923817                 | 5.09 | 81.195          | 1.9095289                 |
| 4.10 | 30.170          | 1.4795774                 | 4.60 | 49.742          | 1.6967246                 | 5.10 | 82.011          | 1.9138719                 |
| 4.11 | 30.473          | 1.4839203                 | 4.61 | 50.242          | 1.7010676                 | 5.11 | 82.835          | 1.9182148                 |
| 4.12 | 30.780          | 1.4882633                 | 4.62 | 50.747          | 1.7054105                 | 5.12 | 83.668          | 1.9225577                 |
| 4.13 | 31.089          | 1.4926062                 | 4.63 | 51.257          | 1.7097535                 | 5.13 | 84.509          | 1.9269007                 |
| 4.14 | 31.401          | 1.4969492                 | 4.64 | 51.772          | 1.7140964                 | 5.14 | 85.358          | 1.9312436                 |
| 4.15 | 31.717          | 1.5012921                 | 4.65 | 52.292          | 1.7184393                 | 5.15 | 86.216          | 1.9355866                 |
| 4.16 | 32.036          | 1.5056350                 | 4.66 | 52.818          | 1.7227823                 | 5.16 | 87.082          | 1.9399295                 |
| 4.17 | 32.358          | 1.5099780                 | 4.67 | 53.349          | 1.7271252                 | 5.17 | 87.957          | 1.9442725                 |
| 4.18 | 32.683          | 1.5143209                 | 4.68 | 53.885          | 1.7314682                 | 5.18 | 88.841          | 1.9486154                 |
| 4.19 | 33.011          | 1.5186639                 | 4.69 | 54.427          | 1.7358111                 | 5.19 | 89.734          | 1.9529584                 |
| 4.20 | 33.343          | 1.5230068                 | 4.70 | 54.974          | 1.7401541                 | 5.20 | 90.636          | 1.9573013                 |
| 4.21 | 33.678          | 1.5273498                 | 4.71 | 55.526          | 1.7444970                 | 5.21 | 91.547          | 1.9616443                 |
| 4.22 | 34.017          | 1.5316927                 | 4.72 | 56.084          | 1.7488400                 | 5.22 | 92.467          | 1.9659872                 |
| 4.23 | 34.359          | 1.5360357                 | 4.73 | 56.648          | 1.7531829                 | 5.23 | 93.396          | 1.9703301                 |
| 4.24 | 34.704          | 1.5403786                 | 4.74 | 57.217          | 1.7575258                 | 5.24 | 94.335          | 1.9746731                 |
| 4.25 | 35.053          | 1.5447215                 | 4.75 | 57.792          | 1.7618688                 | 5.25 | 95.283          | 1.9790160                 |
| 4.26 | 35.405          | 1.5490645                 | 4.76 | 58.373          | 1.7662117                 | 5.26 | 96.241          | 1.9833590                 |
| 4.27 | 35.761          | 1.5534074                 | 4.77 | 58.960          | 1.7705547                 | 5.27 | 97.208          | 1.9877019                 |
| 4.28 | 36.120          | 1.5577504                 | 4.78 | 59.552          | 1.7748976                 | 5.28 | 98.185          | 1.9920449                 |
| 4.29 | 36.483          | 1.5620933                 | 4.79 | 60.151          | 1.7792406                 | 5.29 | 99.172          | 1.9963878                 |
| 4.30 | 36.850          | 1.5664363                 | 4.80 | 60.755          | 1.7835835                 | 5.30 | 100.168         | 2.0007308                 |
| 4.31 | 37.220          | 1.5707792                 | 4.81 | 61.366          | 1.7879265                 | 5.31 | 101.175         | 2.0050737                 |
| 4.32 | 37.594          | 1.5751222                 | 4.82 | 61.983          | 1.7922694                 | 5.32 | 102.192         | 2.0094166                 |
| 4.33 | 37.972          | 1.5794651                 | 4.83 | 62.605          | 1.7966123                 | 5.33 | 103.219         | 2.0137596                 |
| 4.34 | 38.354          | 1.5838081                 | 4.84 | 63.235          | 1.8009553                 | 5.34 | 104.256         | 2.0181025                 |
| 4.35 | 38.739          | 1.5881510                 | 4.85 | 63.870          | 1.8052982                 | 5.35 | 105.304         | 2.0224455                 |
| 4.36 | 39.129          | 1.5924939                 | 4.86 | 64.512          | 1.8096412                 | 5.36 | 106.362         | 2.0267884                 |
| 4.37 | 39.522          | 1.5968369                 | 4.87 | 65.160          | 1.8139841                 | 5.37 | 107.431         | 2.0311314                 |
| 4.38 | 39.919          | 1.6011798                 | 4.88 | 65.815          | 1.8183271                 | 5.38 | 108.511         | 2.0354743                 |
| 4.39 | 40.320          | 1.6055228                 | 4.89 | 66.477          | 1.8226700                 | 5.39 | 109.602         | 2.0398173                 |
| 4.40 | 40.725          | 1.6098657                 | 4.90 | 67.145          | 1.8270130                 | 5.40 | 110.703         | 2.0441602                 |
| 4.41 | 41.135          | 1.6142087                 | 4.91 | 67.820          | 1.8313559                 | 5.41 | 111.816         | 2.0485031                 |
| 4.42 | 41.548          | 1.6185516                 | 4.92 | 68.501          | 1.8356989                 | 5.42 | 112.940         | 2.0528461                 |
| 4.43 | 41.966          | 1.6228946                 | 4.93 | 69.190          | 1.8400418                 | 5.43 | 114.075         | 2.0571890                 |
| 4.44 | 42.387          | 1.6272375                 | 4.94 | 69.885          | 1.8443847                 | 5.44 | 115.221         | 2.0615320                 |
| 4.45 | 42.813          | 1.6315804                 | 4.95 | 70.587          | 1.8487277                 | 5.45 | 116.379         | 2.0658749                 |
| 4.46 | 43.244          | 1.6359234                 | 4.96 | 71.297          | 1.8530706                 | 5.46 | 117.549         | 2.0702179                 |
| 4.47 | 43.678          | 1.6402663                 | 4.97 | 72.013          | 1.8574136                 | 5.47 | 118.730         | 2.0745608                 |
| 4.48 | 44.117          | 1.6446093                 | 4.98 | 72.737          | 1.8617565                 | 5.48 | 119.923         | 2.0789038                 |
| 4.49 | 44.561          | 1.6489522                 | 4.99 | 73.468          | 1.8660995                 | 5.49 | 121.129         | 2.0832467                 |
| 4.50 | 45.009          | 1.6532952                 | 5.00 | 74.207          | 1.8704424                 | 5.50 | 122.346         | 2.0875897                 |

Example.  $\frac{e^{4.20}}{2} = 33.343 \quad \log_{10} \left( \frac{e^{4.20}}{2} \right) = 1.5230068.$

[ 140 ]

TABLE XIV. SEMI-EXPONENTIALS.  $\frac{e^x}{2}$  and  $\log_{10} \left( \frac{e^x}{2} \right)$ . CONTINUED

| $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ | $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ | $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ |
|------|-----------------|---------------------------|------|-----------------|---------------------------|------|-----------------|---------------------------|
| 5.50 | 122.346         | 2.0875897                 | 6.00 | 201.714         | 2.3047369                 | 6.50 | 332.571         | 2.5218844                 |
| 5.51 | 123.576         | 2.0919326                 | 6.01 | 203.742         | 2.3090785                 | 6.51 | 335.913         | 2.5262268                 |
| 5.52 | 124.818         | 2.0962755                 | 6.02 | 205.789         | 2.3134222                 | 6.52 | 339.289         | 2.5305699                 |
| 5.53 | 126.072         | 2.1006185                 | 6.03 | 207.858         | 2.3177667                 | 6.53 | 342.699         | 2.5349128                 |
| 5.54 | 127.339         | 2.1049614                 | 6.04 | 209.947         | 2.3221098                 | 6.54 | 346.143         | 2.5392556                 |
| 5.55 | 128.619         | 2.1093044                 | 6.05 | 212.057         | 2.3264527                 | 6.55 | 349.622         | 2.5435988                 |
| 5.56 | 129.911         | 2.1136473                 | 6.06 | 214.188         | 2.3307951                 | 6.56 | 353.135         | 2.5479408                 |
| 5.57 | 131.217         | 2.1179903                 | 6.07 | 216.340         | 2.3351368                 | 6.57 | 356.685         | 2.5522849                 |
| 5.58 | 132.536         | 2.1223332                 | 6.08 | 218.514         | 2.3394793                 | 6.58 | 360.270         | 2.5566281                 |
| 5.59 | 133.868         | 2.1266762                 | 6.09 | 220.711         | 2.3438220                 | 6.59 | 363.890         | 2.5609701                 |
| 5.60 | 135.213         | 2.1310191                 | 6.10 | 222.929         | 2.3481666                 | 6.60 | 367.547         | 2.5653129                 |
| 5.61 | 136.572         | 2.1353620                 | 6.11 | 225.169         | 2.3525086                 | 6.61 | 371.241         | 2.5696560                 |
| 5.62 | 137.945         | 2.1397050                 | 6.12 | 227.432         | 2.3568516                 | 6.62 | 374.973         | 2.5740000                 |
| 5.63 | 139.331         | 2.1440479                 | 6.13 | 229.718         | 2.3611951                 | 6.63 | 378.741         | 2.5783424                 |
| 5.64 | 140.731         | 2.1483909                 | 6.14 | 232.027         | 2.3655386                 | 6.64 | 382.547         | 2.5826849                 |
| 5.65 | 142.146         | 2.1527338                 | 6.15 | 234.359         | 2.3698817                 | 6.65 | 386.391         | 2.5870270                 |
| 5.66 | 143.574         | 2.1570768                 | 6.16 | 236.714         | 2.3742240                 | 6.66 | 390.275         | 2.5913708                 |
| 5.67 | 145.017         | 2.1614197                 | 6.17 | 239.093         | 2.3785669                 | 6.67 | 394.197         | 2.5957134                 |
| 5.68 | 146.475         | 2.1657627                 | 6.18 | 241.496         | 2.3829100                 | 6.68 | 398.160         | 2.6000576                 |
| 5.69 | 147.947         | 2.1701056                 | 6.19 | 243.923         | 2.3872527                 | 6.69 | 402.161         | 2.6044000                 |
| 5.70 | 149.434         | 2.1744485                 | 6.20 | 246.375         | 2.3915966                 | 6.70 | 406.202         | 2.6087420                 |
| 5.71 | 150.936         | 2.1787915                 | 6.21 | 248.851         | 2.3959394                 | 6.71 | 410.285         | 2.6130856                 |
| 5.72 | 152.452         | 2.1831344                 | 6.22 | 251.352         | 2.4002824                 | 6.72 | 414.409         | 2.6174292                 |
| 5.73 | 153.985         | 2.1874774                 | 6.23 | 253.877         | 2.4046234                 | 6.73 | 418.574         | 2.6217721                 |
| 5.74 | 155.532         | 2.1918203                 | 6.24 | 256.429         | 2.4089672                 | 6.74 | 422.780         | 2.6261144                 |
| 5.75 | 157.095         | 2.1961633                 | 6.25 | 259.006         | 2.4133099                 | 6.75 | 427.030         | 2.6304584                 |
| 5.76 | 158.674         | 2.2005062                 | 6.26 | 261.609         | 2.4176526                 | 6.76 | 431.321         | 2.6348006                 |
| 5.77 | 160.269         | 2.2048492                 | 6.27 | 264.239         | 2.4219970                 | 6.77 | 435.656         | 2.6391436                 |
| 5.78 | 161.880         | 2.2091921                 | 6.28 | 266.894         | 2.4263388                 | 6.78 | 440.034         | 2.6434862                 |
| 5.79 | 163.507         | 2.2135351                 | 6.29 | 269.576         | 2.4306813                 | 6.79 | 444.457         | 2.6478298                 |
| 5.80 | 165.150         | 2.2178780                 | 6.30 | 272.285         | 2.4350237                 | 6.80 | 448.923         | 2.6521719                 |
| 5.81 | 166.810         | 2.2222209                 | 6.31 | 275.022         | 2.4393675                 | 6.81 | 453.435         | 2.6565151                 |
| 5.82 | 168.486         | 2.2265639                 | 6.32 | 277.786         | 2.4437104                 | 6.82 | 457.993         | 2.6608589                 |
| 5.83 | 170.179         | 2.2309068                 | 6.33 | 280.578         | 2.4480536                 | 6.83 | 462.595         | 2.6652009                 |
| 5.84 | 171.890         | 2.2352498                 | 6.34 | 283.398         | 2.4523967                 | 6.84 | 467.244         | 2.6695437                 |
| 5.85 | 173.617         | 2.2395927                 | 6.35 | 286.246         | 2.4567394                 | 6.85 | 471.940         | 2.6738868                 |
| 5.86 | 175.362         | 2.2439357                 | 6.36 | 289.123         | 2.4610826                 | 6.86 | 476.683         | 2.6782296                 |
| 5.87 | 177.124         | 2.2482786                 | 6.37 | 292.029         | 2.4654260                 | 6.87 | 481.474         | 2.6825728                 |
| 5.88 | 178.905         | 2.2526216                 | 6.38 | 294.964         | 2.4697690                 | 6.88 | 486.312         | 2.6869150                 |
| 5.89 | 180.703         | 2.2569645                 | 6.39 | 297.928         | 2.4741114                 | 6.89 | 491.200         | 2.6912584                 |
| 5.90 | 182.519         | 2.2613074                 | 6.40 | 300.922         | 2.4784540                 | 6.90 | 496.137         | 2.6956016                 |
| 5.91 | 184.353         | 2.2656504                 | 6.41 | 303.947         | 2.4827979                 | 6.91 | 501.123         | 2.6999443                 |
| 5.92 | 186.206         | 2.2699933                 | 6.42 | 307.002         | 2.4871412                 | 6.92 | 506.160         | 2.7042878                 |
| 5.93 | 188.077         | 2.2743363                 | 6.43 | 310.087         | 2.4914836                 | 6.93 | 511.246         | 2.7086299                 |
| 5.94 | 189.967         | 2.2786792                 | 6.44 | 313.203         | 2.4958260                 | 6.94 | 516.386         | 2.7129744                 |
| 5.95 | 191.877         | 2.2830222                 | 6.45 | 316.352         | 2.5001705                 | 6.95 | 521.575         | 2.7173168                 |
| 5.96 | 193.805         | 2.2873651                 | 6.46 | 319.530         | 2.5045116                 | 6.96 | 526.816         | 2.7216589                 |
| 5.97 | 195.753         | 2.2917081                 | 6.47 | 322.742         | 2.5088555                 | 6.97 | 532.112         | 2.7260030                 |
| 5.98 | 197.720         | 2.2960510                 | 6.48 | 325.985         | 2.5131977                 | 6.98 | 537.459         | 2.7303454                 |
| 5.99 | 199.707         | 2.3003939                 | 6.49 | 329.262         | 2.5175416                 | 6.99 | 542.860         | 2.7346878                 |
| 6.00 | 201.714         | 2.3047369                 | 6.50 | 332.571         | 2.5218844                 | 7.00 | 548.317         | 2.7390317                 |

Example.  $\frac{e^{5.60}}{2} = 135.213 \quad \log_{10} \left( \frac{e^{5.60}}{2} \right) = 2.1310191$   
[141]

TABLE XIV. SEMI-EXPONENTIALS.  $\frac{e^x}{2}$  and  $\log_{10} \left( \frac{e^x}{2} \right)$ . CONTINUED

| $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ | $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ | $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ |
|------|-----------------|---------------------------|------|-----------------|---------------------------|------|-----------------|---------------------------|
| 7.00 | 548.317         | 2.7390317                 | 7.50 | 904.021         | 2.9561785                 | 8.00 | 1490.479        | 3.1733259                 |
| 7.01 | 553.827         | 2.7433741                 | 7.51 | 913.107         | 2.9605217                 | 8.01 | 1505.457        | 3.1776683                 |
| 7.02 | 559.393         | 2.7477170                 | 7.52 | 922.284         | 2.9648647                 | 8.02 | 1520.589        | 3.1820118                 |
| 7.03 | 565.015         | 2.7520600                 | 7.53 | 931.553         | 2.9692076                 | 8.03 | 1535.870        | 3.1863546                 |
| 7.04 | 570.694         | 2.7564033                 | 7.54 | 940.915         | 2.9735504                 | 8.04 | 1551.306        | 3.1906976                 |
| 7.05 | 576.429         | 2.7607458                 | 7.55 | 950.371         | 2.9778932                 | 8.05 | 1566.895        | 3.1950399                 |
| 7.06 | 582.223         | 2.7650893                 | 7.56 | 959.923         | 2.9822364                 | 8.06 | 1582.645        | 3.1993836                 |
| 7.07 | 588.074         | 2.7694320                 | 7.57 | 969.570         | 2.9865792                 | 8.07 | 1598.552        | 3.2037268                 |
| 7.08 | 593.984         | 2.7737747                 | 7.58 | 979.314         | 2.9909220                 | 8.08 | 1614.617        | 3.2080694                 |
| 7.09 | 599.954         | 2.7781180                 | 7.59 | 989.157         | 2.9952653                 | 8.09 | 1630.841        | 3.2124116                 |
| 7.10 | 605.984         | 2.7824612                 | 7.60 | 999.098         | 2.9996080                 | 8.10 | 1647.234        | 3.2167554                 |
| 7.11 | 612.074         | 2.7868039                 | 7.61 | 1009.139        | 3.0039510                 | 8.11 | 1663.789        | 3.2210981                 |
| 7.12 | 618.225         | 2.7911467                 | 7.62 | 1019.281        | 3.0082939                 | 8.12 | 1680.510        | 3.2254412                 |
| 7.13 | 624.439         | 2.7954901                 | 7.63 | 1029.525        | 3.0126369                 | 8.13 | 1697.400        | 3.2297842                 |
| 7.14 | 630.714         | 2.7998335                 | 7.64 | 1039.872        | 3.0169799                 | 8.14 | 1714.458        | 3.2341270                 |
| 7.15 | 637.053         | 2.8041755                 | 7.65 | 1050.323        | 3.0213229                 | 8.15 | 1731.690        | 3.2384703                 |
| 7.16 | 643.456         | 2.8085189                 | 7.66 | 1060.879        | 3.0256659                 | 8.16 | 1749.092        | 3.2428130                 |
| 7.17 | 649.922         | 2.8128612                 | 7.67 | 1071.541        | 3.0300088                 | 8.17 | 1766.672        | 3.2471560                 |
| 7.18 | 656.454         | 2.8172043                 | 7.68 | 1082.310        | 3.0343517                 | 8.18 | 1784.427        | 3.2514988                 |
| 7.19 | 663.052         | 2.8215476                 | 7.69 | 1093.187        | 3.0386944                 | 8.19 | 1802.364        | 3.2558425                 |
| 7.20 | 669.715         | 2.8258901                 | 7.70 | 1104.174        | 3.0430376                 | 8.20 | 1820.476        | 3.2601848                 |
| 7.21 | 676.446         | 2.8302331                 | 7.71 | 1115.271        | 3.0473806                 | 8.21 | 1838.774        | 3.2645284                 |
| 7.22 | 683.245         | 2.8345765                 | 7.72 | 1126.480        | 3.0517234                 | 8.22 | 1857.251        | 3.2688706                 |
| 7.23 | 690.111         | 2.8389189                 | 7.73 | 1137.801        | 3.0560663                 | 8.23 | 1875.914        | 3.2732129                 |
| 7.24 | 697.047         | 2.8432620                 | 7.74 | 1149.236        | 3.0604092                 | 8.24 | 1894.770        | 3.2775566                 |
| 7.25 | 704.052         | 2.8476047                 | 7.75 | 1160.786        | 3.0647523                 | 8.25 | 1913.812        | 3.2818994                 |
| 7.26 | 711.128         | 2.8519478                 | 7.76 | 1172.452        | 3.0690950                 | 8.26 | 1933.047        | 3.2862424                 |
| 7.27 | 718.275         | 2.8562908                 | 7.77 | 1184.236        | 3.0734383                 | 8.27 | 1952.473        | 3.2905850                 |
| 7.28 | 725.404         | 2.8606338                 | 7.78 | 1196.137        | 3.0777810                 | 8.28 | 1972.098        | 3.2949284                 |
| 7.29 | 732.785         | 2.8649766                 | 7.79 | 1208.159        | 3.0821242                 | 8.29 | 1991.913        | 3.2992704                 |
| 7.30 | 740.150         | 2.8693197                 | 7.80 | 1220.301        | 3.0864670                 | 8.30 | 2011.936        | 3.3036142                 |
| 7.31 | 747.589         | 2.8736629                 | 7.81 | 1232.565        | 3.0908098                 | 8.31 | 2032.158        | 3.3079575                 |
| 7.32 | 755.102         | 2.8780056                 | 7.82 | 1244.953        | 3.0951531                 | 8.32 | 2052.580        | 3.3123000                 |
| 7.33 | 762.691         | 2.8823487                 | 7.83 | 1257.465        | 3.0994961                 | 8.33 | 2073.206        | 3.3166425                 |
| 7.34 | 770.356         | 2.8866915                 | 7.84 | 1270.102        | 3.1038386                 | 8.34 | 2094.045        | 3.3209860                 |
| 7.35 | 778.098         | 2.8910343                 | 7.85 | 1282.867        | 3.1081818                 | 8.35 | 2115.092        | 3.3253293                 |
| 7.36 | 785.918         | 2.8953772                 | 7.86 | 1295.760        | 3.1125246                 | 8.36 | 2136.347        | 3.3296718                 |
| 7.37 | 793.817         | 2.8997205                 | 7.87 | 1308.783        | 3.1168677                 | 8.37 | 2157.819        | 3.3340150                 |
| 7.38 | 801.795         | 2.9040633                 | 7.88 | 1321.936        | 3.1212105                 | 8.38 | 2179.505        | 3.3383578                 |
| 7.39 | 809.853         | 2.9084062                 | 7.89 | 1335.222        | 3.1255535                 | 8.39 | 2201.409        | 3.3427008                 |
| 7.40 | 817.992         | 2.9127491                 | 7.90 | 1348.641        | 3.1298964                 | 8.40 | 2223.533        | 3.3470436                 |
| 7.41 | 826.213         | 2.9170920                 | 7.91 | 1362.195        | 3.1342394                 | 8.41 | 2245.881        | 3.3513868                 |
| 7.42 | 834.517         | 2.9214351                 | 7.92 | 1375.886        | 3.1385826                 | 8.42 | 2268.452        | 3.3557296                 |
| 7.43 | 842.904         | 2.9257782                 | 7.93 | 1389.713        | 3.1429254                 | 8.43 | 2291.250        | 3.3600725                 |
| 7.44 | 851.375         | 2.9301209                 | 7.94 | 1403.680        | 3.1472680                 | 8.44 | 2314.277        | 3.3644154                 |
| 7.45 | 859.932         | 2.9344641                 | 7.95 | 1417.787        | 3.1516110                 | 8.45 | 2337.536        | 3.3687583                 |
| 7.46 | 868.574         | 2.9388068                 | 7.96 | 1432.036        | 3.1559539                 | 8.46 | 2361.030        | 3.3731014                 |
| 7.47 | 877.303         | 2.9431496                 | 7.97 | 1446.429        | 3.1602971                 | 8.47 | 2384.752        | 3.3774433                 |
| 7.48 | 886.120         | 2.9474925                 | 7.98 | 1460.966        | 3.1646402                 | 8.48 | 2408.725        | 3.3817872                 |
| 7.49 | 895.026         | 2.9518356                 | 7.99 | 1475.648        | 3.1689827                 | 8.49 | 2432.926        | 3.3861290                 |
| 7.50 | 904.021         | 2.9561785                 | 8.00 | 1490.479        | 3.1733259                 | 8.50 | 2457.383        | 3.3904730                 |

Example.  $\frac{e^{7.10}}{2} = 605.984$   $\log_{10} \left( \frac{e^{7.10}}{2} \right) = 2.7824612$ .

TABLE XIV. SEMI-EXPONENTIALS.  $\frac{e^x}{2}$  and  $\log_{10} \left( \frac{e^x}{2} \right)$ . CONTINUED

| $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ | $x$  | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ | $x$   | $\frac{e^x}{2}$ | $\log_{10} \frac{e^x}{2}$ |
|------|-----------------|---------------------------|------|-----------------|---------------------------|-------|-----------------|---------------------------|
| 8.50 | 2457.383        | 3.3904730                 | 9.00 | 4051.543        | 3.6076204                 | 9.50  | 6679.863        | 3.8247676                 |
| 8.51 | 2482.082        | 3.3948162                 | 9.01 | 4092.263        | 3.6119636                 | 9.51  | 6746.988        | 3.8291101                 |
| 8.52 | 2507.027        | 3.3991590                 | 9.02 | 4133.388        | 3.6163062                 | 9.52  | 6814.805        | 3.8334534                 |
| 8.53 | 2532.221        | 3.4035016                 | 9.03 | 4174.929        | 3.6206491                 | 9.53  | 6883.295        | 3.8377964                 |
| 8.54 | 2557.672        | 3.4078448                 | 9.04 | 4216.889        | 3.6249922                 | 9.54  | 6952.475        | 3.8421394                 |
| 8.55 | 2583.380        | 3.4121882                 | 9.05 | 4259.264        | 3.6293345                 | 9.55  | 7022.345        | 3.8464822                 |
| 8.56 | 2609.341        | 3.4165308                 | 9.06 | 4302.076        | 3.6336780                 | 9.56  | 7092.923        | 3.8508252                 |
| 8.57 | 2635.562        | 3.4208732                 | 9.07 | 4345.302        | 3.6380200                 | 9.57  | 7164.203        | 3.8551679                 |
| 8.58 | 2662.052        | 3.4252166                 | 9.08 | 4388.982        | 3.6423638                 | 9.58  | 7236.210        | 3.8595112                 |
| 8.59 | 2688.810        | 3.4295601                 | 9.09 | 4433.098        | 3.6467073                 | 9.59  | 7308.929        | 3.8638537                 |
| 8.60 | 2715.830        | 3.4339026                 | 9.10 | 4477.646        | 3.6510498                 | 9.60  | 7382.390        | 3.8681970                 |
| 8.61 | 2743.126        | 3.4382458                 | 9.11 | 4522.647        | 3.6553927                 | 9.61  | 7456.583        | 3.8725398                 |
| 8.62 | 2770.693        | 3.4425884                 | 9.12 | 4568.100        | 3.6597356                 | 9.62  | 7531.526        | 3.8768830                 |
| 8.63 | 2798.535        | 3.4469308                 | 9.13 | 4614.016        | 3.6640791                 | 9.63  | 7607.221        | 3.8812260                 |
| 8.64 | 2826.665        | 3.4512744                 | 9.14 | 4660.383        | 3.6684216                 | 9.64  | 7683.672        | 3.8855688                 |
| 8.65 | 2855.070        | 3.4556167                 | 9.15 | 4707.211        | 3.6727637                 | 9.65  | 7760.882        | 3.8899111                 |
| 8.66 | 2883.767        | 3.4599602                 | 9.16 | 4754.528        | 3.6771074                 | 9.66  | 7838.890        | 3.8942546                 |
| 8.67 | 2912.745        | 3.4643025                 | 9.17 | 4802.308        | 3.6814500                 | 9.67  | 7917.680        | 3.8985980                 |
| 8.68 | 2942.023        | 3.4686462                 | 9.18 | 4850.577        | 3.6857934                 | 9.68  | 7997.247        | 3.9029406                 |
| 8.69 | 2971.592        | 3.4729891                 | 9.19 | 4899.328        | 3.6901365                 | 9.69  | 8077.622        | 3.9072835                 |
| 8.70 | 3001.456        | 3.4773320                 | 9.20 | 4948.563        | 3.6944792                 | 9.70  | 8158.802        | 3.9116264                 |
| 8.71 | 3031.621        | 3.4816749                 | 9.21 | 4998.284        | 3.6988209                 | 9.71  | 8240.792        | 3.9159690                 |
| 8.72 | 3062.088        | 3.4860178                 | 9.22 | 5048.532        | 3.7031652                 | 9.72  | 8323.623        | 3.9203124                 |
| 8.73 | 3092.852        | 3.4903592                 | 9.23 | 5099.272        | 3.7075082                 | 9.73  | 8407.262        | 3.9246546                 |
| 8.74 | 3123.948        | 3.4947038                 | 9.24 | 5150.519        | 3.7118510                 | 9.74  | 8491.770        | 3.9289982                 |
| 8.75 | 3155.337        | 3.4990458                 | 9.25 | 5202.272        | 3.7161930                 | 9.75  | 8577.112        | 3.9333411                 |
| 8.76 | 3187.054        | 3.5033896                 | 9.26 | 5254.569        | 3.7205370                 | 9.76  | 8663.316        | 3.9376842                 |
| 8.77 | 3219.085        | 3.5077325                 | 9.27 | 5307.367        | 3.7248791                 | 9.77  | 8750.384        | 3.9420270                 |
| 8.78 | 3251.440        | 3.5120756                 | 9.28 | 5360.716        | 3.7292228                 | 9.78  | 8838.326        | 3.9463700                 |
| 8.79 | 3284.114        | 3.5164182                 | 9.29 | 5414.587        | 3.7335654                 | 9.79  | 8927.154        | 3.9507131                 |
| 8.80 | 3317.122        | 3.5207614                 | 9.30 | 5469.009        | 3.7379086                 | 9.80  | 9016.875        | 3.9550560                 |
| 8.81 | 3350.460        | 3.5251044                 | 9.31 | 5523.975        | 3.7422517                 | 9.81  | 9107.481        | 3.9593983                 |
| 8.82 | 3384.133        | 3.5294474                 | 9.32 | 5579.491        | 3.7465946                 | 9.82  | 9199.026        | 3.9637418                 |
| 8.83 | 3418.141        | 3.5337900                 | 9.33 | 5635.563        | 3.7509373                 | 9.83  | 9291.480        | 3.9680850                 |
| 8.84 | 3452.496        | 3.5381332                 | 9.34 | 5692.203        | 3.7552804                 | 9.84  | 9384.860        | 3.9724278                 |
| 8.85 | 3487.197        | 3.5424766                 | 9.35 | 5749.405        | 3.7596229                 | 9.85  | 9479.163        | 3.9767701                 |
| 8.86 | 3522.243        | 3.5468192                 | 9.36 | 5807.194        | 3.7639664                 | 9.86  | 9574.444        | 3.9811136                 |
| 8.87 | 3557.631        | 3.5511609                 | 9.37 | 5865.555        | 3.7683091                 | 9.87  | 9670.678        | 3.9854569                 |
| 8.88 | 3593.395        | 3.5555050                 | 9.38 | 5924.507        | 3.7726522                 | 9.88  | 9767.860        | 3.9897994                 |
| 8.89 | 3629.512        | 3.5598482                 | 9.39 | 5984.054        | 3.7769956                 | 9.89  | 9866.020        | 3.9941420                 |
| 8.90 | 3665.986        | 3.5641908                 | 9.40 | 6044.191        | 3.7813382                 | 9.90  | 9965.186        | 3.9984854                 |
| 8.91 | 3702.820        | 3.5685326                 | 9.41 | 6104.922        | 3.7856801                 | 9.91  | 10065.350       | 4.0028289                 |
| 8.92 | 3740.045        | 3.5728768                 | 9.42 | 6166.290        | 3.7900240                 | 9.92  | 10166.494       | 4.0071712                 |
| 8.93 | 3777.635        | 3.5772201                 | 9.43 | 6228.269        | 3.7943674                 | 9.93  | 10268.667       | 4.0115141                 |
| 8.94 | 3815.597        | 3.5815626                 | 9.44 | 6290.860        | 3.7987100                 | 9.94  | 10371.873       | 4.0158572                 |
| 8.95 | 3853.937        | 3.5859044                 | 9.45 | 6354.080        | 3.8030526                 | 9.95  | 10476.107       | 4.0201999                 |
| 8.96 | 3892.678        | 3.5902486                 | 9.46 | 6417.943        | 3.8073958                 | 9.96  | 10581.397       | 4.0245430                 |
| 8.97 | 3931.795        | 3.5945909                 | 9.47 | 6482.450        | 3.8117392                 | 9.97  | 10687.745       | 4.0288860                 |
| 8.98 | 3971.316        | 3.5989344                 | 9.48 | 6547.591        | 3.8160816                 | 9.98  | 10795.160       | 4.0332290                 |
| 8.99 | 4011.228        | 3.6032773                 | 9.49 | 6613.388        | 3.8204240                 | 9.99  | 10903.652       | 4.0375721                 |
| 9.00 | 4051.543        | 3.6076204                 | 9.50 | 6679.863        | 3.8247676                 | 10.00 | 11013.233       | 4.0419148                 |

Example.  $\frac{e^{8.90}}{2} = 3665.986$   $\log_{10} \left( \frac{e^{8.90}}{2} \right) = 3.5641908$ .



TABLE XV

REAL HYPERBOLIC FUNCTIONS.  $f(x + io) = u + io$ 

| $\theta$ | $\sinh \theta$ | $\cosh \theta$ | $\tanh \theta$ | $\coth \theta$ | $\operatorname{sech} \theta$ | $\operatorname{cosech} \theta$ | $\theta$ |
|----------|----------------|----------------|----------------|----------------|------------------------------|--------------------------------|----------|
| 0.00     | 0.00           | 1.00           | 0.00           | $\infty$       | 1.00                         | $\infty$                       | 0.00     |
| 0.01     | 0.010000       | 1.000050       | 0.010000       | 100.           | 0.9999                       | 100.                           | 0.01     |
| 0.02     | 0.020001       | 1.000200       | 0.020000       | 50.            | 0.9998                       | 50.                            | 0.02     |
| 0.03     | 0.030005       | 1.000450       | 0.029999       | 33.34          | 0.9995                       | 33.333                         | 0.03     |
| 0.04     | 0.040011       | 1.000800       | 0.03998        | 25.013         | 0.9992                       | 24.99                          | 0.04     |
| 0.05     | 0.050021       | 1.001250       | 0.04996        | 20.016         | 0.9987                       | 19.992                         | 0.05     |
| 0.06     | 0.060036       | 1.001801       | 0.05993        | 16.686         | 0.9982                       | 16.657                         | 0.06     |
| 0.07     | 0.070057       | 1.002451       | 0.06989        | 14.308         | 0.9975                       | 14.274                         | 0.07     |
| 0.08     | 0.080085       | 1.003202       | 0.07983        | 12.527         | 0.9968                       | 12.487                         | 0.08     |
| 0.09     | 0.090122       | 1.004053       | 0.08976        | 11.141         | 0.9959                       | 11.097                         | 0.09     |
| 0.10     | 0.100167       | 1.005004       | 0.09967        | 10.033         | 0.9950                       | 9.983                          | 0.10     |
| 0.11     | 0.110222       | 1.006056       | 0.10956        | 9.128          | 0.9940                       | 9.073                          | 0.11     |
| 0.12     | 0.120288       | 1.007209       | 0.11943        | 8.373          | 0.9928                       | 8.314                          | 0.12     |
| 0.13     | 0.130366       | 1.008462       | 0.12927        | 7.735          | 0.9916                       | 7.669                          | 0.13     |
| 0.14     | 0.140458       | 1.009816       | 0.13909        | 7.189          | 0.9902                       | 7.120                          | 0.14     |
| 0.15     | 0.150563       | 1.011271       | 0.14888        | 6.716          | 0.9888                       | 6.642                          | 0.15     |
| 0.16     | 0.160684       | 1.012827       | 0.15865        | 6.303          | 0.9873                       | 6.223                          | 0.16     |
| 0.17     | 0.170820       | 1.014485       | 0.16838        | 5.939          | 0.9857                       | 5.854                          | 0.17     |
| 0.18     | 0.180974       | 1.016244       | 0.17808        | 5.615          | 0.9840                       | 5.525                          | 0.18     |
| 0.19     | 0.191145       | 1.018104       | 0.18775        | 5.325          | 0.9822                       | 5.232                          | 0.19     |
| 0.20     | 0.201336       | 1.020067       | 0.19737        | 5.067          | 0.9803                       | 4.967                          | 0.20     |
| 0.21     | 0.211547       | 1.022131       | 0.20696        | 4.832          | 0.9784                       | 4.726                          | 0.21     |
| 0.22     | 0.221779       | 1.024298       | 0.21652        | 4.618          | 0.9763                       | 4.509                          | 0.22     |
| 0.23     | 0.232033       | 1.026567       | 0.22603        | 4.425          | 0.9742                       | 4.310                          | 0.23     |
| 0.24     | 0.242311       | 1.028939       | 0.23549        | 4.246          | 0.9719                       | 4.127                          | 0.24     |
| 0.25     | 0.252612       | 1.031413       | 0.24492        | 4.083          | 0.9695                       | 3.959                          | 0.25     |
| 0.26     | 0.262939       | 1.033991       | 0.25430        | 3.932          | 0.9671                       | 3.803                          | 0.26     |
| 0.27     | 0.273292       | 1.036672       | 0.26363        | 3.793          | 0.9646                       | 3.659                          | 0.27     |
| 0.28     | 0.283673       | 1.039457       | 0.27290        | 3.664          | 0.9620                       | 3.525                          | 0.28     |
| 0.29     | 0.294082       | 1.042346       | 0.28214        | 3.544          | 0.9591                       | 3.400                          | 0.29     |
| 0.30     | 0.304520       | 1.045339       | 0.29131        | 3.433          | 0.9566                       | 3.284                          | 0.30     |
| 0.31     | 0.314989       | 1.048436       | 0.30043        | 3.328          | 0.9537                       | 3.175                          | 0.31     |
| 0.32     | 0.325489       | 1.051638       | 0.30951        | 3.231          | 0.9511                       | 3.072                          | 0.32     |
| 0.33     | 0.336022       | 1.054946       | 0.31852        | 3.140          | 0.9479                       | 2.976                          | 0.33     |
| 0.34     | 0.346589       | 1.058359       | 0.32748        | 3.053          | 0.9447                       | 2.885                          | 0.34     |
| 0.35     | 0.357190       | 1.061878       | 0.33637        | 2.973          | 0.9416                       | 2.800                          | 0.35     |
| 0.36     | 0.367827       | 1.065503       | 0.34522        | 2.897          | 0.9385                       | 2.719                          | 0.36     |
| 0.37     | 0.378500       | 1.069234       | 0.35399        | 2.825          | 0.9353                       | 2.642                          | 0.37     |
| 0.38     | 0.389212       | 1.073073       | 0.36271        | 2.757          | 0.9319                       | 2.569                          | 0.38     |
| 0.39     | 0.399962       | 1.077019       | 0.37136        | 2.693          | 0.9285                       | 2.500                          | 0.39     |
| 0.40     | 0.410752       | 1.081072       | 0.37995        | 2.632          | 0.9250                       | 2.434                          | 0.40     |
| 0.41     | 0.421584       | 1.085234       | 0.38847        | 2.574          | 0.9215                       | 2.372                          | 0.41     |
| 0.42     | 0.432457       | 1.089504       | 0.39693        | 2.519          | 0.9178                       | 2.312                          | 0.42     |
| 0.43     | 0.443374       | 1.093883       | 0.40532        | 2.467          | 0.9141                       | 2.256                          | 0.43     |
| 0.44     | 0.454335       | 1.098372       | 0.41365        | 2.417          | 0.9103                       | 2.201                          | 0.44     |
| 0.45     | 0.465342       | 1.102970       | 0.42190        | 2.370          | 0.9066                       | 2.149                          | 0.45     |
| 0.46     | 0.476395       | 1.107679       | 0.43009        | 2.325          | 0.9025                       | 2.099                          | 0.46     |
| 0.47     | 0.487490       | 1.112498       | 0.43820        | 2.282          | 0.8988                       | 2.051                          | 0.47     |
| 0.48     | 0.498646       | 1.117429       | 0.44624        | 2.241          | 0.8949                       | 2.006                          | 0.48     |
| 0.49     | 0.509845       | 1.122471       | 0.45421        | 2.202          | 0.8909                       | 1.961                          | 0.49     |

Example.  $\sinh 0.25 = 0.252612$ .

TABLE XV

REAL HYPERBOLIC FUNCTIONS.  $f(x + io) = u + io$ . CONTINUED

| $\theta$ | Sinh $\theta$ | Cosh $\theta$ | Tanh $\theta$ | Coth $\theta$ | Sech $\theta$ | Cosech $\theta$ | $\theta$ |
|----------|---------------|---------------|---------------|---------------|---------------|-----------------|----------|
| 0.50     | 0.521095      | 1.127626      | 0.46211       | 2.164         | 0.8868        | 1.919           | 0.50     |
| 0.51     | 0.532398      | 1.132893      | 0.46995       | 2.128         | 0.8827        | 1.878           | 0.51     |
| 0.52     | 0.543754      | 1.138274      | 0.47769       | 2.093         | 0.8785        | 1.839           | 0.52     |
| 0.53     | 0.555104      | 1.143769      | 0.48538       | 2.060         | 0.8743        | 1.801           | 0.53     |
| 0.54     | 0.566629      | 1.149378      | 0.49299       | 2.028         | 0.8700        | 1.765           | 0.54     |
| 0.55     | 0.578152      | 1.155101      | 0.50052       | 1.998         | 0.8658        | 1.730           | 0.55     |
| 0.56     | 0.589732      | 1.160941      | 0.50797       | 1.969         | 0.8614        | 1.696           | 0.56     |
| 0.57     | 0.601371      | 1.166896      | 0.51536       | 1.940         | 0.8570        | 1.663           | 0.57     |
| 0.58     | 0.613070      | 1.172968      | 0.52266       | 1.913         | 0.8525        | 1.631           | 0.58     |
| 0.59     | 0.624831      | 1.179158      | 0.52990       | 1.887         | 0.8480        | 1.601           | 0.59     |
| 0.60     | 0.636654      | 1.185465      | 0.53704       | 1.862         | 0.8435        | 1.571           | 0.60     |
| 0.61     | 0.648540      | 1.191891      | 0.54413       | 1.838         | 0.8390        | 1.542           | 0.61     |
| 0.62     | 0.660492      | 1.198436      | 0.55112       | 1.814         | 0.8344        | 1.514           | 0.62     |
| 0.63     | 0.672509      | 1.205101      | 0.55805       | 1.792         | 0.8298        | 1.487           | 0.63     |
| 0.64     | 0.684594      | 1.211887      | 0.56490       | 1.770         | 0.8251        | 1.461           | 0.64     |
| 0.65     | 0.696748      | 1.218793      | 0.57166       | 1.749         | 0.8205        | 1.435           | 0.65     |
| 0.66     | 0.708970      | 1.225822      | 0.57836       | 1.729         | 0.8158        | 1.410           | 0.66     |
| 0.67     | 0.721264      | 1.232973      | 0.58498       | 1.709         | 0.8110        | 1.387           | 0.67     |
| 0.68     | 0.733630      | 1.240247      | 0.59152       | 1.690         | 0.8065        | 1.363           | 0.68     |
| 0.69     | 0.746070      | 1.247646      | 0.59798       | 1.672         | 0.8015        | 1.340           | 0.69     |
| 0.70     | 0.758584      | 1.255169      | 0.60437       | 1.655         | 0.7967        | 1.318           | 0.70     |
| 0.71     | 0.771174      | 1.262818      | 0.61067       | 1.637         | 0.7919        | 1.297           | 0.71     |
| 0.72     | 0.783840      | 1.270593      | 0.61691       | 1.621         | 0.7870        | 1.276           | 0.72     |
| 0.73     | 0.796586      | 1.278495      | 0.62306       | 1.605         | 0.7821        | 1.255           | 0.73     |
| 0.74     | 0.809411      | 1.286525      | 0.62914       | 1.590         | 0.7773        | 1.235           | 0.74     |
| 0.75     | 0.822317      | 1.294683      | 0.63516       | 1.574         | 0.7724        | 1.216           | 0.75     |
| 0.76     | 0.835305      | 1.302971      | 0.64108       | 1.559         | 0.7675        | 1.197           | 0.76     |
| 0.77     | 0.848377      | 1.311390      | 0.64693       | 1.545         | 0.7625        | 1.178           | 0.77     |
| 0.78     | 0.861533      | 1.319939      | 0.65271       | 1.532         | 0.7576        | 1.160           | 0.78     |
| 0.79     | 0.874776      | 1.328621      | 0.65842       | 1.518         | 0.7527        | 1.143           | 0.79     |
| 0.80     | 0.888106      | 1.337435      | 0.66403       | 1.505         | 0.7477        | 1.125           | 0.80     |
| 0.81     | 0.901525      | 1.346383      | 0.66959       | 1.493         | 0.7427        | 1.109           | 0.81     |
| 0.82     | 0.915034      | 1.355466      | 0.67507       | 1.481         | 0.7377        | 1.092           | 0.82     |
| 0.83     | 0.928635      | 1.364684      | 0.68047       | 1.469         | 0.7327        | 1.076           | 0.83     |
| 0.84     | 0.942328      | 1.374039      | 0.68580       | 1.458         | 0.7278        | 1.061           | 0.84     |
| 0.85     | 0.956116      | 1.383531      | 0.69107       | 1.447         | 0.7228        | 1.045           | 0.85     |
| 0.86     | 0.969999      | 1.393161      | 0.69626       | 1.436         | 0.7178        | 1.030           | 0.86     |
| 0.87     | 0.983980      | 1.402931      | 0.70137       | 1.425         | 0.7128        | 1.016           | 0.87     |
| 0.88     | 0.998058      | 1.412841      | 0.70642       | 1.415         | 0.7078        | 1.002           | 0.88     |
| 0.89     | 1.012237      | 1.422893      | 0.71139       | 1.405         | 0.7028        | 0.988           | 0.89     |
| 0.90     | 1.026517      | 1.433086      | 0.71629       | 1.396         | 0.6978        | 0.973           | 0.90     |
| 0.91     | 1.040899      | 1.443423      | 0.72114       | 1.387         | 0.6928        | 0.960           | 0.91     |
| 0.92     | 1.055386      | 1.453905      | 0.72591       | 1.377         | 0.6878        | 0.947           | 0.92     |
| 0.93     | 1.069978      | 1.464531      | 0.73060       | 1.368         | 0.6828        | 0.934           | 0.93     |
| 0.94     | 1.084677      | 1.475305      | 0.73522       | 1.360         | 0.6778        | 0.921           | 0.94     |
| 0.95     | 1.099484      | 1.486225      | 0.73979       | 1.351         | 0.6728        | 0.909           | 0.95     |
| 0.96     | 1.114402      | 1.497295      | 0.74427       | 1.343         | 0.6678        | 0.897           | 0.96     |
| 0.97     | 1.129431      | 1.508514      | 0.74870       | 1.335         | 0.6629        | 0.885           | 0.97     |
| 0.98     | 1.144573      | 1.519884      | 0.75306       | 1.327         | 0.6579        | 0.873           | 0.98     |
| 0.99     | 1.159829      | 1.531406      | 0.75736       | 1.320         | 0.6529        | 0.862           | 0.99     |

Example.  $\cosh 0.55 = 1.155101$ .

TABLE XV

REAL HYPERBOLIC FUNCTIONS.  $f(x + io) = u + io$ . CONTINUED

| $\theta$ | $\sinh \theta$ | $\cosh \theta$ | $\tanh \theta$ | $\coth \theta$ | $\operatorname{sech} \theta$ | $\operatorname{cosech} \theta$ | $\theta$ |
|----------|----------------|----------------|----------------|----------------|------------------------------|--------------------------------|----------|
| 1.00     | 1.175201       | 1.543081       | 0.76159        | 1.3130         | 0.6480                       | 0.8509                         | 1.00     |
| 1.01     | 1.190691       | 1.554910       | 0.76576        | 1.3059         | 0.6431                       | 0.8395                         | 1.01     |
| 1.02     | 1.206300       | 1.566895       | 0.76987        | 1.2989         | 0.6382                       | 0.8290                         | 1.02     |
| 1.03     | 1.222029       | 1.579036       | 0.77391        | 1.2921         | 0.6333                       | 0.8183                         | 1.03     |
| 1.04     | 1.237881       | 1.591336       | 0.77789        | 1.2855         | 0.6284                       | 0.8078                         | 1.04     |
| 1.05     | 1.253857       | 1.603794       | 0.78181        | 1.2791         | 0.6235                       | 0.7975                         | 1.05     |
| 1.06     | 1.269958       | 1.616413       | 0.78566        | 1.2728         | 0.6186                       | 0.7874                         | 1.06     |
| 1.07     | 1.286185       | 1.629194       | 0.78946        | 1.2666         | 0.6138                       | 0.7777                         | 1.07     |
| 1.08     | 1.302542       | 1.642138       | 0.79320        | 1.2607         | 0.6090                       | 0.7677                         | 1.08     |
| 1.09     | 1.319029       | 1.655245       | 0.79688        | 1.2549         | 0.6042                       | 0.7581                         | 1.09     |
| 1.10     | 1.335647       | 1.668519       | 0.80050        | 1.2492         | 0.5993                       | 0.7487                         | 1.10     |
| 1.11     | 1.352400       | 1.681959       | 0.80406        | 1.2437         | 0.5945                       | 0.7393                         | 1.11     |
| 1.12     | 1.369287       | 1.695567       | 0.80757        | 1.2382         | 0.5898                       | 0.7302                         | 1.12     |
| 1.13     | 1.386312       | 1.709345       | 0.81102        | 1.2330         | 0.5850                       | 0.7215                         | 1.13     |
| 1.14     | 1.403475       | 1.723294       | 0.81441        | 1.2279         | 0.5803                       | 0.7125                         | 1.14     |
| 1.15     | 1.420778       | 1.737415       | 0.81775        | 1.2229         | 0.5755                       | 0.7038                         | 1.15     |
| 1.16     | 1.438224       | 1.751710       | 0.82104        | 1.2180         | 0.5708                       | 0.6953                         | 1.16     |
| 1.17     | 1.455813       | 1.766180       | 0.82427        | 1.2132         | 0.5662                       | 0.6869                         | 1.17     |
| 1.18     | 1.473548       | 1.780826       | 0.82745        | 1.2085         | 0.5616                       | 0.6786                         | 1.18     |
| 1.19     | 1.491430       | 1.795651       | 0.83058        | 1.2040         | 0.5569                       | 0.6705                         | 1.19     |
| 1.20     | 1.509461       | 1.810656       | 0.83365        | 1.1995         | 0.5523                       | 0.6625                         | 1.20     |
| 1.21     | 1.527644       | 1.825841       | 0.83668        | 1.1952         | 0.5477                       | 0.6546                         | 1.21     |
| 1.22     | 1.545979       | 1.841209       | 0.83965        | 1.1910         | 0.5431                       | 0.6468                         | 1.22     |
| 1.23     | 1.564468       | 1.856761       | 0.84258        | 1.1868         | 0.5385                       | 0.6392                         | 1.23     |
| 1.24     | 1.583115       | 1.872499       | 0.84546        | 1.1828         | 0.5340                       | 0.6317                         | 1.24     |
| 1.25     | 1.601919       | 1.888424       | 0.84828        | 1.1789         | 0.5296                       | 0.6242                         | 1.25     |
| 1.26     | 1.620884       | 1.904538       | 0.85106        | 1.1750         | 0.5251                       | 0.6170                         | 1.26     |
| 1.27     | 1.640010       | 1.920842       | 0.85380        | 1.1712         | 0.5206                       | 0.6098                         | 1.27     |
| 1.28     | 1.659301       | 1.937339       | 0.85648        | 1.1675         | 0.5162                       | 0.6026                         | 1.28     |
| 1.29     | 1.678758       | 1.954029       | 0.85913        | 1.1640         | 0.5118                       | 0.5957                         | 1.29     |
| 1.30     | 1.698382       | 1.970914       | 0.86172        | 1.1605         | 0.5074                       | 0.5888                         | 1.30     |
| 1.31     | 1.718177       | 1.987997       | 0.86428        | 1.1570         | 0.5030                       | 0.5820                         | 1.31     |
| 1.32     | 1.738143       | 2.005278       | 0.86678        | 1.1537         | 0.4987                       | 0.5753                         | 1.32     |
| 1.33     | 1.758283       | 2.022760       | 0.86925        | 1.1504         | 0.4944                       | 0.5687                         | 1.33     |
| 1.34     | 1.778599       | 2.040445       | 0.87167        | 1.1472         | 0.4901                       | 0.5623                         | 1.34     |
| 1.35     | 1.799093       | 2.058333       | 0.87405        | 1.1441         | 0.4858                       | 0.5559                         | 1.35     |
| 1.36     | 1.819766       | 2.076427       | 0.87639        | 1.1410         | 0.4816                       | 0.5495                         | 1.36     |
| 1.37     | 1.840622       | 2.094729       | 0.87869        | 1.1380         | 0.4773                       | 0.5433                         | 1.37     |
| 1.38     | 1.861662       | 2.113240       | 0.88095        | 1.1351         | 0.4732                       | 0.5372                         | 1.38     |
| 1.39     | 1.882887       | 2.131963       | 0.88317        | 1.1323         | 0.4690                       | 0.5311                         | 1.39     |
| 1.40     | 1.904302       | 2.150898       | 0.88535        | 1.1295         | 0.4649                       | 0.5252                         | 1.40     |
| 1.41     | 1.925906       | 2.170049       | 0.88749        | 1.1268         | 0.4608                       | 0.5192                         | 1.41     |
| 1.42     | 1.947703       | 2.189417       | 0.88960        | 1.1241         | 0.4568                       | 0.5134                         | 1.42     |
| 1.43     | 1.969695       | 2.209004       | 0.89167        | 1.1215         | 0.4527                       | 0.5077                         | 1.43     |
| 1.44     | 1.991884       | 2.228812       | 0.89370        | 1.1189         | 0.4486                       | 0.5020                         | 1.44     |
| 1.45     | 2.014272       | 2.248842       | 0.89569        | 1.1165         | 0.4446                       | 0.4964                         | 1.45     |
| 1.46     | 2.036862       | 2.269098       | 0.89765        | 1.1140         | 0.4407                       | 0.4909                         | 1.46     |
| 1.47     | 2.059655       | 2.289580       | 0.89958        | 1.1116         | 0.4367                       | 0.4855                         | 1.47     |
| 1.48     | 2.082654       | 2.310292       | 0.90147        | 1.1093         | 0.4329                       | 0.4802                         | 1.48     |
| 1.49     | 2.105861       | 2.331234       | 0.90332        | 1.1070         | 0.4290                       | 0.4749                         | 1.49     |

Example.  $\tanh 1.25 = 0.84828$ .

TABLE XV

REAL HYPERBOLIC FUNCTIONS.  $f(x + io) = u + io$ . CONTINUED

| $\theta$ | $\sinh \theta$ | $\cosh \theta$ | $\tanh \theta$ | $\coth \theta$ | $\operatorname{sech} \theta$ | $\operatorname{cosech} \theta$ | $\theta$ |
|----------|----------------|----------------|----------------|----------------|------------------------------|--------------------------------|----------|
| 1.50     | 2.129279       | 2.352410       | 0.90515        | 1.1048         | 0.4251                       | 0.4697                         | 1.50     |
| 1.51     | 2.152910       | 2.373820       | 0.90694        | 1.1026         | 0.4212                       | 0.4645                         | 1.51     |
| 1.52     | 2.176757       | 2.395469       | 0.90870        | 1.1005         | 0.4174                       | 0.4594                         | 1.52     |
| 1.53     | 2.200821       | 2.417356       | 0.91042        | 1.0984         | 0.4137                       | 0.4543                         | 1.53     |
| 1.54     | 2.225105       | 2.439486       | 0.91212        | 1.0963         | 0.4099                       | 0.4494                         | 1.54     |
| 1.55     | 2.249611       | 2.461859       | 0.91379        | 1.0944         | 0.4062                       | 0.4444                         | 1.55     |
| 1.56     | 2.274343       | 2.484479       | 0.91542        | 1.0924         | 0.4025                       | 0.4398                         | 1.56     |
| 1.57     | 2.299302       | 2.507347       | 0.91703        | 1.0905         | 0.3988                       | 0.4350                         | 1.57     |
| 1.58     | 2.324490       | 2.530465       | 0.91860        | 1.0886         | 0.3952                       | 0.4302                         | 1.58     |
| 1.59     | 2.349912       | 2.553837       | 0.92015        | 1.0868         | 0.3916                       | 0.4255                         | 1.59     |
| 1.60     | 2.375568       | 2.577464       | 0.92167        | 1.0850         | 0.3879                       | 0.4209                         | 1.60     |
| 1.61     | 2.401462       | 2.601349       | 0.92316        | 1.0832         | 0.3844                       | 0.4164                         | 1.61     |
| 1.62     | 2.427596       | 2.625495       | 0.92462        | 1.0815         | 0.3809                       | 0.4119                         | 1.62     |
| 1.63     | 2.453973       | 2.649902       | 0.92606        | 1.0798         | 0.3774                       | 0.4075                         | 1.63     |
| 1.64     | 2.480595       | 2.674575       | 0.92747        | 1.0782         | 0.3739                       | 0.4031                         | 1.64     |
| 1.65     | 2.507465       | 2.699515       | 0.92886        | 1.0766         | 0.3704                       | 0.3988                         | 1.65     |
| 1.66     | 2.534586       | 2.724725       | 0.93022        | 1.0750         | 0.3670                       | 0.3945                         | 1.66     |
| 1.67     | 2.561960       | 2.750207       | 0.93155        | 1.0735         | 0.3636                       | 0.3903                         | 1.67     |
| 1.68     | 2.589591       | 2.775965       | 0.93286        | 1.0719         | 0.3602                       | 0.3862                         | 1.68     |
| 1.69     | 2.617481       | 2.802000       | 0.93415        | 1.0704         | 0.3569                       | 0.3820                         | 1.69     |
| 1.70     | 2.645632       | 2.828315       | 0.93541        | 1.0691         | 0.3536                       | 0.3780                         | 1.70     |
| 1.71     | 2.674048       | 2.854914       | 0.93665        | 1.0676         | 0.3503                       | 0.3740                         | 1.71     |
| 1.72     | 2.702731       | 2.881797       | 0.93786        | 1.0662         | 0.3470                       | 0.3700                         | 1.72     |
| 1.73     | 2.731685       | 2.908969       | 0.93906        | 1.0649         | 0.3438                       | 0.3661                         | 1.73     |
| 1.74     | 2.760912       | 2.936432       | 0.94023        | 1.0636         | 0.3405                       | 0.3622                         | 1.74     |
| 1.75     | 2.790414       | 2.964188       | 0.94138        | 1.0623         | 0.3373                       | 0.3584                         | 1.75     |
| 1.76     | 2.820196       | 2.992241       | 0.94250        | 1.0610         | 0.3342                       | 0.3546                         | 1.76     |
| 1.77     | 2.850260       | 3.020593       | 0.94361        | 1.0597         | 0.3310                       | 0.3508                         | 1.77     |
| 1.78     | 2.880609       | 3.049247       | 0.94470        | 1.0585         | 0.3279                       | 0.3471                         | 1.78     |
| 1.79     | 2.911246       | 3.078206       | 0.94576        | 1.0573         | 0.3248                       | 0.3435                         | 1.79     |
| 1.80     | 2.942174       | 3.107473       | 0.94681        | 1.0562         | 0.3218                       | 0.3399                         | 1.80     |
| 1.81     | 2.973397       | 3.137051       | 0.94783        | 1.0550         | 0.3187                       | 0.3363                         | 1.81     |
| 1.82     | 3.004916       | 3.166942       | 0.94884        | 1.0539         | 0.3158                       | 0.3328                         | 1.82     |
| 1.83     | 3.036737       | 3.197150       | 0.94983        | 1.0528         | 0.3128                       | 0.3293                         | 1.83     |
| 1.84     | 3.068860       | 3.227678       | 0.95080        | 1.0517         | 0.3098                       | 0.3258                         | 1.84     |
| 1.85     | 3.101291       | 3.258528       | 0.95175        | 1.0507         | 0.3069                       | 0.3224                         | 1.85     |
| 1.86     | 3.134032       | 3.289705       | 0.95268        | 1.0497         | 0.3040                       | 0.3191                         | 1.86     |
| 1.87     | 3.167086       | 3.321210       | 0.95359        | 1.0487         | 0.3011                       | 0.3157                         | 1.87     |
| 1.88     | 3.200457       | 3.353047       | 0.95449        | 1.0477         | 0.2982                       | 0.3125                         | 1.88     |
| 1.89     | 3.234148       | 3.385220       | 0.95537        | 1.0467         | 0.2954                       | 0.3092                         | 1.89     |
| 1.90     | 3.268163       | 3.417732       | 0.95624        | 1.0458         | 0.2926                       | 0.3059                         | 1.90     |
| 1.91     | 3.302504       | 3.450585       | 0.95709        | 1.0448         | 0.2897                       | 0.3028                         | 1.91     |
| 1.92     | 3.337176       | 3.483783       | 0.95792        | 1.0439         | 0.2870                       | 0.2997                         | 1.92     |
| 1.93     | 3.372181       | 3.517329       | 0.95873        | 1.0430         | 0.2843                       | 0.2965                         | 1.93     |
| 1.94     | 3.407524       | 3.551227       | 0.95953        | 1.0422         | 0.2816                       | 0.2935                         | 1.94     |
| 1.95     | 3.443207       | 3.585481       | 0.96032        | 1.0413         | 0.2789                       | 0.2904                         | 1.95     |
| 1.96     | 3.479234       | 3.620093       | 0.96109        | 1.0405         | 0.2762                       | 0.2874                         | 1.96     |
| 1.97     | 3.515610       | 3.655067       | 0.96185        | 1.0397         | 0.2736                       | 0.2844                         | 1.97     |
| 1.98     | 3.552337       | 3.690406       | 0.96259        | 1.0389         | 0.2710                       | 0.2815                         | 1.98     |
| 1.99     | 3.589419       | 3.726115       | 0.96331        | 1.0380         | 0.2684                       | 0.2786                         | 1.99     |

Example.  $\coth 1.70 = 1.0691$ .

TABLE XV

REAL HYPERBOLIC FUNCTIONS.  $f(x + io) = u + io$ . CONTINUED

| $\theta$ | $\sinh \theta$ | $\cosh \theta$ | $\tanh \theta$ | $\coth \theta$ | $\operatorname{sech} \theta$ | $\operatorname{cosech} \theta$ | $\theta$ |
|----------|----------------|----------------|----------------|----------------|------------------------------|--------------------------------|----------|
| 2.00     | 3.626860       | 3.762196       | 0.96403        | 1.0373         | 0.2658                       | 0.2757                         | 2.00     |
| 2.01     | 3.66466        | 3.79865        | 0.96473        | 1.0365         | 0.2632                       | 0.2729                         | 2.01     |
| 2.02     | 3.70283        | 3.83549        | 0.96541        | 1.0358         | 0.2607                       | 0.2701                         | 2.02     |
| 2.03     | 3.74138        | 3.87271        | 0.96608        | 1.0351         | 0.2582                       | 0.2673                         | 2.03     |
| 2.04     | 3.78029        | 3.91032        | 0.96675        | 1.0344         | 0.2557                       | 0.2645                         | 2.04     |
| 2.05     | 3.81958        | 3.94832        | 0.96740        | 1.0337         | 0.2533                       | 0.2618                         | 2.05     |
| 2.06     | 3.85926        | 3.98671        | 0.96803        | 1.0330         | 0.2508                       | 0.2596                         | 2.06     |
| 2.07     | 3.89932        | 4.02550        | 0.96865        | 1.0323         | 0.2484                       | 0.2565                         | 2.07     |
| 2.08     | 3.93977        | 4.06470        | 0.96926        | 1.0317         | 0.2460                       | 0.2538                         | 2.08     |
| 2.09     | 3.98061        | 4.10430        | 0.96986        | 1.0310         | 0.2436                       | 0.2512                         | 2.09     |
| 2.10     | 4.02186        | 4.14431        | 0.97045        | 1.0305         | 0.2413                       | 0.2486                         | 2.10     |
| 2.11     | 4.06350        | 4.18474        | 0.97103        | 1.0298         | 0.2389                       | 0.2461                         | 2.11     |
| 2.12     | 4.10555        | 4.22558        | 0.97159        | 1.0293         | 0.2366                       | 0.2436                         | 2.12     |
| 2.13     | 4.14801        | 4.26685        | 0.97215        | 1.0286         | 0.2344                       | 0.2411                         | 2.13     |
| 2.14     | 4.19089        | 4.30855        | 0.97269        | 1.0280         | 0.2321                       | 0.2386                         | 2.14     |
| 2.15     | 4.23419        | 4.35067        | 0.97323        | 1.0275         | 0.2298                       | 0.2362                         | 2.15     |
| 2.16     | 4.27791        | 4.39323        | 0.97375        | 1.0269         | 0.2276                       | 0.2338                         | 2.16     |
| 2.17     | 4.32205        | 4.43623        | 0.97426        | 1.0264         | 0.2254                       | 0.2314                         | 2.17     |
| 2.18     | 4.36663        | 4.47967        | 0.97477        | 1.0259         | 0.2232                       | 0.2290                         | 2.18     |
| 2.19     | 4.41165        | 4.52356        | 0.97526        | 1.0254         | 0.2211                       | 0.2267                         | 2.19     |
| 2.20     | 4.45711        | 4.56791        | 0.97574        | 1.0249         | 0.2189                       | 0.2244                         | 2.20     |
| 2.21     | 4.50301        | 4.61271        | 0.97622        | 1.0243         | 0.2168                       | 0.2221                         | 2.21     |
| 2.22     | 4.54936        | 4.65797        | 0.97668        | 1.0239         | 0.2147                       | 0.2198                         | 2.22     |
| 2.23     | 4.59617        | 4.70370        | 0.97714        | 1.0234         | 0.2126                       | 0.2176                         | 2.23     |
| 2.24     | 4.64344        | 4.74989        | 0.97758        | 1.0229         | 0.2105                       | 0.2154                         | 2.24     |
| 2.25     | 4.69117        | 4.79657        | 0.97803        | 1.0225         | 0.2085                       | 0.2132                         | 2.25     |
| 2.26     | 4.73937        | 4.84372        | 0.97847        | 1.0220         | 0.2064                       | 0.2110                         | 2.26     |
| 2.27     | 4.78804        | 4.89136        | 0.97888        | 1.0216         | 0.2044                       | 0.2089                         | 2.27     |
| 2.28     | 4.83720        | 4.93948        | 0.97929        | 1.0211         | 0.2024                       | 0.2067                         | 2.28     |
| 2.29     | 4.88683        | 4.98810        | 0.97970        | 1.0207         | 0.2005                       | 0.2047                         | 2.29     |
| 2.30     | 4.93696        | 5.03722        | 0.98010        | 1.0203         | 0.1985                       | 0.2026                         | 2.30     |
| 2.31     | 4.98758        | 5.08684        | 0.98049        | 1.0199         | 0.1966                       | 0.2005                         | 2.31     |
| 2.32     | 5.03870        | 5.13697        | 0.98087        | 1.0195         | 0.1947                       | 0.1985                         | 2.32     |
| 2.33     | 5.09032        | 5.18762        | 0.98124        | 1.0191         | 0.1928                       | 0.1965                         | 2.33     |
| 2.34     | 5.14245        | 5.23879        | 0.98161        | 1.0187         | 0.1909                       | 0.1945                         | 2.34     |
| 2.35     | 5.19510        | 5.29047        | 0.98198        | 1.0184         | 0.1890                       | 0.1925                         | 2.35     |
| 2.36     | 5.24827        | 5.34269        | 0.98233        | 1.0180         | 0.1872                       | 0.1905                         | 2.36     |
| 2.37     | 5.30196        | 5.39544        | 0.98268        | 1.0177         | 0.1854                       | 0.1886                         | 2.37     |
| 2.38     | 5.35618        | 5.44873        | 0.98302        | 1.0173         | 0.1835                       | 0.1867                         | 2.38     |
| 2.39     | 5.41093        | 5.50256        | 0.98335        | 1.0169         | 0.1817                       | 0.1848                         | 2.39     |
| 2.40     | 5.46623        | 5.55695        | 0.98368        | 1.0166         | 0.1800                       | 0.1829                         | 2.40     |
| 2.41     | 5.52207        | 5.61189        | 0.98399        | 1.0163         | 0.1782                       | 0.1811                         | 2.41     |
| 2.42     | 5.57847        | 5.66739        | 0.98431        | 1.0159         | 0.1765                       | 0.1793                         | 2.42     |
| 2.43     | 5.63542        | 5.72346        | 0.98462        | 1.0156         | 0.1747                       | 0.1775                         | 2.43     |
| 2.44     | 5.69294        | 5.78010        | 0.98492        | 1.0153         | 0.1730                       | 0.1757                         | 2.44     |
| 2.45     | 5.75103        | 5.83732        | 0.98522        | 1.0150         | 0.1713                       | 0.1739                         | 2.45     |
| 2.46     | 5.80969        | 5.89512        | 0.98551        | 1.0147         | 0.1696                       | 0.1721                         | 2.46     |
| 2.47     | 5.86893        | 5.95352        | 0.98579        | 1.0144         | 0.1680                       | 0.1704                         | 2.47     |
| 2.48     | 5.92876        | 6.01250        | 0.98607        | 1.0141         | 0.1663                       | 0.1687                         | 2.48     |
| 2.49     | 5.98918        | 6.07209        | 0.98635        | 1.0138         | 0.1647                       | 0.1670                         | 2.49     |

Example.  $\operatorname{sech} 2.00 = 0.2658$ .

TABLE XV

REAL HYPERBOLIC FUNCTIONS.  $f(x + io) = u + io$ . CONTINUED

| $\theta$ | Sinh $\theta$ | Cosh $\theta$ | Tanh $\theta$ | Coth $\theta$ | Sech $\theta$ | Cosech $\theta$ | $\theta$ |
|----------|---------------|---------------|---------------|---------------|---------------|-----------------|----------|
| 2.5      | 6.05020       | 6.13229       | 0.98661       | 1.0136        | 0.1631        | 0.1653          | 2.5      |
| 2.6      | 6.69473       | 6.76901       | 0.98903       | 1.0111        | 0.1477        | 0.1494          | 2.6      |
| 2.7      | 7.40626       | 7.47347       | 0.99101       | 1.0091        | 0.1338        | 0.1350          | 2.7      |
| 2.8      | 8.19192       | 8.25273       | 0.99263       | 1.0074        | 0.1212        | 0.1221          | 2.8      |
| 2.9      | 9.05956       | 9.11458       | 0.99396       | 1.0061        | 0.1097        | 0.1104          | 2.9      |
| 3.0      | 10.01787      | 10.06766      | 0.99505       | 1.0050        | 0.0937        | 0.09982         | 3.0      |
| 3.1      | 11.07645      | 11.12150      | 0.99595       | 1.0041        | 0.0899        | 0.0903          | 3.1      |
| 3.2      | 12.24588      | 12.28665      | 0.99668       | 1.0033        | 0.0814        | 0.0816          | 3.2      |
| 3.3      | 13.53788      | 13.57476      | 0.99728       | 1.0027        | 0.0736        | 0.0739          | 3.3      |
| 3.4      | 14.96536      | 14.99874      | 0.99778       | 1.0022        | 0.0667        | 0.0668          | 3.4      |
| 3.5      | 16.54263      | 16.57282      | 0.99818       | 1.0018        | 0.0604        | 0.0604          | 3.5      |
| 3.6      | 18.28546      | 18.31278      | 0.99851       | 1.0015        | 0.0546        | 0.0547          | 3.6      |
| 3.7      | 20.21129      | 20.23601      | 0.99878       | 1.0012        | 0.0494        | 0.0495          | 3.7      |
| 3.8      | 22.33941      | 22.36178      | 0.99900       | 1.0010        | 0.0447        | 0.0448          | 3.8      |
| 3.9      | 24.69110      | 24.71135      | 0.99918       | 1.0008        | 0.0405        | 0.0405          | 3.9      |
| 4.0      | 27.28992      | 27.30823      | 0.99933       | 1.0007        | 0.0366        | 0.0366          | 4.0      |
| 4.1      | 30.16186      | 30.17843      | 0.99945       | 1.0006        | 0.0331        | 0.0332          | 4.1      |
| 4.2      | 33.33567      | 33.35066      | 0.99955       | 1.0005        | 0.0300        | 0.0300          | 4.2      |
| 4.3      | 36.84311      | 36.85668      | 0.99963       | 1.0004        | 0.0271        | 0.0271          | 4.3      |
| 4.4      | 40.71930      | 40.73157      | 0.99970       | 1.0003        | 0.0245        | 0.0245          | 4.4      |
| 4.5      | 45.00301      | 45.01412      | 0.99975       | 1.0003        | 0.0222        | 0.0222          | 4.5      |
| 4.6      | 49.73713      | 49.74718      | 0.99980       | 1.0002        | 0.0201        | 0.0201          | 4.6      |
| 4.7      | 54.96904      | 54.97813      | 0.99983       | 1.0002        | 0.0182        | 0.0182          | 4.7      |
| 4.8      | 60.75109      | 60.75932      | 0.99986       | 1.0001        | 0.0165        | 0.0165          | 4.8      |
| 4.9      | 67.14117      | 67.14861      | 0.99989       | 1.0001        | 0.0149        | 0.0149          | 4.9      |
| 5.0      | 74.20321      | 74.20995      | 0.99991       | 1.0001        | 0.0135        | 0.0135          | 5.0      |
| 5.1      | 82.0079       | 82.0140       | 0.99993       | 1.00007       | 0.01219       | 0.01219         | 5.1      |
| 5.2      | 90.6334       | 90.6389       | 0.99993       | 1.00007       | 0.01103       | 0.01103         | 5.2      |
| 5.3      | 100.1659      | 100.1709      | 0.99994       | 1.00006       | 0.00998       | 0.00998         | 5.3      |
| 5.4      | 110.7009      | 110.7055      | 0.99995       | 1.00005       | 0.00903       | 0.00903         | 5.4      |
| 5.5      | 122.3439      | 122.3480      | 0.99996       | 1.00004       | 0.00818       | 0.00818         | 5.5      |
| 5.6      | 135.2114      | 135.2150      | 0.99997       | 1.00003       | 0.00740       | 0.00740         | 5.6      |
| 5.7      | 149.4320      | 149.4354      | 0.99998       | 1.00002       | 0.00669       | 0.00669         | 5.7      |
| 5.8      | 165.1483      | 165.1513      | 0.99998       | 1.00002       | 0.00606       | 0.00606         | 5.8      |
| 5.9      | 182.5174      | 182.5201      | 0.99998       | 1.00002       | 0.00548       | 0.00548         | 5.9      |
| 6.0      | 201.7132      | 201.7156      | 0.99999       | 1.00001       | 0.00496       | 0.00496         | 6.0      |
| 6.1      | 222.9278      | 222.9300      | 1.0000        | 1.0000        | 0.00449       | 0.00449         | 6.1      |
| 6.2      | 246.3735      | 246.3755      | 1.0000        | 1.0000        | 0.00406       | 0.00406         | 6.2      |
| 6.3      | 272.2850      | 272.2869      | 1.0000        | 1.0000        | 0.00367       | 0.00367         | 6.3      |
| 6.4      | 300.9217      | 300.9233      | 1.0000        | 1.0000        | 0.00332       | 0.00332         | 6.4      |
| 6.5      | 332.5701      | 332.5716      | 1.0000        | 1.0000        | 0.00301       | 0.00301         | 6.5      |
| 6.6      | 367.5469      | 367.5483      | 1.0000        | 1.0000        | 0.00272       | 0.00272         | 6.6      |
| 6.7      | 406.2023      | 406.2035      | 1.0000        | 1.0000        | 0.00246       | 0.00246         | 6.7      |
| 6.8      | 448.9231      | 448.9242      | 1.0000        | 1.0000        | 0.00223       | 0.00223         | 6.8      |
| 6.9      | 496.1369      | 496.1379      | 1.0000        | 1.0000        | 0.00202       | 0.00202         | 6.9      |
| 7.0      | 548.3161      | 548.3170      | 1.0000        | 1.0000        | 0.00182       | 0.00182         | 7.0      |
| 7.1      | 605.9831      | 605.9839      | 1.0000        | 1.0000        | 0.00165       | 0.00165         | 7.1      |
| 7.2      | 669.7150      | 669.7158      | 1.0000        | 1.0000        | 0.00149       | 0.00149         | 7.2      |
| 7.3      | 740.1496      | 740.1503      | 1.0000        | 1.0000        | 0.00135       | 0.00135         | 7.3      |
| 7.4      | 817.9919      | 817.9925      | 1.0000        | 1.0000        | 0.00122       | 0.00122         | 7.4      |
| 7.5      | 904.0209      | 904.0215      | 1.0000        | 1.0000        | 0.00111       | 0.00111         | 7.5      |

Example. cosech 2.50 = 0.1653.

[ 149 ]

TABLE XVI. SUBDIVISIONS OF A DEGREE—AUXILIARY TABLE

| °    | '     | " | °    | '     | " | °     | '       | " | °  | '      | " | °  | '      | " | °  | '      | " |
|------|-------|---|------|-------|---|-------|---------|---|----|--------|---|----|--------|---|----|--------|---|
| 0.01 | 00.36 |   | 0.41 | 24.36 |   | 0.81  | 48.36   |   | 01 | 0.0167 |   | 41 | 0.6833 |   | 21 | 0.0058 |   |
| 0.02 | 01.12 |   | 0.42 | 25.12 |   | 0.82  | 49.12   |   | 02 | 0.0333 |   | 42 | 0.7000 |   | 22 | 0.0061 |   |
| 0.03 | 01.48 |   | 0.43 | 25.48 |   | 0.83  | 49.48   |   | 03 | 0.0500 |   | 43 | 0.7167 |   | 23 | 0.0064 |   |
| 0.04 | 02.24 |   | 0.44 | 26.24 |   | 0.84  | 50.24   |   | 04 | 0.0667 |   | 44 | 0.7333 |   | 24 | 0.0067 |   |
| 0.05 | 03.00 |   | 0.45 | 27.00 |   | 0.85  | 51.00   |   | 05 | 0.0833 |   | 45 | 0.7500 |   | 25 | 0.0069 |   |
|      |       |   |      |       |   |       |         |   |    |        |   |    |        |   |    |        |   |
| 0.06 | 03.36 |   | 0.46 | 27.36 |   | 0.86  | 51.36   |   | 06 | 0.1000 |   | 46 | 0.7667 |   | 26 | 0.0072 |   |
| 0.07 | 04.12 |   | 0.47 | 28.12 |   | 0.87  | 52.12   |   | 07 | 0.1167 |   | 47 | 0.7833 |   | 27 | 0.0075 |   |
| 0.08 | 04.48 |   | 0.48 | 28.48 |   | 0.88  | 52.48   |   | 08 | 0.1333 |   | 48 | 0.8000 |   | 28 | 0.0078 |   |
| 0.09 | 05.24 |   | 0.49 | 29.24 |   | 0.89  | 53.24   |   | 09 | 0.1500 |   | 49 | 0.8167 |   | 29 | 0.0081 |   |
| 0.10 | 06.00 |   | 0.50 | 30.00 |   | 0.90  | 54.00   |   | 10 | 0.1667 |   | 50 | 0.8333 |   | 30 | 0.0083 |   |
|      |       |   |      |       |   |       |         |   |    |        |   |    |        |   |    |        |   |
| 0.11 | 06.36 |   | 0.51 | 30.36 |   | 0.91  | 54.36   |   | 11 | 0.1833 |   | 51 | 0.8500 |   | 31 | 0.0086 |   |
| 0.12 | 07.12 |   | 0.52 | 31.12 |   | 0.92  | 55.12   |   | 12 | 0.2000 |   | 52 | 0.8667 |   | 32 | 0.0089 |   |
| 0.13 | 07.48 |   | 0.53 | 31.48 |   | 0.93  | 55.48   |   | 13 | 0.2167 |   | 53 | 0.8833 |   | 33 | 0.0092 |   |
| 0.14 | 08.24 |   | 0.54 | 32.24 |   | 0.94  | 56.24   |   | 14 | 0.2333 |   | 54 | 0.9000 |   | 34 | 0.0094 |   |
| 0.15 | 09.00 |   | 0.55 | 33.00 |   | 0.95  | 57.00   |   | 15 | 0.2500 |   | 55 | 0.9167 |   | 35 | 0.0097 |   |
|      |       |   |      |       |   |       |         |   |    |        |   |    |        |   |    |        |   |
| 0.16 | 09.36 |   | 0.56 | 33.36 |   | 0.96  | 57.36   |   | 16 | 0.2667 |   | 56 | 0.9333 |   | 36 | 0.0100 |   |
| 0.17 | 10.12 |   | 0.57 | 34.12 |   | 0.97  | 58.12   |   | 17 | 0.2833 |   | 57 | 0.9500 |   | 37 | 0.0103 |   |
| 0.18 | 10.48 |   | 0.58 | 34.48 |   | 0.98  | 58.48   |   | 18 | 0.3000 |   | 58 | 0.9667 |   | 38 | 0.0106 |   |
| 0.19 | 11.24 |   | 0.59 | 35.24 |   | 0.99  | 59.24   |   | 19 | 0.3167 |   | 59 | 0.9833 |   | 39 | 0.0108 |   |
| 0.20 | 12.00 |   | 0.60 | 36.00 |   | 1.00  | 60.00   |   | 20 | 0.3333 |   | 60 | 1.0000 |   | 40 | 0.0111 |   |
|      |       |   |      |       |   |       |         |   |    |        |   |    |        |   |    |        |   |
| 0.21 | 12.36 |   | 0.61 | 36.36 |   | ....  | ....    |   | 21 | 0.3500 |   | 01 | 0.0003 |   | 41 | 0.0114 |   |
| 0.22 | 13.12 |   | 0.62 | 37.12 |   | ....  | ....    |   | 22 | 0.3667 |   | 02 | 0.0006 |   | 42 | 0.0117 |   |
| 0.23 | 13.48 |   | 0.63 | 37.48 |   | ....  | ....    |   | 23 | 0.3833 |   | 03 | 0.0008 |   | 43 | 0.0119 |   |
| 0.24 | 14.24 |   | 0.64 | 38.24 |   | ....  | ....    |   | 24 | 0.4000 |   | 04 | 0.0011 |   | 44 | 0.0122 |   |
| 0.25 | 15.00 |   | 0.65 | 39.00 |   | ....  | ....    |   | 25 | 0.4167 |   | 05 | 0.0014 |   | 45 | 0.0125 |   |
|      |       |   |      |       |   |       |         |   |    |        |   |    |        |   |    |        |   |
| 0.26 | 15.36 |   | 0.66 | 39.36 |   | ....  | ....    |   | 26 | 0.4333 |   | 06 | 0.0017 |   | 46 | 0.0128 |   |
| 0.27 | 16.12 |   | 0.67 | 40.12 |   | ....  | ....    |   | 27 | 0.4500 |   | 07 | 0.0019 |   | 47 | 0.0131 |   |
| 0.28 | 16.48 |   | 0.68 | 40.48 |   | ....  | ....    |   | 28 | 0.4667 |   | 08 | 0.0022 |   | 48 | 0.0133 |   |
| 0.29 | 17.24 |   | 0.69 | 41.24 |   | ....  | ....    |   | 29 | 0.4833 |   | 09 | 0.0025 |   | 49 | 0.0136 |   |
| 0.30 | 18.00 |   | 0.70 | 42.00 |   | ....  | ....    |   | 30 | 0.5000 |   | 10 | 0.0028 |   | 50 | 0.0139 |   |
|      |       |   |      |       |   |       |         |   |    |        |   |    |        |   |    |        |   |
| 0.31 | 18.36 |   | 0.71 | 42.36 |   | 0.001 | 00.03.6 |   | 31 | 0.5167 |   | 11 | 0.0031 |   | 51 | 0.0142 |   |
| 0.32 | 19.12 |   | 0.72 | 43.12 |   | 0.002 | 00.07.2 |   | 32 | 0.5333 |   | 12 | 0.0033 |   | 52 | 0.0144 |   |
| 0.33 | 19.48 |   | 0.73 | 43.48 |   | 0.003 | 00.10.8 |   | 33 | 0.5500 |   | 13 | 0.0036 |   | 53 | 0.0147 |   |
| 0.34 | 20.24 |   | 0.74 | 44.24 |   | 0.004 | 00.14.4 |   | 34 | 0.5667 |   | 14 | 0.0039 |   | 54 | 0.0150 |   |
| 0.35 | 21.00 |   | 0.75 | 45.00 |   | 0.005 | 00.18   |   | 35 | 0.5833 |   | 15 | 0.0042 |   | 55 | 0.0153 |   |
|      |       |   |      |       |   |       |         |   |    |        |   |    |        |   |    |        |   |
| 0.36 | 21.36 |   | 0.76 | 45.36 |   | 0.006 | 00.21.6 |   | 36 | 0.6000 |   | 16 | 0.0044 |   | 56 | 0.0156 |   |
| 0.37 | 22.12 |   | 0.77 | 46.12 |   | 0.007 | 00.25.2 |   | 37 | 0.6167 |   | 17 | 0.0047 |   | 57 | 0.0158 |   |
| 0.38 | 22.48 |   | 0.78 | 46.48 |   | 0.008 | 00.28.8 |   | 38 | 0.6333 |   | 18 | 0.0050 |   | 58 | 0.0161 |   |
| 0.39 | 23.24 |   | 0.79 | 47.24 |   | 0.009 | 00.32.4 |   | 39 | 0.6500 |   | 19 | 0.0053 |   | 59 | 0.0164 |   |
| 0.40 | 24.00 |   | 0.80 | 48.00 |   | 0.010 | 00.36   |   | 40 | 0.6667 |   | 20 | 0.0055 |   | 60 | 0.0167 |   |

Examples.  $0^{\circ}.41 = 0^{\circ}.24'.36''$        $0^{\circ}.41'.00'' = 0^{\circ}.6833.$   
 $0^{\circ}.005 = 0^{\circ}.00'.18''$        $0^{\circ}.00'.46'' = 0^{\circ}.0128.$

## EXPLANATORY TEXT



THEY TROTAKE

# EXPLANATORY TEXT

## INTRODUCTION

THE Tables in this book are designed primarily for presenting hyperbolic functions of a complex variable either in the rectangular coördinate form of that variable ( $x + iy$ ) or the polar coördinate form ( $\rho/\delta$ ). They are also designed secondarily for presenting circular functions of a complex variable. A few formulas are added as aids to the conversion of such functions. The most extensive range offered is in Tables VII to XIV inclusive, between which, the functions  $\sinh(x + iy)$ ,  $\cosh(x + iy)$ ,  $\tanh(x + iy)$ , expressed in the result either in rectangular coördinates  $u + iv$  or in polar coördinate quantities  $r/\gamma$ , may be obtained between the limits of 0 and  $\pm 10$  of  $x$ , and between the limits of 0 and  $\pm \alpha$  for  $y$ . It is shown, moreover, to be an easy matter to extend the range of  $x$  beyond the offered range of  $\pm 10$ , should such an extension be required. The practical need for tabulated values of hyperbolic functions of  $(x + iy)$  beyond the range of  $x = \pm 10$  appears to be so small that any such extension is left to the reader.

As the author's applications for financial assistance in the computation of the Tables were unsuccessful, the steps in  $x$  and  $y$  (0.05 and 0.07854 respectively) are larger than were originally intended; *i.e.*, for reducing the work of the user to the lowest practicable limits. Consequently, interpolation must ordinarily be resorted to, when three or more significant digits are needed in the results. Such interpolations require an appreciable amount of time to effect in two dimensions; *i.e.*, for both  $x$  and  $y$ . In order to render such interpolation unnecessary for ordinary engineering purposes, where three, or at most four, significant digits may be needed, a separate atlas of 23 large-scale charts, 45 cm.  $\times$  45 cm. over ruled areas, has been prepared, and is published as an adjunct to these Tables. The necessary interpolation can very swiftly be made on the charts, by inspection.

## COMPLEX QUANTITIES

The following brief outline of complex quantities is offered in view of their fundamental importance in connection with the Tables, for the assistance of those who have studied elementary mathematics, but who may not have become familiar with complex numbers. For a more comprehensive discussion of complex quantities, the reader must be referred to special treatises on the subject.

Ordinary numerical quantities, or the numbers dealt with in ordinary arithmetic, may be considered to range between zero and either positive or negative infinity, by indefinitely small gradations. Such numbers may be represented geometrically by distances, in either direction, from a zero point on an infinite straight line. Thus in Fig. 1, we may consider that the straight line  $-XOX$  extends from minus infinity on the left, to plus infinity on the right,  $O$  being the zero point. The point  $x_1$  would then represent  $+1$ , and so on. That is, the number  $+1$  may be regarded as represented on the line  $-XOX$  either by the position of the point  $x_1$  with respect to the zero point  $O$ ;

or, as the vector  $Ox_1$ ; *i e.*, the straight line drawn from the origin  $O$  to the point  $x_1$  and forming a part of the reference line —  $XOX$ . Under these assumptions, the ordinary numbers of arithmetic may be represented geometrically as vectors; but such vectors are confined to a single straight-line direction from  $O$  towards  $X$  for positive numbers, and from  $O$  towards —  $X$  for negative numbers.

Complex quantities, or complex numbers, cannot be completely represented by reference to a single direction, or to vectors along one and the same straight line. They may, however, be represented geometrically by the position, in an infinite plane, of a movable point with respect to a fixed point as origin. Thus, in Fig. 1, the plane  $XOY$  is the plane of reference, and the fixed point  $O$  is the origin. Then any point  $P_1$  in the plane represents a complex number, and any complex number may be represented by a point on the plane.

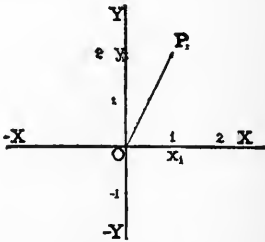


FIG. 1. — Complex quantity  $1 + i2$ .

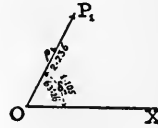


FIG. 2. — Plane Vector  $2.236 e^{i63.106}$  or  $\rho e^{i\delta}$ , designated by  $2.236 / 63^\circ 26'$ .

A complex number may be specified either in rectangular coördinates, or in polar coördinates, as may be preferred. Thus, the same vector  $OP_1$  is represented in Fig. 1 to rectangular coördinates, and in Fig. 2 to polar coördinates. In Fig. 1, the  $X$  axis —  $XOX$  passing through the origin  $O$  is the fundamental reference axis, and the  $Y$  axis —  $YOY$ , perpendicular thereto in the reference plane, immediately follows. Then the point  $P_1$ , measuring  $+1$  along  $OX$ , and  $+2$  along  $OY$ , may be defined by the expression  $(1 + i2)$ , where the symbol  $i$  signifies measurement along the subordinate axis. It is shown in mathematical treatises that  $i = \sqrt{-1}$ . The vector  $OP_1$  of Fig. 1 may therefore be expressed as  $(1 + \sqrt{-1} \cdot 2)$  and a vector from  $O$  to any point in the plane may be represented by  $x + \sqrt{-1} y = x + iy$ , where  $x$  and  $y$  may have any positive or negative numerical values, including zero.

In pursuance of time-honored terminology, the axis —  $XOX$  is sometimes called the “real” axis, and —  $YOY$  the “imaginary,” axis; so that the  $x$ -component of a complex number becomes the “real component,” and the  $y$ -component the “imaginary component.” The symbol  $i$  still stands for the *imaginary* component. In mathematics as applied to electrical engineering, the symbol  $i$  commonly designates electric current-strength, and so, in order to prevent the possibility of confusion, the symbol  $j$  is frequently substituted as the sign of the imaginary. Under such a convention, the plane-vector, or complex quantity,  $OP_1$ , would be represented as  $1 + j2$ . As, however, in this book we necessarily consider complex quantities from a broader viewpoint than that offered by electrical engineering, we shall use the symbol  $i$  to denote the imaginary component, perpendicularly rotated with respect to the fundamental  $X$  axis.

## EXPLANATORY TEXT

Complex quantities may also be expressed in polar coördinates, as in Fig. 2, where the fundamental reference axis  $OX$  is drawn in the positive direction in the reference plane, from the origin  $O$ , and the circular angle  $\delta_1$  is measured in the positive or counter-clockwise direction from  $OX$  to  $OP_1$ . The vector  $OP_1$  is then specified in polar coördinates by its length  $\rho_1$  and by its angle  $\delta_1$ . The length  $\rho_1$  is called the *modulus* of the vector, and the angle  $\delta_1$  is called the *argument*. This argument may be expressed in circular radians, in degrees-minutes-seconds, quadrants, or any other recognized unit of circular angle. Thus, in Fig. 2, the vector  $OP_1$  may be represented to polar coördinates symbolically by  $\rho_1/\delta_1$  or, using numbers, by  $2.236/63^\circ.26'$ , where 2.236 is the modulus to the same scale of linear measure as in Fig. 1, and  $63^\circ.26'$  is the argument.

If one and the same complex quantity be expressed both in rectangular and polar coördinates, as follows:

$$x + iy = \rho/\delta \quad (1)$$

it is evident that  $x = \rho \cos \delta$ ,  $y = \rho \sin \delta$ ,  $y/x = \tan \delta$ , and  $\rho = \sqrt{x^2 + y^2}$ , relations which enable the coördinates to be changed, at will, from one form to the other. Thus in Figs 1 and 2,  $x_1 = 1$ ,  $y_1 = 2$ ,  $\rho_1 = \sqrt{5} = 2.236$ , and  $\delta_1 = \tan^{-1}(2) = 63^\circ.26'$ .

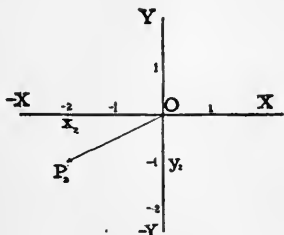


FIG. 3. — Complex quantity  $-2 - i1$ .

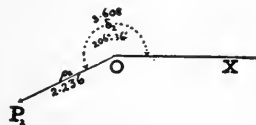


FIG. 4. — Plane-Vector  $2.236 \angle 206^\circ.34'$  or  $2.236 \angle 206^\circ.34'$ .

Similarly, Figs. 3 and 4 represent the complex quantity or plane vector  $OP_2$  to rectangular and polar coördinates respectively. Here  $x_2 = -2$ ,  $y_2 = -1$ ,  $\rho_2 = \sqrt{5} = 2.236$  and  $\delta_2 = 206^\circ.34'$ .

### ADDITION OF COMPLEX QUANTITIES

One vector quantity is added to another, by drawing it in the reference plane from the extremity of the latter as origin, and then drawing a vector from the origin to its

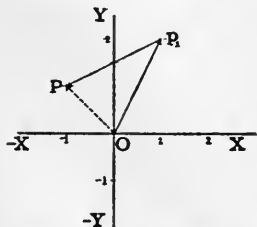


FIG. 5. — Addition of two complex quantities  
 $(1 + i2) + (-2 - i1) = -1 + i1$ .

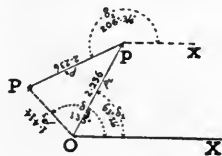


FIG. 6. — Complex Addition, Polar coördinates.  
 $\rho_1/\delta_1 + \rho_2/\delta_2 = \rho_3/\delta_3$   
 $2.236/63^\circ.26' + 2.236/206^\circ.34' = 1.414/135^\circ$   
 $Op + pP = OP$

free end. The last named vector is the required sum. Thus, in Fig. 5, the complex quantity  $OP_2 = -2 - i1$  of Fig 3 is added to the complex quantity  $OP_1 = 1 + i2$

## EXPLANATORY TEXT

of Fig. 1, giving the resultant vector  $OP = -1 + i1$ . Fig. 6 shows the corresponding operation with polar coordinate vectors. Here  $OP_2 = 2.236 / 206^\circ.34'$  of Fig. 4 is added to  $OP_1 = 2.236 / 63^\circ.26'$  of Fig. 2, to produce  $OP = 1.414 / 135^\circ = \rho_3 / \delta_3$  of Fig. 6.

On the drawing-board, the graphic process of adding vectors is as easily effected when they are expressed in polar as in rectangular coordinates. But the arithmetical addition is much more easily made with rectangular coordinates. The rule is: find the vector sum by taking first the sum of the reals, and then the sum of the imaginaries; or

$$\begin{aligned}(x_1 + iy_1) + (x_2 + iy_2) + \dots + (x_n + iy_n) &= (x_1 + x_2 + \dots + x_n) \\ &+ i(y_1 + y_2 + \dots + y_n) = \Sigma x + i\Sigma y. \quad (2)\end{aligned}$$

In the case of Figs. 5 and 6:

$$(1 + i2) + (-2 - i1) = (+1 - 2) + i(2 - 1) = -1 + i1 = \sqrt{2} / 135^\circ.$$

## SUBTRACTION OF COMPLEX QUANTITIES

Reversing the sign of a rectangular complex quantity means reversing the sign of both its real and imaginary components. Reversing the sign of a polar complex quantity means changing its argument by  $180^\circ$ .

To subtract one complex quantity  $A$  from another  $B$ , reverse the sign of  $A$ , and then add it thus reversed to  $B$ , by the rules of addition.

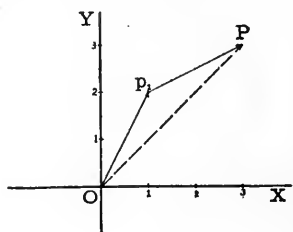


FIG. 7. — Complex Subtraction  
 $(1 + i2) - (-2 - i1) \dots 3 + i3 = OP$ .

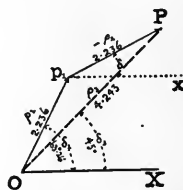


FIG. 8. — Complex Subtraction, Polar Coordinates  
 $\rho_1 / \delta_1 - \rho_2 / \delta_2 = \rho_3 / \delta_3$   
 $2.236 / 63^\circ.26' - 2.236 / 206^\circ.34' = 4.243 / 45^\circ$   
 $Op_1 + p_1P = OP$ .

In Figs. 7 and 8, the vector  $P_2$  of Figs. 3 and 4 is subtracted from the vector  $P_1$  of Figs. 1 and 2. In Fig. 7, we have

$$\begin{aligned}OP_1 - OP_2 &= OP. \\ (1 + i2) - (-2 - i1) &= (1 + i2) + (2 + i1). \\ &= 3 + i3.\end{aligned}$$

In Fig. 8,

$$\begin{aligned}\rho_1 / \delta_1 - \rho_2 / \delta_2 &= \rho_3 / \delta_3 \\ 2.236 / 63^\circ.26' - 2.236 / 206^\circ.34' &= 2.236 / 63^\circ.26' + 2.236 / 26^\circ.34'. \\ &= 4.243 / 45^\circ.\end{aligned}$$

Here again the process of complex subtraction, which is only a slight modification of complex addition, is very easily made on the drawing board by purely geometric processes, whether the quantities are rectangular or polar. If, however, the process is to be conducted algebraically, it is much more easily conducted with rectangular coordinates.

## EXPLANATORY TEXT

### MULTIPLICATION OF COMPLEX QUANTITIES

Two rectangular complex quantities may be multiplied algebraically by the ordinary rules of algebra, remembering that  $i^2 = -1$ . Thus

$$(x_1 + iy_1)(x_2 + iy_2) = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1). \quad (3)$$

In Fig. 9, the vector  $OP_1$  of Figs. 1 and 2 is multiplied by the vector  $OP_2$  of Figs. 3 and 4. The product is the broken line  $OP_3$ .

$$\text{For } (1 + i2) \times (-2 - i1) = (-2 + 2) - i(1 + 4) = -i5.$$

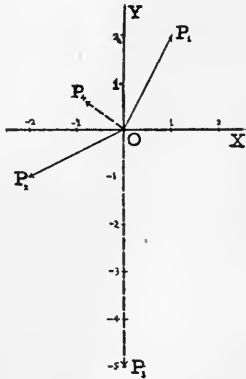


FIG. 9. — Product and Quotient of Complex Quantities  
Rectangular Coördinates

$$(1 + i2) \times (-2 - i1) = -i5 = OP_3$$

$$(-2 + i1) \div (+1 + i2) = -0.8 + i0.6 = OP_4$$

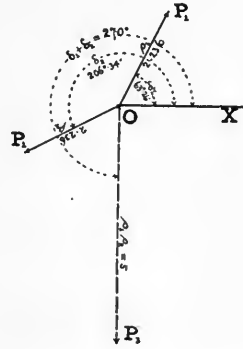


FIG. 10. — Product of two Complex Quantities, Polar Coördinates  
 $2.236 / 63^\circ.26' \times 2.236 / 206^\circ.34' = 5 / 270^\circ = OP_3$

If the two quantities to be multiplied are polar; then

$$\rho_1 / \delta_1 \times \rho_2 / \delta_2 = \rho_1 \rho_2 / \delta_1 + \delta_2. \quad (4)$$

Or the rule is form the product of the moduli and add the arguments. Thus in Fig. 10,  $OP_1 = \sqrt{5} / 63^\circ.26'$  and  $OP_2 = \sqrt{5} / 206^\circ.34' \therefore OP_3 = 5 / 270^\circ.00'$ .

### RECIPROCAL OF A COMPLEX QUANTITY

The reciprocal of a rectangular complex quantity can be reduced to the standard algebraic form, by multiplying both numerator and denominator by the same complex quantity with reversed imaginary. Thus:

$$\frac{1}{x + iy} = \frac{1 \times (x - iy)}{(x + iy)(x - iy)} = \frac{x - iy}{x^2 + y^2} = \left( \frac{x}{x^2 + y^2} \right) - i \left( \frac{y}{x^2 + y^2} \right). \quad (5)$$

For example if  $x + iy = 1 + i2$ ,

$$\frac{1}{1 + i2} = \frac{1}{1 + i2} \left( \frac{1 - i2}{1 - i2} \right) = \frac{1 - i2}{1 + 4} = \frac{1}{5} - \frac{i2}{5} = 0.2 - i0.4.$$

The reciprocal of a polar complex quantity is obtained by taking the reciprocal of its modulus, and reversing its argument. That is

$$\frac{1}{\rho / \delta} = \frac{1}{\rho} \frac{\rho}{\delta} = \frac{1}{\rho} \sqrt{\delta}. \quad (6)$$

[ 157 ]

## EXPLANATORY TEXT

For example:

$$\frac{1}{\sqrt{5} / 63^{\circ}.26'} = \frac{1}{\sqrt{5}} \sqrt{63^{\circ}.26'}.$$

### QUOTIENT OF COMPLEX QUANTITIES

To find the quotient of a complex quantity  $A$  divided by another  $B$ , form the reciprocal of  $B$  and then multiply this reciprocal by  $A$ .

Thus to find  $(x_1 + iy_1)/(x_2 + iy_2)$

$$\frac{x_1 + iy_1}{x_2 + iy_2} = \frac{x_1 + iy_1}{x_2 + iy_2} \left( \frac{x_2 - iy_2}{x_2 - iy_2} \right) = \frac{(x_1x_2 + y_1y_2) + i(y_1x_2 - y_2x_1)}{x_2^2 + y_2^2}. \quad (7)$$

For example:

$$\begin{aligned} \frac{OP_2}{OP_1} &= \frac{-2 - i1}{1 + i2} = \frac{-2 - i1}{1 + i2} \left( \frac{1 - i2}{1 - i2} \right) \\ &= \frac{(-2 - 2) + i(4 - 1)}{1 + 4} = \frac{-4 + i3}{5} = -0.8 + i0.6. \end{aligned}$$

Thus, in Fig. 9,  $\frac{OP_2}{OP_1} = OP_4$ .

The quotient of two polar complex quantities is formed by taking the quotient of their moduli and the difference of their arguments. That is

$$\frac{\rho_2 / \delta_2}{\rho_1 / \delta_1} = \frac{\rho_2}{\rho_1} / \delta_2 - \delta_1. \quad (8)$$

Thus in Fig. 11 we have the quotient of  $OP_2$  of Figs. 3 and 4 divided by  $OP_1$  of Figs. 1 and 2, or

$$\frac{\sqrt{5} / 206^{\circ}.34'}{\sqrt{5} / 63^{\circ}.26'} = 1 / 143^{\circ}.08'.$$

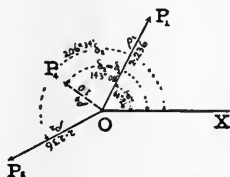


FIG. 11. — Quotient of two Complex Quantities,  
Polar Coördinates.

$$\begin{aligned} 2.236 / 206^{\circ}.34' \div 2.236 / 63^{\circ}.26' &= 1.0 / 143^{\circ}.08' = OP_4 \\ \rho_2 / \delta_2 \div \rho_1 / \delta_1 &= \rho_4 / \delta_2 - \delta_1 \end{aligned}$$

It is thus evident that in order to find either the sums or the differences of complex quantities, it is desirable to have them expressed in rectangular coördinates; while, on the other hand, in order to find products, reciprocals, or quotients, it is preferable to have them expressed in polar coördinates.

## EXPLANATORY TEXT

### POWERS AND ROOTS OF COMPLEX QUANTITIES

It will be evident from the foregoing that

$$(\rho/\delta)^n = \rho^n/n\delta: \text{ and } \sqrt[n]{\rho/\delta} = \sqrt[n]{\rho}/\delta/n \quad (9)$$

operations that are readily executed on polar complex quantities.

### CIRCULAR AND HYPERBOLIC FUNCTIONS GEOMETRICALLY COMPARED

Since the Tables in this book are adapted for the evaluation of both circular and hyperbolic functions of a complex variable; that is, either of  $\sin(x + iy)$ ,  $\cos(x + iy)$  and  $\tan(x + iy)$ ; or of  $\sinh(x + iy)$ ,  $\cosh(x + iy)$  and  $\tanh(x + iy)$ , it may be advisable to consider some propositions in the comparative geometry of the circular and hyperbolic functions, both real and complex.

### REAL CIRCULAR AND HYPERBOLIC FUNCTIONS

The geometry of the real circular functions  $\sin x$ ,  $\cos x$  and  $\tan x$  relates, as is well known, to the motion of a radius vector over a circle. The geometry of the real hyperbolic functions  $\sinh x$ ,  $\cosh x$  and  $\tanh x$  relates to the motion of a radius vector over a rectangular hyperbola. In Fig. 12,  $A b c d E g$  is a circle  $x^2 + y^2 = 1$ , assumed to have unit radius, and center  $O$ . As the radius vector  $OA$  rotates in the positive or counterclockwise direction about the center  $O$ , it describes a circular sector such as  $AOE$ , and a circular angle  $\beta$ , the tangent  $Ef$  being always perpendicular to the radius vector  $OE$ . The magnitude of the circular angle  $\beta$  may be defined in either of two ways, namely:—

(1) By the ratio of the circular arc length  $s$  described during the motion, by the vector's terminal  $E$ , to the constant length  $\rho$  of the radius vector.

(2) By the area of the circular sector  $AOE$  swept out by the radius vector during the motion.

According to definition (1), if the radius vector generates any infinitesimal angle  $d\beta$  circular radians, by moving its terminal over an infinitesimally small circular arc  $ds$

$$\text{then} \quad d\beta = \frac{ds}{\rho} = \frac{ds}{1} \quad \text{circular radians (10)}$$

since the constant radius vector  $\rho$  has been taken equal to unity. Consequently, in passing over any circular arc from distance  $s_1$  to distance  $s_2$ , through a distance  $s_2 - s_1 = s$ , the total circular sector and circular angle generated will of course be:—

$$\beta = \int_{s_1}^{s_2} \frac{ds}{1} = (s_2 - s_1) = s \quad \text{circular radians (11)}$$

or the angle  $\beta$ , as is well known, becomes equal to the length of the circular arc described, when expressed in circular radians.

According to definition (2), if in Fig. 12, the radius vector of unit length moves from the initial position  $OA$  to any position such as  $OE$ , it will sweep out a circular sector  $OEA$ .



## EXPLANATORY TEXT

If the arc  $AE^1$  be measured in the negative or clockwise direction equal in length to the arc  $AE$ , then it is well known that the area of the double sector  $EOE^1$  shaded in Fig. 12, is equal to  $\beta$  units of area because the area of the whole circle is manifestly  $\pi$  units, and the shaded area is  $\frac{2\beta}{2\pi}$  that of the whole circle. Consequently, the magnitude of the angle  $\beta$  expressed in circular radians is numerically twice the area of the circular sector  $AOE$  which it covers when the circle has unit radius.



FIG. 12. — Circular Sector and Real Circular Functions.

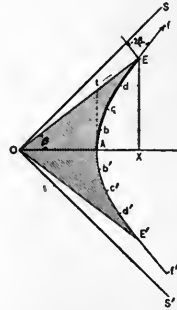


FIG. 13. — Hyperbolic Sector and Real Hyperbolic Functions.

Turning now to the hyperbolic case, let  $A b c d E$  Fig. 13, be an arc of a rectangular hyperbola  $x^2 - y^2 = 1$ , assumed to have unit semi-diameter  $OA$ , and center  $O$ . As the radius vector  $OA$  rotates in the positive or counterclockwise direction with center  $O$ , it describes a hyperbolic sector  $AOE$ , and also what may conventionally be called for convenience a “hyperbolic angle”  $\theta$ .\* The tangent  $Ef$  to the path of the moving terminal  $E$  always makes a circular angle  $\beta$  with the  $Y$  axis; or a circular angle of  $2\beta$  with a perpendicular to the radius vector. The magnitude of the hyperbolic angle  $\theta$  may be defined in either of two ways; namely: —

(1) By the ratio of the hyperbolic arc length  $s$  described during the motion, by the terminal  $E$ , to the integrated mean length of the varying radius vector.

(2) By the area of the hyperbolic sector  $AOE$  Fig. 13, swept out by the radius vector during the motion.†

According to definition (1), if the variable radius vector  $\rho$  generates any infinitesimal hyperbolic angle  $d\theta$  by moving its terminal over an infinitesimally small hyperbolic arc  $ds$ ; then ‡

$$d\theta = \frac{ds}{\rho} \quad \text{hyperbolic radians} \quad (12)$$

\* It should be pointed out that a “hyperbolic angle” in the sense above defined is not the opening between two lines intersecting in a plane; but a quantity otherwise analogous to a circular angle, and the argument  $x$  of the functions  $\sinh x$ ,  $\cosh x$ ,  $\tanh x$ , etc. The use of the term “hyperbolic angle” can only be justified by its convenience of analogy.

† Greenhill’s “Differential and Integral Calculus,” 1896, p. 108.

‡ A demonstration of this proposition has been given by the author in “The Application of Hyperbolic Functions to Electrical Engineering Problems.” Appendix L, p. 250. University of London Press, 1911.

## EXPLANATORY TEXT

Consequently, in passing over any hyperbolic arc from distance  $s_1$  to distance  $s_2$  through a distance  $s_2 - s_1 = s$ , the total hyperbolic sector and hyperbolic angle generated will be

$$\theta = \int_{s_1}^{s_2} \frac{ds}{\rho} = \frac{s}{\rho^1} \quad \text{hyperbolic radians} \quad (13)$$

where  $\rho^1$  is the integrated mean value of  $\rho$  as defined by the last equation. Any infinitesimally small angle, whether circular or hyperbolic, is therefore expressed in corresponding radian measure by one and the same term  $ds/\rho$ ; but whereas, in the case of circular angles, the constancy of the radius vector makes the integral simply  $s/\rho$ , in the case of hyperbolic angles, the variation of the radius vector makes the integral more complex. Fig. 14 represents a circular angle of 1 radian in five sections of 0.2 radian each; while Fig. 15 represents a hyperbolic angle of 1 radian correspondingly divided. The integrated mean radius vector of the full sector  $AOF$  intersects the curve in the point  $f$ , the total length of the arc  $A B C D E F$  being 1.3167 units.

### SINES, COSINES AND TANGENTS OF CIRCULAR AND HYPERBOLIC ANGLES

If, with unit radius, we draw both a circular and a rectangular hyperbolic sector, as in Figs. 12 and 13, and take  $OA$  as the initial line in each; then for any position

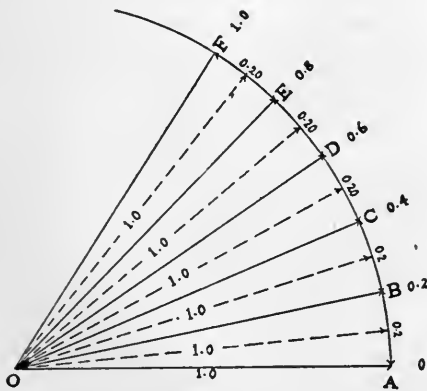


FIG. 14. — Circular Sector of 1 Radian Subdivided into Five Sectors of 0.2 Radian each.

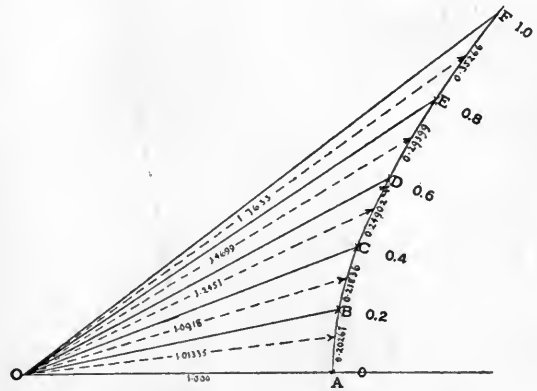


FIG. 15. — Hyperbolic Sector of 1 Radian Subdivided into Five Sectors each of 0.2 Radian.

of the radius vector such as  $OE$ , we shall have in either case the following magnitude conditions: —

The sine will be equal to the length of the perpendicular from the terminal of the radius vector on to the  $X$  axis.

The cosine will be equal to the length of the intercept on the  $X$  axis made by the above-mentioned perpendicular.

The versed sine will be equal to the length  $XA$ , Fig. 12, and  $AX$ , Fig. 13, between the intercept on the  $X$  axis, and the horizontal unit radius.

## EXPLANATORY TEXT

The tangent will be equal to the length of the perpendicular from the radius vector (or radius vector produced) on to unit radius point of the  $X$  axis. Thus in

Fig. 12,  $\sin \beta = XE$ .

Fig. 13,  $\sinh \theta = XE$ .

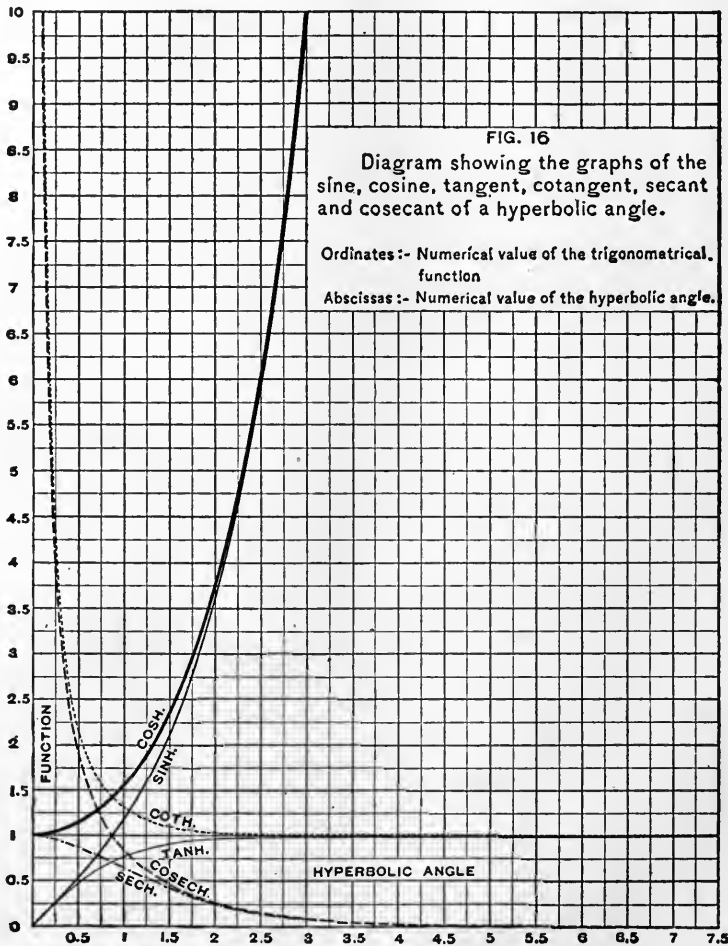
Fig. 12,  $\cos \beta = OX$ .

Fig. 13,  $\cosh \theta = OX$ .

Fig. 12,  $\tan \beta = At$ .

Fig. 13,  $\tanh \theta = At$ .

Whereas the values of  $\sin \beta$ ,  $\cos \beta$  and  $\tan \beta$  fluctuate periodically in sign as  $\beta$  increases from 0 to  $\alpha$ , the values of  $\sinh \theta$ ,  $\cosh \theta$ , and  $\tanh \theta$  do not change sign, the graphs of the real hyperbolic functions being indicated in Fig. 16, as far as  $\theta = 3.0$ .



# EXPLANATORY TEXT

## BISECTION OF CIRCULAR AND HYPERBOLIC ANGLES

If we take any circular angle  $BOC$  Fig. 17, we may of course bisect this angle in either of two ways: —

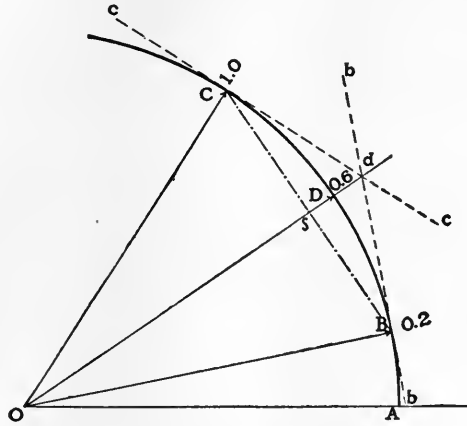


FIG. 17. — Bisection of a circular sector in the well-known manner by a radius vector through the intersection of terminal tangents, or through the midpoint of the chord between terminal points.

(1) By drawing tangents  $bb$ ,  $cc$ , to the curve at the points  $B$ ,  $C$ , respectively, and drawing the straight line  $Od$  from the center  $O$  through the point of intersection  $d$ .

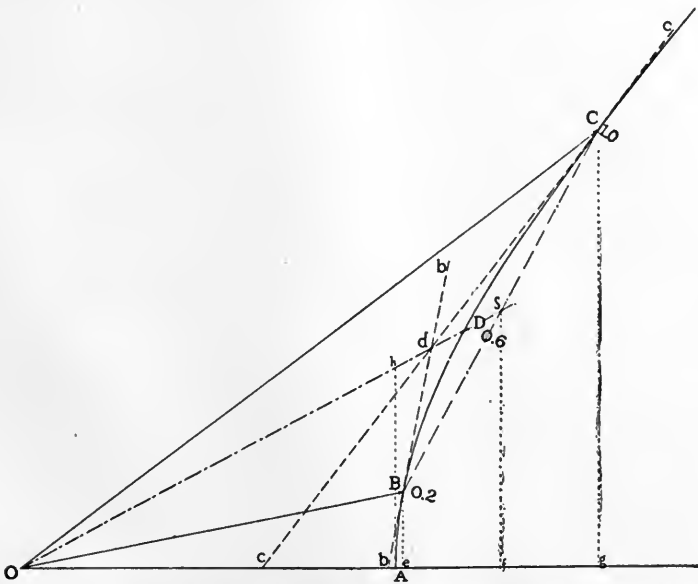


FIG. 18. — Bisection of a hyperbolic sector by a radius vector through the intersection of terminal tangents, or through the midpoint of the chord between terminal points.

(2) By drawing the chord  $BC$ , and marking the radius  $OD$  through the midpoint  $\delta$  of this chord.

Similarly, if we take any hyperbolic angle  $BDC$  Fig. 18, between the points  $B$  and  $C$  of a rectangular hyperbola, we may bisect this angle in either of two ways:—

(1) By drawing tangents  $bb$ ,  $cc$ , to the curve at the points  $B$ ,  $C$ , respectively, and drawing the straight line  $Od$  from the center  $O$  through the point of intersection  $d$ .

(2) By drawing the chord,  $BC$ , and marking the radius  $OD$  through the midpoint  $\delta$  of this chord.\*

## COMPARATIVE GEOMETRY OF COMPLEX CIRCULAR AND HYPERBOLIC FUNCTIONS

We have seen that the real circular functions  $\sin x$ ,  $\cos x$ , may be derived from a circle diagram, and that the real hyperbolic functions  $\sinh x$ ,  $\cosh x$ , may be similarly derived from a rectangular hyperbola diagram. We shall see that both the complex circular functions  $\sin(x + iy)$ ,  $\cos(x + iy)$ , and the complex hyperbolic functions  $\sinh(x + iy)$ ,  $\cosh(x + iy)$ , may be derived from a combination circle and hyperbola diagram.

### COMPLEX CIRCULAR FUNCTIONS

#### CONSTRUCTION FOR $\sin(x \pm iy)$ , AND FOR $\sin^{-1}(u \pm iv)$

In Fig. 19, take  $OA = 1$  along the negative side of the  $Y$  axis. From  $OA$  as initial line, mark off the circular angle  $x = AOB$ . From  $OB$  as initial line, mark off the hyperbolic angle  $y$  and its sector  $BOD$ . Let  $C$  be the foot of the perpendicular from  $D$  on  $OB$  produced. Drop perpendiculars from  $C$  and  $D$  on the axis of reals  $OX$ , at  $c$  and  $d$  respectively. About  $c$  as center, rotate  $cd$  positively through  $90^\circ$  to  $cZ$ . Then will

\* This proposition is proved in Greenhill's "Differential and Integral Calculus," Macmillan & Co., 1896, page 67, Fig. 16, for the particular case when the angle  $AOB$ , in our Fig. 18, is zero. The demonstration of proposition (1) for the general case of Fig. 18 is not difficult; but that found by the author is rather lengthy. The demonstration of the general proposition (2) is, however, brief and direct, as follows:—

Let  $\theta_1$  be the hyperbolic angle of the sector  $AOB$ .

Let  $\theta_2$  be the hyperbolic angle of the sector  $AOC$ .

Then it is required to show that

$$\frac{\delta f}{Of} = \frac{hA}{OA} = \frac{hA}{1} = \tanh \frac{(\theta_1 + \theta_2)}{2}.$$

But from an inspection of the Figure,

$$Be = \sinh \theta_1, \quad Cg = \sinh \theta_2,$$

$$Oe = \cosh \theta_1, \quad Og = \cosh \theta_2,$$

so that

$$f\delta = \frac{eB + gC}{2} = \frac{\sinh \theta_1 + \sinh \theta_2}{2}$$

and

$$Of = \frac{Oe + Og}{2} = \frac{\cosh \theta_1 + \cosh \theta_2}{2}.$$

Thus

$$\frac{f\delta}{Of} = \frac{\sinh \theta_1 + \sinh \theta_2}{\cosh \theta_1 + \cosh \theta_2}$$

which is a known equivalent expression for  $\tanh \frac{(\theta_1 + \theta_2)}{2}$ , see Becker and Van Orstrand's "Hyperbolic Functions," 1909, p. XIV, Formula (49).

## EXPLANATORY TEXT

the complex vector  $OZ = Oc + icd$  be the required circular sine of the complex angle  $x + iy$  radians. In the case represented,  $\sin (1 + i 1) = 1.299 + i 0.635 = 1.446 / 26^\circ.05$ . As  $y$  varies,  $Z$  moves along the hyperbola  $bZ$ :—

$$\frac{X^2}{\sin^2 x} - \frac{Y^2}{\cos^2 x} = 1 \quad (14)$$

and as  $x$  varies,  $Z$  moves along the ellipse:—

$$\frac{X^2}{\cosh^2 y} + \frac{Y^2}{\sinh^2 y} = 1. \quad (15)$$

Both the hyperbola and the ellipse have as common foci  $FF'$ , the points  $X = \pm 1$ ,  $Y = 0$ .

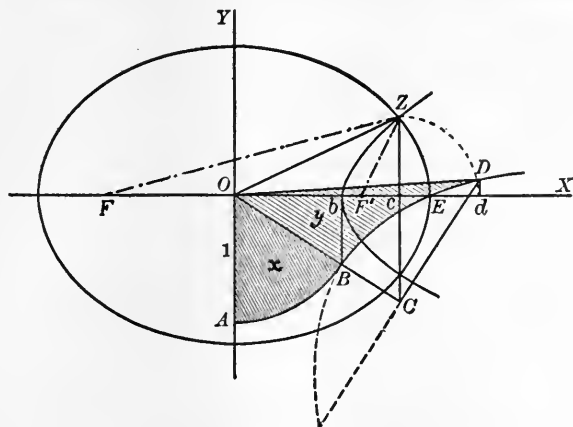


FIG. 19. — Constructions for  $\sin (x + iy)$  and  $\sin^{-1} (u \pm iv)$ .

From the same figure, we have also, if  $Oc = u$  and  $cZ = iv$ ,  
 $\sin^{-1} (u \pm iv) = \sin^{-1} Ob \pm i \cosh^{-1} OE$

$$\begin{aligned} &= \sin^{-1} \left\{ \frac{\sqrt{(1+u)^2 + v^2} - \sqrt{(1-u)^2 + v^2}}{2} \right\} \\ &\pm i \cosh^{-1} \left\{ \frac{\sqrt{(1+u)^2 + v^2} + \sqrt{(1-u)^2 + v^2}}{2} \right\} \end{aligned} \quad (16)$$

since  $Ob = \frac{FZ - F'Z}{2}$  and  $OE = \frac{FZ + F'Z}{2}$ .

### CONSTRUCTIONS FOR $\cos (x + iy)$ AND FOR $\cos^{-1} (u + iv)$

In Fig. 20, take  $OA$  as unit distance along the real or  $X$  axis, in the positive direction. From  $OA$  as initial line, describe the circular angle  $x$ , or the circular sector  $AOB$  of area  $x/2$ . On  $OB$  as initial line, describe the hyperbolic angle  $y$ , or the hyperbolic sector area  $BOD$  of area  $y/2$ . Let  $C$  be the foot of the perpendicular from  $D$  on  $OB$  produced. Drop perpendiculars from  $C$  and  $D$  on the  $X$  axis at  $c$  and  $d$  respectively. With  $c$  as center, rotate the line  $cd$  in the positive direction through  $90^\circ$  into the position  $cZ$ ; so that  $\overline{cZ} = i \cdot \overline{cd}$ . Then the complex quantity  $OZ = Oc + i.cd$  will be the required circular cosine of the complex angle  $(x + iy)$  radians.

## EXPLANATORY TEXT

In the case represented,  $\cos (\mathbf{r}+i \mathbf{r})=0.834-i 0.989=1.293 \sqrt{49^{\circ} .866}$ . As  $y$  varies,  $Z$  moves along the hyperbola  $b Z$  defined by

$$\frac{X^2}{\cos^2 x} - \frac{Y^2}{\sin^2 x} = 1 \quad (17)$$

and as  $x$  varies,  $Z$  moves along the ellipse  $ZE$ , defined by

$$\frac{X^2}{\cosh^2 \gamma} + \frac{Y^2}{\sinh^2 \gamma} = 1. \quad (18)$$

Both the hyperbola and the ellipse have as common foci  $FF'$ , the points  $X = \pm 1, Y = 0$ .

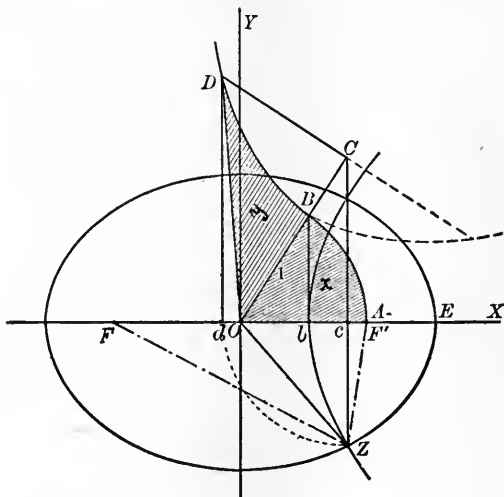


FIG. 20. — Constructions for  $\cos (x \pm iy)$  and  $\cos^{-1} (u \pm iv)$ .

From Fig. 20 we obtain: —

$$\begin{aligned} \cos^{-1} OZ &= \cos^{-1}(u \pm iv) = \cos^{-1} Ob \mp \cosh^{-1} OE \\ &= \cos^{-1} \left\{ \frac{\sqrt{(1+u)^2 + v^2} - \sqrt{(1-u)^2 + v^2}}{2} \right\} \\ &\mp i \cosh^{-1} \left\{ \frac{\sqrt{(1+u)^2 + v^2} + \sqrt{(1-u)^2 + v^2}}{2} \right\} \end{aligned} \quad (19)$$

since  $Ob = \frac{FZ - F'Z}{2}$  and  $OE = \frac{FZ + F'Z}{2}$ .

# COMPLEX HYPERBOLIC FUNCTIONS

### CONSTRUCTIONS FOR $\sinh (x \pm iy)$ AND $\sinh^{-1} (u \pm iv)$

In Fig. 21, take  $OA$  as unit length along the real or  $X$  axis in the positive direction. From  $OA$  as initial line, describe the circular angle  $y$ , or the circular sector  $AOB$  of area  $y/2$ . From  $OB$  as initial line describe the hyperbolic angle  $x$ , or the hyperbolic sector  $BOD$  of area  $x/2$ . Let  $C$  be the foot of the perpendicular from  $D$  on  $OB$  pro-

# EXPLANATORY TEXT

duced. Drop perpendiculars from  $C$  and  $D$  on the  $Y$  axis at  $c$  and  $d$  respectively. With  $c$  as center, rotate the line  $cd$  negatively, or clockwise, through  $90^\circ$  to  $cZ$ . The complex quantity  $OZ = Oc - i.cd$  will be the required hyperbolic sine of the complex angle  $(x + iy)$  radians.

In the case represented,  $\sinh (1 + i 1) = 0.635 + i 1.2985 = 1.446 / 63^\circ.95$ . As  $x$  varies,  $Z$  moves along the hyperbola  $Zbz$ :—

$$\frac{Y^2}{\sin^2 y} - \frac{X^2}{\cos^2 y} = 1 \quad (20)$$

and as  $y$  varies,  $Z$  moves along the ellipse  $XExy$

$$\frac{Y^2}{\cosh^2 x} + \frac{X^2}{\sinh^2 x} = 1. \quad (21)$$

The hyperbola and ellipse are confocal at the points  $F$  and  $f$  defined by  $X = 0$ ,  $Y = \pm 1$ .

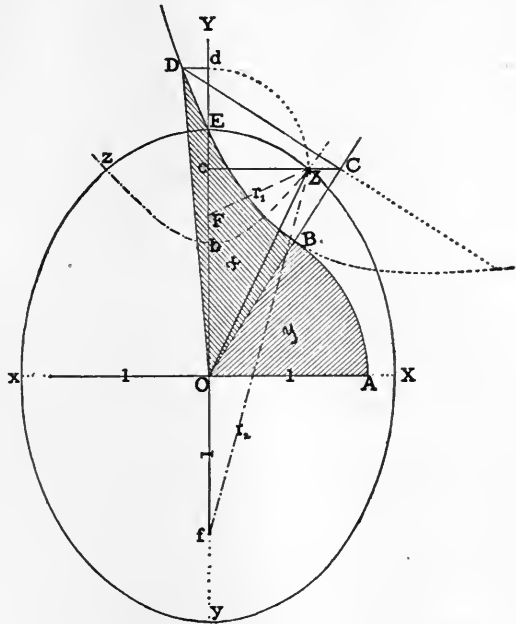


FIG. 21. — Constructions for  $\sinh (x \pm iy)$  and  $\sinh^{-1} (u \pm iv)$ .

From Fig. 21 we also obtain

$$\begin{aligned} \sinh^{-1} (u \pm iv) &= \sinh^{-1} (cZ \pm Oc) = \cosh^{-1} OE \pm i \sinh^{-1} Ob. \\ &= \cosh^{-1} \left\{ \frac{\sqrt{(1+v)^2 + u^2} + \sqrt{(1-v)^2 + u^2}}{2} \right\} \\ &\quad \pm i \sinh^{-1} \left\{ \frac{\sqrt{(1+v)^2 + u^2} - \sqrt{(1-v)^2 + u^2}}{2} \right\} \end{aligned} \quad (22)$$

$$\text{since } OE = \frac{fZ + FZ}{2} \text{ and } Ob = \frac{fZ - FZ}{2}.$$



# EXPLANATORY TEXT

## CONSTRUCTIONS FOR $\cosh (x + iy)$ AND $\cosh^{-1} (u + iv)$

In Fig. 22, take  $OA$  as unit distance along the real or  $X$  axis in the positive direction. From  $OA$ , as initial line, describe the circular angle  $y$ , or the circular sector  $AOB$  of area  $y/2$ . From  $OB$ , as initial line, describe the hyperbolic angle  $x$ , or the hyperbolic sector  $BOD$  of area  $x/2$ . Let  $C$  be the foot of the perpendicular from  $D$  on  $OB$  produced. Drop perpendiculars from  $C$  and  $D$  on the  $X$  axis at  $c$  and  $d$  respectively. About  $c$ , as

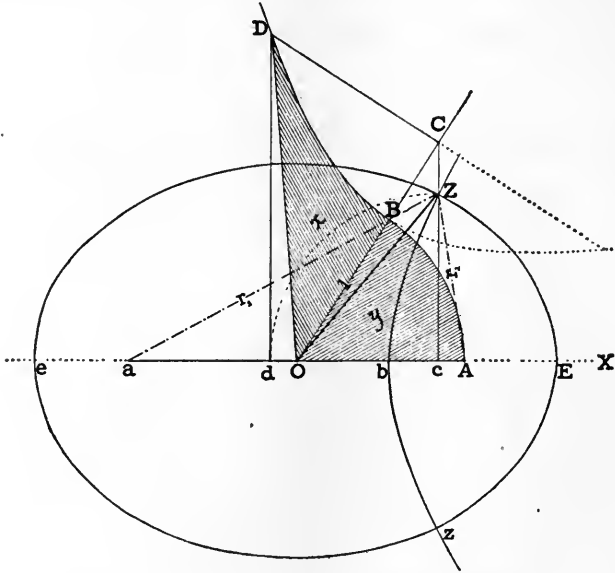


FIG. 22. — Constructions for  $\cosh (x + iy)$  and  $\cosh^{-1} (u + iv)$ .

center, rotate the line  $cd$  negatively, or clockwise, through  $90^\circ$  to  $cZ$ ; so that  $cZ = -i.cd$ . Then the complex quantity  $OZ = Oc - i.cd$  will be the required cosine of the complex angle  $(x + iy)$  radians.

In the case represented,  $\cosh (1 + i 1) = 0.834 + i 0.989 = 1.293 / 49^\circ.866$ . As  $x$  varies,  $Z$  moves along the hyperbola  $Zbz$

$$\frac{X^2}{\cos^2 y} - \frac{Y^2}{\sin^2 y} = 1. \quad (23)$$

As  $y$  varies,  $Z$  moves along the ellipse  $EZez$

$$\frac{X^2}{\cosh^2 x} + \frac{Y^2}{\sinh^2 x} = 1. \quad (24)$$

The ellipse and hyperbola are confocal at the points  $A, a$ , defined by  $X = \pm 1, Y = 0$ .

# EXPLANATORY TEXT

From the same figure. If  $Oc = u$  and  $cZ = iv$

$$\begin{aligned} \cosh^{-1}(u \pm iv) &= \cosh^{-1}(Oc \pm i.cZ) = \cosh^{-1}OE \pm i \cos^{-1}Ob \\ &= \cosh^{-1}\left(\frac{r_1 + r_2}{2}\right) \pm i \cos^{-1}\left(\frac{r_1 - r_2}{2}\right) \\ &= \cosh^{-1}\left\{\frac{\sqrt{(1+u)^2 + v^2} + \sqrt{(1-u)^2 + v^2}}{2}\right\} \\ &\quad \pm i \cos^{-1}\left\{\frac{\sqrt{(1+u)^2 + v^2} - \sqrt{(1-u)^2 + v^2}}{2}\right\}. \quad (25) \end{aligned}$$

## CONSTRUCTIONS FOR $\tan(x \pm iy)$ AND $\tan^{-1}(u \pm iv)$

In Fig. 23, lay off along the  $X$  axis a point  $A$  distant  $\tan x$  from  $O$ , and also a point  $B$  such that  $OB = \cot x$ . Draw a circle through  $A$  and  $B$  having its center on  $OX$  at  $C$ . The distance  $OC$  measures  $\cot 2x$  and the radius of the circle is  $\operatorname{cosec} 2x$ . Any circle thus drawn will intersect the  $Y$  axis at two points  $e$  and  $f$  which are at unit dis-

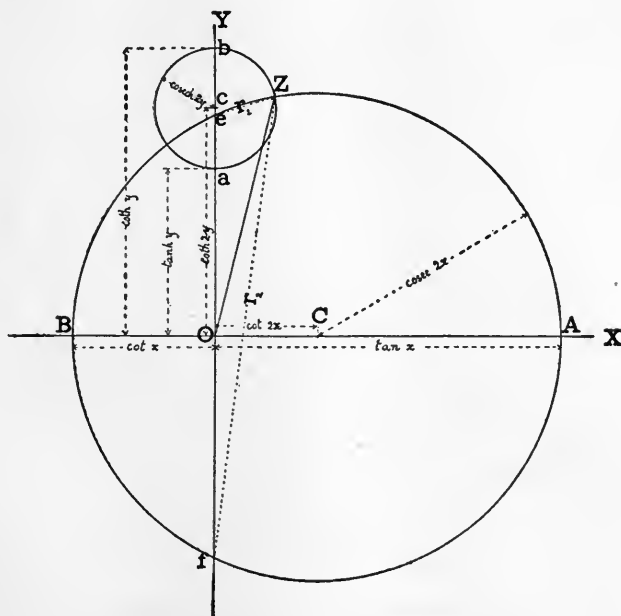


FIG. 23. — Constructions for  $\tan(x \pm iy)$  and  $\tan^{-1}(u \pm iv)$ .

tances from  $O$ . Then lay off the  $Y$  axis two points  $a$  and  $b$ , distant respectively  $\tanh y$  and  $\coth y$  from  $O$ . With center  $c$  on the  $Y$  axis, draw a circle through  $a$  and  $b$ . The distance  $Oc$  will be  $\coth 2y$ , and the radius of the circle will be  $\operatorname{cosech} 2y$ . Let  $Z$  be the point of intersection of the two circles. Then  $OZ$  is the required tangent of  $(x + iy)$ . If  $x$  is kept constant but  $y$  is varied, the point  $Z$  moves over the circle  $AZB$ . If on the other hand  $y$  is kept constant, but  $x$  is varied,  $Z$  will move around the circle  $aZb$  and will make one complete revolution for each increase of  $\pi$  units in  $x$ .

## EXPLANATORY TEXT

In the case represented,  $\tan (1 + i 1) = 0.2718 + i 1.084 = 1.118 / 75^{\circ}.916$ .

From Fig. 23 it is evident that the angle  $AeO$  is equal to  $x$ , and angle  $eAO$  is thus the complement of  $x$ . Hence half the angle between  $r_1$  and  $r_2$  is the complement of  $x$ . Moreover  $y = \log_e \sqrt{r_2/r_1}$ . Therefore, if  $OZ = u + iv$ ,

$$\tan^{-1}(u \pm iv) = \left\{ \frac{\pi - \tan^{-1}\left(\frac{u}{\pm v - 1}\right) + \tan^{-1}\left(\frac{u}{\pm v + 1}\right)}{2} \right\} + \frac{i}{2} \log_e \sqrt{\frac{(1 \pm v)^2 + u^2}{(1 \mp v)^2 + u^2}}. \quad (26)$$

### CONSTRUCTIONS FOR $\tanh(x \pm iy)$ AND $\tanh^{-1}(u \pm iv)$

In Fig. 24 mark off on the axis of reals  $xOX$  two points  $T$  and  $X$  such that the former is distant by  $\tanh x$  and the latter by  $\coth x$  from the origin  $O$ . Find the point  $C$  midway between  $T$  and  $X$ . Incidentally, this point will be distant  $\coth 2x$  from  $O$ . With

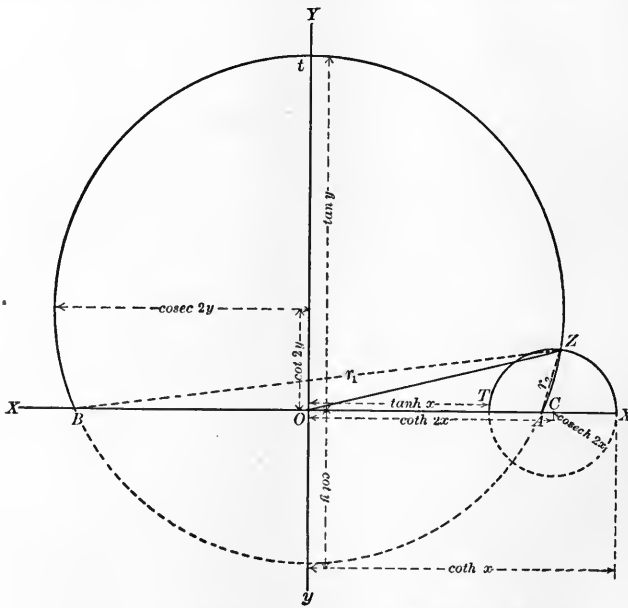


FIG. 24. — Constructions for  $\tanh(x \pm iy)$  and  $\tanh^{-1}(u \pm iv)$

center  $C$  and radius  $CT = CX = \operatorname{cosech} 2x$ , draw the circle  $TXZ$ . Mark off on the axis of imaginaries  $yOY$ , two points  $t$  and  $y$  such that the former is distant by  $\tan y$  and the latter by  $\cot y$  from the origin  $O$ . Find the point  $c$  midway between them. Incidentally, this point will be distant  $\cot 2y$  from  $O$ . With center  $c$  and radius  $ct = cy = \operatorname{cosec} 2y$ , draw the circle  $ByAt$ . This circle will cut the axis of reals at two points  $A$  and  $B$  distant each one unit from  $O$ . It will also intersect the circle  $TXZ$  perpendicularly at  $Z$ . Connect  $OZ$ . This vector  $OZ$  is the required hyperbolic tangent of the complex angle  $(x + iy)$  radians.

## EXPLANATORY TEXT

In the case represented,  $\tanh (1 + i) = 1.084 + i 0.2718 = 1.118 / 14^{\circ}.084$ . As  $x$  varies,  $Z$  moves along the circle  $AtB$ . As  $y$  varies,  $Z$  moves along the circle  $TZX$ , performing one complete revolution for each  $\pi$  units of increase in  $y$ .

From the same Figure, if  $OZ = u \pm iv = op \pm ipZ$ , we have  $\tanh^{-1}(u \pm iv) = x \pm iy$ .

In this case  $x = \log_e \sqrt{r_1/r_2}$

$$\text{or } x = \frac{1}{2} \log_e (r_1/r_2). \quad (27)$$

and  $y = \frac{\pi - \alpha}{2}$  where  $\alpha$  is the circular angle at  $Z$  between the radii vectores  $r$ , and  $r_2$ .

Also

$$\alpha = \tan^{-1} \left( \frac{u+1}{\pm v} \right) - \tan^{-1} \left( \frac{u-1}{\pm v} \right). \quad (28)$$

Hence

$$\tanh^{-1}(u \pm iv) = \frac{1}{2} \log_e \sqrt{\frac{(1+u)^2 + v^2}{(1-u)^2 + v^2}} + i \left\{ \frac{\pi - \tan^{-1} \left( \frac{u+1}{\pm v} \right) + \tan^{-1} \left( \frac{u-1}{\pm v} \right)}{2} \right\} \quad (29)$$

## DEGREE OF PRECISION OF TABLES

### INTRODUCTION

If a numerical quantity, freed from decimals, is correctly expressed to within say 1 part in 1000; *i.e.*, 1 part in  $10^3$ , then this degree of precision may conveniently be described as precision of the third order. In general, therefore, if a numerical quantity be correctly expressed to within 1 part in  $10^n$ , where  $n$  is any real positive number, its precision is of the  $n$ th order. The weekly statement of the financial assets of a bank might be expressed as \$186,257,361.26 which, assuming that it is to be taken as being numerically correct to a single cent, represents 18,625,736,126 cents, an apparent precision of 1 in  $10^{10.27}$ , or of the 10.27th order. Physical and astronomical precisions are less ostensibly pretentious, however, and rarely exceed the 6th order. Engineering computations are commonly satisfied with a precision of the third order; although, on rare occasions, the order required may be the highest that physics can attain.

The degree of precision corresponding to retaining a specified number of significant digits correct within unity, in Tables, can only be stated approximately; since it varies with the values of the digits. Thus, if we have tables containing entries each of three significant digits, correct to the last digit, the lowest entry may be 100 and the highest 999. The precision would therefore be 1 in 100 in the former case, and practically 1 in 1000 in the latter. That is the order of precision would vary between the second and the third. The average precision might be stated as of the 2.5th order. Such tables of  $n$  significant digits lay claim to an average precision of the  $(n - 1/2)$ th order.

Many tables are, however, employed in which the last digit is stated to be correct to the nearest digit; that is within half of unity. On that understanding, the precision of say a three-digit table would vary between  $1/2$  in 100 to  $1/2$  in 999 or between the 2.3rd and the 3.3rd order, with a mean of the 2.8th order. Consequently, we may say that such tables, giving  $n$  significant digits, lay claim to an average precision of the  $(n - 0.2)$ th order.

## EXPLANATORY TEXT

### DEGREE OF PRECISION PRESENTED IN THE FOLLOWING TABLES

The tables of complex hyperbolic functions here presented have been prepared with a view to giving five decimal places regularly. This means five significant digits when the values of the results lie between 0.1 and unity, six significant digits when they lie between 1 and 10, four when they lie between 0.1 and 0.01 and so on. Tables I to VI inclusive were computed with the aid of five-figure logarithms of real hyperbolic functions, so that their degree of precision is necessarily limited to, and must on the average fall below that of such logarithm tables, which, as we have seen, is of the 4.8th

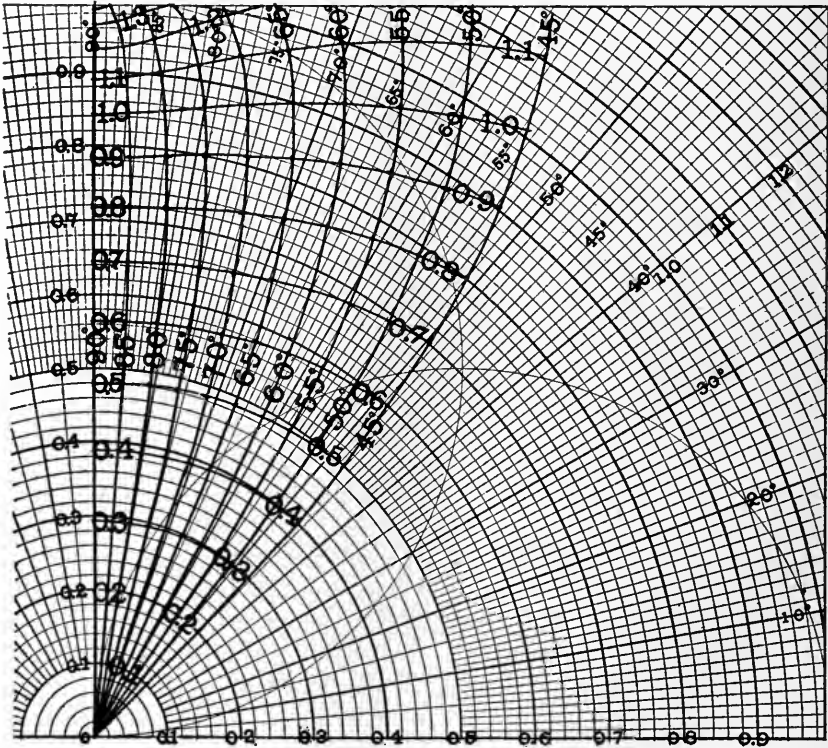


FIG. 25. —  $\sinh(\rho/\delta)$  expressed in polar coordinates.

order. Exclusive of such mistakes as may exist, they do not claim a degree of precision beyond the 4.5th order.

Tables VII to XIII inclusive were, however, computed for the most part from Ligowski's gudermannian angles which are tabulated by him for each thousandth of a hyperbolic radian, to the nearest hundredth of a second of circular arc. The logarithms of the corresponding real hyperbolic functions were then found in the eight-place tables of Bauschinger and Peters, which offer such logarithms for each and every second of circular arc. The results were computed in the formulas to at least six significant

## EXPLANATORY TEXT

digits and the sixth was then frequently discarded to meet the needs of the five-decimal table. Consequently in this group of tables, excluding such errors as may exist, the precision is on the average of the 4.8th order, and rises to the 5.8th order, when the value of the result lies between 1 and 10. The average precision of the second group of tables is thus about half an order greater than that of the first group.

### PRECISION OF THE CHARTS IN THE ATLAS

The charts of the accompanying Atlas have been prepared with a view to offering three digits in the deduced quantity, if reasonable care be taken in their use. This represents an average degree of precision of the 2.5th order; or about equal to that fur-

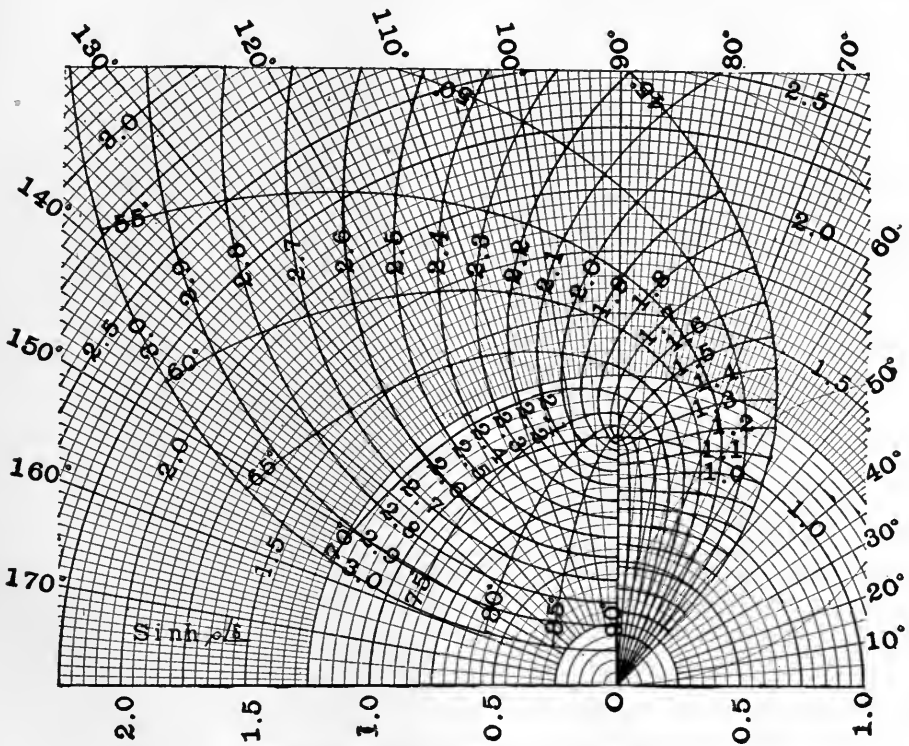


FIG. 26. —  $\sinh(\rho/\delta)$  expressed in polar coordinates.

nished by an ordinary 25 cm. slide rule. When a higher degree of precision than this is needed, arithmetical interpolation in the Tables must ordinarily be resorted to; but even then it is desirable to obtain a preliminary approximate value from the Atlas, in order to furnish a check against gross error.

# EXPLANATORY TEXT

## GRAPHIC REPRESENTATIONS

Figs. 25 and 26 present the results obtained from Table I to true polar coördinates. Each intersection of the curves corresponds to an entry in the table. Fig. 25 relates to pages 2, 3, and Fig. 26 to the rest of the table. The curves of constant  $\rho$  intersect those of constant  $\delta$  perpendicularly. That is, each intersection occurs theoretically at right angles. If, however, an attempt is made to prepare plates corresponding to Figs. 25 and 26 on a large scale, for a reasonable degree of precision, in rapid interpolation by graphical inspection, difficulties present themselves. Firstly, it has been found impracticable to procure polar coördinate ruled sheets large enough. Secondly, regular polar coördinate charts of the type presented in Figs. 25 and 26 necessarily offer very little graphical interpolation precision at small radial distances from the origin of co-

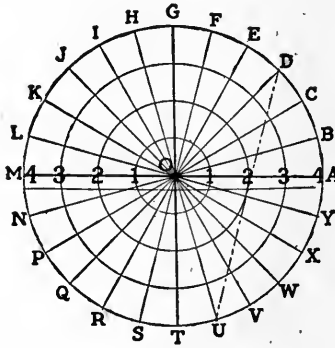


FIG. 27. — Polar Coördinate Diagram Regular Presentation on Circular Sheet.

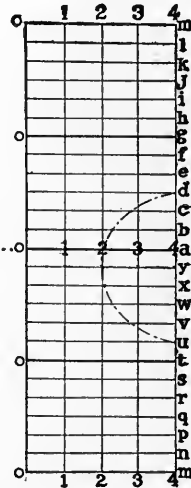


FIG. 28. — Polar Coördinate Diagram Squared Presentation on Rectangular Sheet.

ordinates, where the radial lines, sharply converging, crowd the diagram. On the other hand, they offer relatively great apparent interpolation precision at large distances from the origin, as the radii diverge. In preparing interpolation charts, therefore, the author has devised the scheme of using squared paper sheets for presenting such polar coördinate quantities as appear in Figs. 25 and 26. Fig. 27 represents the regular polar coördinate  $r/\gamma$  diagram, in which the lines of constant  $r$  are circles concentric at  $O$ , and the lines of constant  $\gamma$  are radii diverging from  $O$ . Fig. 28 represents the corresponding squared polar coördinate diagram, in which the lines of constant  $r$  are the vertical straight lines 1-1, 2-2, 3-3, and 4-4. The lines of constant  $\gamma$  are the parallel horizontal lines, while the origin point  $O$  in Fig. 27 becomes expanded into the original straight line  $o-o$  in Fig. 28. The straight, broken line  $DU$  is transformed into the curved line  $du$ , and, in general, orthogonally intersecting curves of one diagram do not transform into orthogonally intersecting curves on the other.

# EXPLANATORY TEXT

## INTERPOLATION CHARTS

Plates IA, IB, and IC of the Atlas correspond to Table I and present, to squared polar coördinates, the results in that table. Each intersection of curves in the plates corresponds to one entry in the table. Plate IA includes the entries on pages 2 and 3 of the table; while Plates IB and IC include the entries in the remainder of the table. The curves of constant  $\rho$  and constant  $\delta$  intersect one another at various angles, but the method of interpolation requires little explanation. The entering quantity will fall within some particular curvilinear parallelogram. The respective opposite sides may be subdivided into tenths in any of the three following ways: (1) by direct inspectional estimate, (2) by graphical subdivision on a sheet of tracing paper laid over the chart, (3) by means of a radiating decimal scale of lines, prepared in advance, on tracing paper or thin celluloid. It is not, in general, worth the effort of attempting a closer subdivision than tenths of the sides of any parallelogram. The point of intersection of lines parallel to the sides, through the correct decimal points, is then to be marked on the covering tracing paper, or held with a blunt pointer, such as a knitting needle, on the chart itself, and the rectangular coördinates of this point read off from the parallel ruling or background of the plate. That is, the charts are always used with the entering variable on the curvilinear coördinates, and with the result found on the rectilinear framework in the background; except when inverse functions are sought, and the procedure consequently reversed.

TABLE I

$$\sinh (\rho / \delta) = r / \gamma$$

### POLAR HYPERBOLIC SINES OF A POLAR VARIABLE

Table I, pages 2 to 7, gives the hyperbolic sine of vectors up to 3.0 in modulus, by steps of 0.1, for each degree of argument from 45° to 90°. The results are also expressed in polar coördinates, as plane vectors, corresponding to the relation:—

$$\sinh (\rho / \delta) = r / \gamma \tag{30}$$

$$\text{or} \quad \rho / \delta = \sinh^{-1} (r / \gamma). \tag{31}$$

The graphs of the results to true polar coördinates appear in Figs. 25 and 26, where the curves of constant  $\rho$  always intersect orthogonally the curves of constant  $\delta$ ; so that at any point of intersection the angles of intersection are right angles. In Plates IA, IB, and IC of the Atlas, the same graphs are given to squared polar coördinates, the disadvantages of the distortion being more than outweighed by the advantages in facility of graphic interpolation. In these charts the curves of constant  $\rho$  do not intersect the curves of constant  $\delta$  orthogonally.



## EXPLANATORY TEXT

### INTERPOLATION. FIRST CASE. IN MODULUS ONLY

If Table I is entered with a vector quantity of more than one decimal in modulus and of some exact degree of argument, such as  $2.76/70^\circ$ ; then the result will lie nearly on the line between the results for  $2.7/70^\circ$  and  $2.8/70^\circ$ ; namely, between  $1.2031/136.489$  and  $1.2136/143^\circ.005$ . A first approximation may be obtained by proportional parts between them, thus:—

$$\begin{array}{l}
 \text{Required } \sinh 2.76/70^\circ \\
 \text{by Table, } \sinh 2.80/70^\circ = 1.2136/143^\circ.005. \\
 \text{by Table, } \sinh 2.70/70^\circ = 1.2031/136^\circ.489. \\
 \hline
 \text{Difference } 0.10/70^\circ = 0.0105 \quad /6^\circ.516. \\
 \text{Proportion for } 0.06/70^\circ = 0.0063 \quad /3^\circ.910. \\
 \sinh 2.70/70^\circ = 1.2031/136^\circ.489. \\
 \hline
 \text{Result } \sinh 2.76/70^\circ = 1.2094/140^\circ.399. \\
 \hline
 \text{The true value is} \quad \underline{\underline{1.2086/140^\circ.366.}}
 \end{array}$$

### INTERPOLATION BY THE USE OF TAYLOR'S THEOREM

When more precise interpolation is required than that by simple intermediate proportion, we may use Taylor's theorem in the following form; since

$$\frac{d(\sinh \theta)}{d\theta} = \cosh \theta, \quad \frac{d^2(\sinh \theta)}{d\theta^2} = \sinh \theta, \quad \text{etc.}$$

$$\sinh(\theta + \Delta\theta) = \sinh \theta + \Delta\theta \cosh \theta + \frac{(\Delta\theta)^2}{2!} \sinh \theta + \frac{(\Delta\theta)^3}{3!} \cosh \theta + \dots \quad (32)$$

Let  $\theta = \rho/\underline{\delta}$  and  $\Delta\theta = \Delta\rho/\underline{\delta}$ .

Then

$$\sinh\{(\rho + \Delta\rho)/\underline{\delta}\} = \sinh(\rho/\underline{\delta}) + \Delta\rho/\underline{\delta} \cdot \cosh(\rho/\underline{\delta}) + \frac{(\Delta\rho)^2}{2!} / \underline{\delta} \cdot \sinh(\rho/\underline{\delta}) + \dots \quad (33)$$

The number of correction terms to be retained depends on the interval, and on the degree of precision desired. It is seldom that more than two correction terms have to be retained. Thus in the case already considered:—

$\sinh(2.76/70^\circ) = \sinh(2.7/70^\circ) + 0.06/70^\circ \cdot \cosh(2.7/70^\circ) + 0.0018/140^\circ \cdot \sinh(2.7/70^\circ)$ .  
By Table II, page 13,  $\cosh(2.7/70^\circ) = 1.3422/153^\circ.322$ . Consequently dealing first with the first correction term only:

$$\begin{aligned}
 \sinh(2.76/70^\circ) &= 1.2031/136^\circ.489 + 0.06/70^\circ \times 1.3422/153^\circ.322 \\
 &= 1.2031/136^\circ.489 + 0.08053/223^\circ.322 \\
 &= 1.2031/136^\circ.489 (1 + 0.066937/86^\circ.833) \\
 &= 1.2031/136^\circ.489 (1 + 0.003698 + i 0.06684) \\
 &= 1.2031/136^\circ.489 (1.00592/3^\circ.810) \\
 &= 1.2102/140^\circ.299.
 \end{aligned}$$

## EXPLANATORY TEXT

Taking next the second correction term into account.

$$\begin{aligned}
 \sinh (2.76 / 70^{\circ}) &= 1.2102 / 140^{\circ}.299 + 0.0018 / 140^{\circ} \times 1.2031 / 136^{\circ}.489 \\
 &= 1.2102 / 140^{\circ}.299 + 0.002166 / 276^{\circ}.489 \\
 &= 1.2102 / 140^{\circ}.299 (1 + 0.001789 / 136^{\circ}.190) \\
 &= 1.2102 / 140^{\circ}.299 (1 - 0.00129 + i 0.00124) \\
 &= 1.2102 / 140^{\circ}.299 (0.99871 + i 0.00124) \\
 &= 1.2102 / 140^{\circ}.299 (0.99871 / 0^{\circ}.067) \\
 &= 1.2086 / 140^{\circ}.366.
 \end{aligned} \tag{34}$$

The correct result is 1.2086 / 140^{\circ}.366.

## SECOND AND GENERAL CASE. INTERPOLATION BOTH IN MODULUS AND ARGUMENT

Let the entered quantity be  $\sinh (1.025 / 80.75^{\circ})$ .

We have from Table I the four nearest results as follows:—

$$\begin{array}{ll}
 \sinh 1.0 / 80^{\circ} = 0.85125 / 83^{\circ}.489. & \sinh 1.1 / 80^{\circ} = 0.90416 / 84^{\circ}.286. \\
 \sinh 1.0 / 81^{\circ} = 0.84940 / 84^{\circ}.156. & \sinh 1.1 / 81^{\circ} = 0.90172 / 84^{\circ}.877.
 \end{array}$$

$$\begin{array}{ll}
 \text{Difference for } 1^{\circ} &= -0.00185 + 0^{\circ}.667. & \text{Diff. for } 1^{\circ} &= -0.00244 + 0^{\circ}.591. \\
 \text{Proportion for } 0.75^{\circ} &= -0.001388 + 0.500. & & -0.00183 + 0.443. \\
 \sinh 1.0 / 80^{\circ}.75 &= 0.84986 / 83^{\circ}.989. & \sinh 1.1 / 80^{\circ}.75 &= 0.90233 / 84^{\circ}.729. \\
 \sinh 1.1 / 80.75 &= 0.90233 / 84^{\circ}.729. & & \\
 \text{Difference for } 0.1 &= +0.05247 / 0^{\circ}.740. & & \\
 \text{Proportion for } 0.025 &= +0.01312 / 0^{\circ}.185. & & \\
 \sinh 1.025 / 80^{\circ}.75 &= 0.86298 / 84^{\circ}.174. & & \\
 \text{The true value} &= 0.86372 / 84^{\circ}.166. & &
 \end{array} \tag{35}$$

## DUAL INTERPOLATION BY THE USE OF TAYLOR'S THEOREM

Let the nearest tabular function be  $\sinh \theta = \sinh (\rho / \delta)$  and the required function  $\sinh (\theta + \Delta\theta) = \sinh \{(\rho + \Delta\rho) / \delta + \Delta\delta\}$ .

$$\text{Then } (\rho + \Delta\rho) / \delta + \Delta\delta = \rho / \delta + (\Delta\rho + i\rho\Delta\delta). \tag{36}$$

where the increment  $\Delta\rho + i\rho\Delta\delta$  is taken with reference to the vector axis  $\rho / \delta$ . Referring this increment to the initial axis of reference,

$$\theta + \Delta\theta = (\rho + \Delta\rho) / \delta + \Delta\delta = \rho / \delta + \sqrt{(\Delta\rho)^2 + (\rho\Delta\delta)^2} / \delta + \tan^{-1} \left( \frac{\rho\Delta\delta}{\Delta\rho} \right). \tag{37}$$

$$\text{So that } \Delta\theta = \sqrt{(\Delta\rho)^2 + (\rho\Delta\delta)^2} / \delta + \tan^{-1} (\rho\Delta\delta / \Delta\rho). \tag{38}$$

When, however,  $\Delta\delta$  is not very small, the last formula may contain an appreciable error, and the following method of deducing  $\Delta\theta$ , using rectangular complex quantities, is to be preferred.

# EXPLANATORY TEXT

$$\text{Let } \theta = \rho/\delta = x + iy.$$

$$\text{and } \theta + \Delta\theta = (\rho + \Delta\rho)/\delta + \Delta\delta = x + \Delta x + i(y + \Delta y). \quad (39)$$

$$\text{Then } \Delta\theta = \Delta x + i\Delta y \quad (40)$$

$$= \sqrt{(\Delta x)^2 + (\Delta y)^2} \tan^{-1} (\Delta y/\Delta x). \quad (41)$$

We then have by Taylor's theorem, as before,

$$\sinh(\theta + \Delta\theta) = \sinh \theta + \Delta\theta \cdot \cosh \theta + \frac{(\Delta\theta)^2}{2} \cdot \sinh \theta + \frac{(\Delta\theta)^3}{3 \cdot 2} \cdot \cosh \theta + \dots \quad (42)$$

a series in which two correcting terms only need ordinarily be retained. Thus, in the example last considered,  $\theta = 1.0/80^\circ$  and  $\theta + \Delta\theta = 1.025/80^\circ.75$ . If we form  $\Delta\theta$  by the use of (37), we have  $\Delta\rho = 0.025$ ,  $\Delta\delta = 0^\circ.75 = 0.01309$  radian,  $\rho\Delta\delta = 0.01309$ .

$$\begin{aligned} \Delta\theta &= \sqrt{(0.025)^2 + (0.01309)^2} / 80^\circ + \tan^{-1} (0.01309/0.025) \\ &= 0.02822 / 80^\circ + 27^\circ.637 \\ &= 0.02822 / 107^\circ.637. \end{aligned}$$

If we form  $\Delta\theta$  by the use of the rigid formula (41)

$$\theta + \Delta\theta = 1.025/80^\circ.75 = 0.164761 + i 1.0116715.$$

$$\theta = 1.0/80^\circ = 0.173648 + i 0.9848078.$$

$$\begin{aligned} \Delta\theta &= -0.008887 + i 0.0268637 \\ &= 0.028295 / 108^\circ.306. \end{aligned}$$

Entering now the correction formula (42), we find in the tables:

$$\sinh 1.0/80^\circ = 0.85125/83^\circ.489, \quad \cosh 1.0/80^\circ = 0.57991/14^\circ.521.$$

so that

$$\begin{aligned} \sinh 1.025/80^\circ.75 &= \sinh 1.0/80^\circ + 0.028295 / 108^\circ.306 \times \cosh 1.0/80^\circ \\ &\quad + \frac{(0.028295)^2}{2} / 216^\circ.612 \times \sinh 1.0/80^\circ \\ &\quad + \frac{(0.028295)^3}{6} / 324^\circ.918 \times \cosh 1.0/80^\circ \\ &= 0.85125/83^\circ.489 + 0.028295 / 108^\circ.306 \times 0.57991/14^\circ.521 \\ &\quad + 0.0004003 / 216^\circ.612 \times 0.85125/83^\circ.489 \\ &\quad + 0.000001 / 324^\circ.918 \times 0.57991/14^\circ.521. \end{aligned}$$

It is evident that in conformity with the precision of the tables, only the first two correction terms need be included. Taking the first into account, we have:

$$\begin{aligned} \sinh 1.025/80^\circ.75 &= 0.85125/83^\circ.489 + 0.028295/108^\circ.306 \times 0.57991/14^\circ.521 \\ &= 0.85125/83^\circ.489 + 0.016408/122^\circ.827 \\ &= 0.85125/83^\circ.489 (1 + 0.019276/39^\circ.338) \\ &= 0.85125/83^\circ.489 (1 + 0.014909 + i 0.012219) \\ &= 0.85125/83^\circ.489 (1.014909 + i 0.012219) \\ &= 0.85125/83^\circ.489 \times 1.01498/0^\circ.690 \\ &= 0.86400/84^\circ.179. \end{aligned}$$

## EXPLANATORY TEXT

Taking up the second correction term:—

$$\begin{aligned}
 \sinh 1.025/80^\circ.75 &= 0.86400/84^\circ.179 + 0.0004003/216^\circ.612 \times 0.85125/83^\circ.489 \\
 &= 0.86400/84^\circ.179 + 0.0003405/300^\circ.101 \\
 &= 0.86400/84^\circ.179 (1 + 0.000395/215^\circ.922) \\
 &= 0.86400/84^\circ.179 (1 - 0.00032 - i0.000232) \\
 &= 0.86400/84^\circ.179 (0.99968 - i0.000232) \\
 &= 0.86400/84^\circ.179 \times 0.99968 \sqrt{0^\circ.013} \\
 &= \underline{0.863727/84^\circ.166.}
 \end{aligned}$$

The true value is 0.86372 /84°.166.

## CONCLUSIONS

In general, dual interpolation by simple proportion, as in (35), will give a result of the third order of precision. In order to secure precision of the fourth order, interpolation by the use of Taylor's theorem as in (42) may be required.

### EXTENSION OF TABLE BY USE OF FORMULA FOR $2\theta$

Although Table I is only carried as far as 3.0 in modulus ( $\rho = 3$ ); yet it may be used with a little additional calculation in conjunction with Table II, for obtaining the hyp. sines of plane vector quantities of moduli up to 6.0, by means of the formula:

$$\sinh 2\theta = 2 \sinh \theta \cosh \theta \quad (43)$$

Example: Required  $\sinh 5.0/77^\circ$ , a quantity outside of Table I. Here  $\theta = 2.5/77^\circ$  is within the limits of the Table; so that

$$\begin{aligned}
 \sinh 5.0/77^\circ &= 2 \times \sinh 2.5/77^\circ \times \cosh 2.5/77^\circ \quad (44) \\
 &= 2 \times 0.87843/120^\circ.891 \times 0.96459/156^\circ.524 \\
 &= 2 \times 0.87843 \times 0.96459/277^\circ.415 \\
 &= 1.75686 \times 0.96459/277^\circ.415 \\
 &= \underline{1.69465/277^\circ.415.}
 \end{aligned}$$

This method ordinarily calls for interpolation both in  $\sinh \theta$  and  $\cosh \theta$ . For this reason, it may be preferable to obtain the required result by the use of either Table VII or Table X, the limits of which are less restricted.

## TABLE II

$$\cosh (\rho/\delta) = r/\gamma$$

### POLAR HYPERBOLIC COSINES OF A POLAR VARIABLE

Table II gives the value of  $\cosh \rho/\delta$  between the limits of  $\rho = 0$  and  $\rho = 3.0$  by steps of 0.1, and the limits  $\delta = 45^\circ$  and  $\delta = 90^\circ$ , by steps of  $1^\circ$ . The graphs of these quantities, to squared polar coördinates, appear in Plates IIA and IIB of the Atlas.

## EXPLANATORY TEXT

### INTERPOLATION BY SIMPLE PROPORTION

In general, as in the case of Table I, a very fair degree of precision in interpolation can be obtained by taking first simple proportional parts in argument, and then simple proportional parts in modulus.

Example: Required  $\cosh (0.93105 / 57^{\circ}.518)$ .

We have from Table II: —

|                                                          |                                                              |
|----------------------------------------------------------|--------------------------------------------------------------|
| $\cosh 0.9 / 57^{\circ} = 0.88922 / 23^{\circ}.140.$     | $\cosh 1.0 / 57^{\circ} = 0.87976 / 28^{\circ}.917.$         |
| $\cosh 0.9 / 58^{\circ} = 0.87602 / 23^{\circ}.003.$     | $\cosh 1.0 / 58^{\circ} = 0.86350 / 28^{\circ}.823.$         |
| Difference for $1^{\circ} = -0.01320 / 0^{\circ}.137.$   | Difference for $1^{\circ} = -0.01626 / 0^{\circ}.094.$       |
| Diff. for $0.518^{\circ} = -0.00685 / 0^{\circ}.071.$    | Diff. for $0.518^{\circ} = -0.00844 / 0^{\circ}.049.$        |
| $\cosh 0.9 / 57^{\circ}.518 = 0.88237 / 23^{\circ}.069.$ | $\cosh 1.0 / 57^{\circ}.518 = 0.87132 / 28^{\circ}.868.$     |
|                                                          | $\cosh 0.9 / 57^{\circ}.518 = 0.88237 / 23^{\circ}.069.$     |
|                                                          | Difference for $0.1 = -0.01105 / 5^{\circ}.799.$             |
|                                                          | Diff. for $0.3105 = -0.00343 / 1^{\circ}.800.$               |
|                                                          | $\cosh 0.93105 / 57^{\circ}.518 = 0.87894 / 24^{\circ}.869.$ |
|                                                          | The correct value is <u><u>0.87837 / 24.803.</u></u>         |

### INTERPOLATION OF TAYLOR'S THEOREM

When a higher degree of precision is required than can be expected from simple proportional parts, we may use Taylor's Theorem in the following form: —

$$\cosh (\theta + \Delta\theta) = \cosh \theta + \Delta\theta \sinh \theta + \frac{(\Delta\theta)^2}{2!} \cosh \theta + \frac{(\Delta\theta)^3}{3!} \sinh \theta + \dots \quad (45)$$

Example: Required  $\cosh 0.93105 / 57^{\circ}.518$

having given in Table II  $\cosh 0.9 / 57^{\circ} = 0.88922 / 23^{\circ}.140$

and in Table I  $\sinh 0.9 / 57^{\circ} = 0.85414 / 64^{\circ}.218.$

$$0.93105 / 57^{\circ}.518 = 0.500 + i 0.785398.$$

$$0.900 / 57^{\circ} = 0.49018 + i 0.754804.$$

$$\Delta\theta = 0.00982 + i 0.030594$$

$$= 0.03214 / 72^{\circ}.196.$$

$$\begin{aligned} \cosh 0.93105 / 57^{\circ}.518 &= \cosh 0.9 / 57^{\circ} + 0.03214 / 72^{\circ}.196 \times 0.85414 / 64^{\circ}.218 \\ &+ 0.00052 / 144^{\circ}.392 \times 0.88922 / 23^{\circ}.140. \end{aligned}$$

## EXPLANATORY TEXT

It is evident that for the Tables here considered only two correction terms need be included. Taking up the first correction term,

$$\begin{aligned}
 \cosh 0.93105 / 57^{\circ}.518 &= 0.88922 / 23^{\circ}.140 + 0.03214 / 72^{\circ}.196 \times 0.85414 / 64^{\circ}.218 \\
 &= 0.88922 / 23^{\circ}.140 + 0.02745 / 136^{\circ}.414 \\
 &= 0.88922 / 23^{\circ}.140 (1 + 0.03087 / 113^{\circ}.274) \\
 &= 0.88922 / 23^{\circ}.140 (1 - 0.01220 + i 0.02835) \\
 &= 0.88922 / 23^{\circ}.140 (0.98780 + i 0.02835) \\
 &= 0.88922 / 23^{\circ}.140 \times 0.98780 (1 + i 0.02870) \\
 &= 0.88922 / 23^{\circ}.140 \times 0.98780 \times 1.00041 / 1^{\circ}.645 \\
 &= 0.87873 / 24^{\circ}.785.
 \end{aligned}$$

Taking up the second correction term: —

$$\begin{aligned}
 \cosh 0.93105 / 57^{\circ}.518 &= 0.87873 / 24^{\circ}.785 + 0.00052 / 144^{\circ}.392 \times 0.88922 / 23^{\circ}.140 \\
 &= 0.87873 / 24^{\circ}.785 + 0.00046 / 167^{\circ}.532 \\
 &= 0.87873 / 24^{\circ}.785 (1 + 0.000524 / 142^{\circ}.747) \\
 &= 0.87873 / 24^{\circ}.785 (1 - 0.000416 + i 0.000317) \\
 &= 0.87873 / 24^{\circ}.785 (0.999584 + i 0.000317) \\
 &= 0.87873 \times 0.999584 / 24^{\circ}.785 (1 + i 0.00032) \\
 &= 0.87837 / 24^{\circ}.785^{\circ} \times 1 / 0.018^{\circ} \\
 &= \underline{0.87837 / 24^{\circ}.803.}
 \end{aligned}$$

The correct value is 0.87837 / 24^{\circ}.803.

## GRAPHICAL INTERPOLATION

For rapid but less precise work, interpolation may be made by proportional parts on Plate IIA or Plate IIB, without arithmetical computation.

### TABLE III

$$\tanh (\rho / \delta) = r / \gamma$$

#### POLAR HYPERBOLIC TANGENTS OF A POLAR VARIABLE

Table III gives in polar coördinates the value of  $\tanh \rho / \delta$  between the limits  $\rho = 0$  and  $\rho = 3.0$  by steps of 0.1, and the limits  $\delta = 45^{\circ}$  and  $\delta = 90^{\circ}$ , by steps of  $1^{\circ}$ . The graphs of these quantities, to squared polar coördinates, appear in Plates IIIA and IIIB of the Atlas.

#### INTERPOLATION BY SIMPLE PROPORTION

In general, as in the cases of Tables I and II, a very fair degree of precision can be obtained by taking first simple proportional parts in argument and then simple proportional parts in modulus.

Example: Required  $\tanh (0.93105 / 57^{\circ}.518)$ .

[ 181 ]

## EXPLANATORY TEXT

We have from Table III: —

$$\tanh 0.9/57^\circ = 0.96056/41^\circ.078.$$

$$\tanh 0.9/58^\circ = 0.97069/42^\circ.111.$$

$$\text{Difference for } 1^\circ = 0.01013/1^\circ.033.$$

$$\text{Diff. for } 0.518^\circ = 0.00525/0^\circ.535.$$

$$\tanh 0.9/57^\circ.518 = 0.96581/41^\circ.613.$$

$$\tanh 1.0/57^\circ = 1.06648/37^\circ.035.$$

$$\tanh 1.0/58^\circ = 1.08054/38^\circ.004.$$

$$\text{Difference for } 1^\circ = 0.01406/0^\circ.969.$$

$$\text{Diff. for } 0.518^\circ = 0.00728/0^\circ.502.$$

$$\tanh 1.0/57^\circ.518 = 1.07376/37^\circ.537.$$

$$\tanh 0.9/57^\circ.518 = 0.96581/41^\circ.613.$$

$$\text{Difference for } 0.1 = 0.10795/-4^\circ.076.$$

$$\text{" for } 0.3105 = 0.03352/-1^\circ.266.$$

$$\text{Inferred value of } \tanh 0.93105/57^\circ.518 = 0.99933/40^\circ.347.$$

$$\text{Correct value of } \tanh 0.93105/57^\circ.518 = 1.0000/40^\circ.395.$$

## INTERPOLATION BY TAYLOR'S THEOREM

For a higher degree of interpolation precision than by simple proportion, we may use Taylor's theorem in the following form: —

$$\begin{aligned} \tanh(\theta + \Delta\theta) &= \tanh\theta + \Delta\theta \operatorname{sech}^2\theta - \frac{(\Delta\theta)^2}{2!} \cdot 2 \operatorname{sech}^2\theta \tanh\theta \\ &\quad - \frac{(\Delta\theta)^3}{3!} \cdot 2 \operatorname{sech}^2\theta (\operatorname{sech}^2\theta - 2 \tanh^2\theta) + \dots \end{aligned} \quad (46)$$

Example: Required  $\tanh 0.93105/57^\circ.518$ .

having given in Table I  $\sinh 0.9/57^\circ = 0.85414/64.218$ .

II  $\cosh 0.9/57^\circ = 0.88922/23^\circ.140$ .

III  $\tanh 0.9/57^\circ = 0.96056/41^\circ.078$ .

Here  $\Delta\theta = 0.03214/72^\circ.196$ , as given by (41). Hence by Taylor's theorem as far as the second correction term inclusive,

$$\begin{aligned} \tanh 0.93105/57^\circ.518 &= \tanh 0.9/57^\circ + \frac{0.03214/72.196}{(0.88922)^2/46.280} \\ &\quad - \frac{(0.03214)^2/144^\circ.392}{(0.88922)^2/46^\circ.280} \times 0.96056/41^\circ.078. \end{aligned}$$

Taking up the first correction term: —

$$\begin{aligned} \tanh 0.93105/57^\circ.518 &= 0.96056/41^\circ.078 + \frac{0.03214}{0.79071} /25^\circ.916 \\ &= 0.96056/41^\circ.078 + 0.04065/25^\circ.916 \\ &= 0.96056/41^\circ.078 (1 + 0.04232\sqrt{15^\circ.162}) \\ &= 0.96056/41^\circ.078 (1 + 0.04084 - i0.01107) \\ &= 0.96056/41^\circ.078 (1.04084 - i0.01107) \\ &= 0.96056/41.078 (1.04090\sqrt{0^\circ.609}) \\ &= 0.99985/40^\circ.469. \end{aligned}$$

## EXPLANATORY TEXT

Taking up next the second correction term: —

$$\begin{aligned}
 \tanh 0.93105 / 57^{\circ}.518 &= 0.99985 / 40^{\circ}.469 - \frac{0.00103 / 144^{\circ}.392 \times 0.96056 / 41^{\circ}.078}{(0.88922)^2 / 46^{\circ}.280} \\
 &= 0.99985 / 40^{\circ}.469 + \frac{0.00103 \times 0.96056 \sqrt{40^{\circ}.810}}{0.79071} \\
 &= 0.99985 / 40^{\circ}.469 + 0.00126 \sqrt{40^{\circ}.810} \\
 &= 0.99985 / 40^{\circ}.469 (1 + 0.00126 \sqrt{81.279}) \\
 &= 0.99985 / 40^{\circ}.469 (1 + 0.00019 - i 0.00125) \\
 &= 0.99985 / 40^{\circ}.469 (1.00019 - i 0.00125) \\
 &= 0.99985 / 40^{\circ}.469 \times 1.00019 \sqrt{0^{\circ}.072} \\
 &= \frac{1.0000}{\dots\dots\dots} / 40^{\circ}.397. \\
 \text{Correct value} &= \frac{1.0000}{\dots\dots\dots} / 40^{\circ}.395.
 \end{aligned}$$

When more than two correction terms have to be retained, it is often easier to determine  $\sinh (\theta + \Delta\theta)$  and  $\cosh (\theta + \Delta\theta)$  by Taylor's theorem, as already described, and then to take their ratio for  $\tanh (\theta + \Delta\theta)$ .

### TABLE IV

POLAR RATIO  $\frac{\sinh \theta}{\theta}$  FOR POLAR VALUES OF  $\theta$

Table IV has been prepared by dividing the values of  $\sinh \theta$  found successively in Table I by their respective values of  $\theta$ . The object of the table is to facilitate the computation of the equivalent T or II of any uniform alternating-current line of known electrical constants.\* That is, the table pertains more particularly to the applications of hyperbolic functions than to the fundamental properties of those functions. The table gives the vector value of  $\frac{\sinh (\rho / \delta)}{\rho / \delta}$  for the range  $\rho = 0$  to  $\rho = 3$  by steps of 0.1, and for  $\delta = 45^{\circ}$  to  $\delta = 90^{\circ}$  by steps of  $1^{\circ}$ . The graphs of the values contained in the tables are plotted to squared polar coordinates in Charts IVA and IVB of the Atlas, for rapid graphic interpolation.

### INTERPOLATION BY SIMPLE PROPORTION

A fair degree of precision in interpolation can ordinarily be obtained by first taking simple proportional parts in argument and then simple proportional parts in modulus.

Example: Required  $\frac{\sinh (1.025 / 80^{\circ}.75)}{1.025 / 80^{\circ}.75}$ .

\*"The Application of Hyperbolic Functions to Electrical Engineering Problems," by A. E. Kennelly, University of London Press, 1914, Chap. III.



# EXPLANATORY TEXT

We have from Table IV the following values of  $\frac{\sinh \theta}{\theta}$ :—

|                                     |                  |                                     |                                   |
|-------------------------------------|------------------|-------------------------------------|-----------------------------------|
| For $1.0/80^\circ = 0.85125$        | $/3^\circ.489.$  | For $1.1/80^\circ = 0.82196$        | $/4^\circ.286.$                   |
| $1.0/81^\circ = 0.84940$            | $/3^\circ.156.$  | $1.1/81^\circ = 0.81975$            | $/3^\circ.877.$                   |
| Difference for $1^\circ = -0.00185$ | $/-0^\circ.333.$ | Difference for $1^\circ = -0.00221$ | $/-0^\circ.409.$                  |
| $0.75^\circ = -0.00139$             | $/-0^\circ.250.$ | $0^\circ.75 = -0.00166$             | $/-0^\circ.307.$                  |
| For $1.0/80^\circ.75 = 0.84986$     | $/3^\circ.239.$  | For $1.1/80^\circ.75 = 0.82030$     | $/3^\circ.979.$                   |
|                                     |                  | For $1.0/80^\circ.75 = 0.84986$     | $/3^\circ.239.$                   |
|                                     |                  | Difference for $0.1 = -0.02956$     | $/0^\circ.740.$                   |
|                                     |                  | " for $0.025 = -0.00739$            | $/0^\circ.185.$                   |
|                                     |                  | For $1.025/80^\circ.75 = 0.84247$   | $/3^\circ.424.$                   |
|                                     |                  | Correct value = <b>0.84265</b>      | <b><math>/3^\circ.416.</math></b> |

When a higher degree of precision is required than can be expected from proportional parts, the proper value of  $\sinh (\theta + \Delta\theta)$  should be obtained by Taylor's theorem as already explained in connection with Table I, and this value divided by  $(\theta + \Delta\theta)$ ; because the expansion of  $\frac{\sinh (\theta + \Delta\theta)}{(\theta + \Delta\theta)}$  directly, by Taylor's theorem, does not lend itself conveniently for computation.

## EXTENSION FOR THE RANGE OF THE TABLE BY THE USE OF FORMULA FOR $2\theta$

Although Table IV is only carried as far as 3.0 in modulus ( $\rho = 3$ ); yet it may be used with a little additional calculation, in conjunction with Table II, for obtaining  $\frac{\sinh \theta}{\theta}$ , for vector values of  $\theta$  with moduli up to 6.0, by means of the formula:—

$$\sinh 2\theta = 2 \sinh \theta. \cosh \theta \quad (47)$$

whence

$$\frac{\sinh 2\theta}{2\theta} = \frac{\sinh \theta}{\theta} \cdot \cosh \theta. \quad (48)$$

Consequently, to find  $\frac{\sinh \theta}{\theta}$  for the double of any quantity within the range of Table IV, find the value of  $\frac{\sinh \theta}{\theta}$  for the quantity, by interpolation directly in Table IV, and multiply the result by the hyperbolic cosine of the quantity as obtained from Table II. Corresponding steps may be taken with Charts II and IV.

Example: Required  $\frac{\sinh (5.0/77^\circ)}{5.0/77^\circ}$ , this being outside of the limits of Table IV; but not outside twice the value therein obtainable.

Here  $\frac{\sinh \theta}{\theta}$  for  $\theta = 2.5/77^\circ$  is by Table IV  $0.35137 /43^\circ.891.$   
and  $\cosh \theta$  " " " " " " II  $0.96459 /156^\circ.524.$

## EXPLANATORY TEXT

$$\begin{aligned}\text{Hence } \frac{\sinh (5.0 / 77^{\circ})}{5.0 / 77^{\circ}} &= 0.35137 / 43^{\circ}.891 \times 0.96459 / 156^{\circ}.524. \\ &= \underline{0.33893 / 200^{\circ}.415.}\end{aligned}$$

This procedure calls for interpolation both in  $\frac{\sin \theta}{\theta}$  and in  $\cosh \theta$ . For this reason it may be preferable to obtain the required result by the use of either Table VII or Table X, the limits of which are less restricted.

### TABLE V

POLAR RATIO  $\frac{\tanh \theta}{\theta}$  FOR POLAR VALUES OF  $\theta$

Table V, like Table IV, has been prepared for electrical engineering applications of hyperbolic functions, rather than for developing these functions alone. It gives the vector value of  $\frac{\tanh (\rho / \delta)}{\rho / \delta}$  for the range  $\rho = 0$  to  $\rho = 3.0$  in modulus, by steps of  $0.1$ , and for the range  $\delta = 45^{\circ}$  to  $\delta = 90^{\circ}$  in argument, by steps of  $1^{\circ}$ . It was computed directly from Table III by dividing the resulting values successively by their respective values of  $\theta$ . The graphs of the values in Table V are presented to squared polar coordinates in Chart V, for rapid graphic interpolation.

### INTERPOLATION BY SIMPLE PROPORTION

Except where a high degree of precision in interpolation is required, it is preferable to interpolate first by simple proportion in argument, and then by simple proportion in modulus; although this order of operations may be inverted.

Example: Required  $\frac{\tanh \theta}{\theta}$  for  $\theta = 0.93105 / 57^{\circ}.518$ .

We have from Table V:—

$$\begin{aligned}\text{For } \theta &= 0.9 / 57^{\circ} = 1.06729 \sqrt{15^{\circ}.922.} \\ \theta &= 0.9 / 58^{\circ} = 1.07854 \sqrt{15^{\circ}.889.}\end{aligned}$$

$$\begin{aligned}\text{Difference for } 1^{\circ} &= 0.01125 \sqrt{-0.033.} \\ \text{" for } 0.518^{\circ} &= 0.00583 \sqrt{-0.017.}\end{aligned}$$

$$\text{For } \theta = 0.9 / 57^{\circ}.518 = 1.07312 \sqrt{15^{\circ}.905.}$$

$$\begin{aligned}\text{For } \theta &= 1.0 / 57^{\circ} = 1.06648 \sqrt{19^{\circ}.965.} \\ \theta &= 1.0 / 58^{\circ} = 1.08054 \sqrt{19^{\circ}.996.}\end{aligned}$$

$$\begin{aligned}\text{Difference for } 1^{\circ} &= 0.01406 \sqrt{0^{\circ}.031.} \\ \text{" for } 0^{\circ}.518 &= 0.00728 \sqrt{0^{\circ}.016.}\end{aligned}$$

$$\begin{aligned}\text{For } \theta &= 1.0 / 57^{\circ}.518 = 1.07376 \sqrt{19^{\circ}.981.} \\ \theta &= 0.9 / 57^{\circ}.518 = 1.07312 \sqrt{15^{\circ}.905.}\end{aligned}$$

$$\begin{aligned}\text{Difference for } 0.1 &= 0.00064 \sqrt{4^{\circ}.076.} \\ \text{" for } 0.03105 &= 0.00020 \sqrt{1^{\circ}.266.}\end{aligned}$$

$$\begin{aligned}\text{For } \theta &= 0.93105 / 57^{\circ}.518 = 1.07332 \sqrt{17^{\circ}.171.} \\ \text{Correct value,} & \quad \underline{1.07406 \sqrt{17^{\circ}.123.}}\end{aligned}$$

[ 185 ]

## EXPLANATORY TEXT

When a higher degree of precision is needed than simple proportion can give, it is preferable to find the proper interpolated value for  $\tanh \theta$  from preceding tables and then to divide by  $\theta$ ; since the function  $\frac{\tanh (\theta + \Delta \theta)}{(\theta + \Delta \theta)}$  does not lend itself to expansion by Taylor's theorem in a simple form.

Tables IV and V jointly, with their respective graphs in the Atlas, enable the equivalent T or II of any uniform alternating-current line in the steady state, at a single frequency, to be completely determined, provided  $\theta$  does not exceed six radians in modulus ( $\delta$  lying between  $45^\circ$  and  $90^\circ$ ); because although in both tables,  $\theta$  is not carried beyond three radians; yet  $\frac{\sinh \theta}{\theta}$  can be found by extension up to six radians, and in the formul-

as for deducing the equivalent T or II,  $\frac{\tanh \theta}{\theta}$  has only to be carried to half the modulus of  $\frac{\sinh \theta}{\theta}$ .

The following example may illustrate the use of Tables IV and V either with or without the aid of the graphic interpolation Charts IV and V of the Atlas. An alternating-current line of uniform electrical constants is 250 km. long and has, at a certain frequency, a total conductor impedance of  $565.711 / 84^\circ.777$  ohms, associated with a total distributed insulation admittance of  $4.3707 \times 10^{-3} / 90^\circ$  mhos. Its hyperbolic angle is therefore  $\sqrt{5.65711 \times 4.3707 \times 10^{-1} / 174^\circ.777} = 1.5724 / 87^\circ.388$  hyperbolic radians. Interpolating either from the tables or the Charts IV and V, we obtain

$$\frac{\sinh \theta}{\theta} = 0.638 / 2^\circ.6 \quad \text{and} \quad \frac{\tanh (\frac{\theta}{2})}{(\frac{\theta}{2})} = \frac{\tanh 0.7862 / 87^\circ.388}{0.7862 / 87^\circ.388} = 1.27 \sqrt{1^\circ.5}.$$

If we multiply the conductor impedance by  $\frac{\sinh \theta}{\theta}$ , we have

$565.711 / 84^\circ.777 \times 0.638 / 2^\circ.6 = 360.69 / 87^\circ.377$  ohms, and if we multiply half the

insulation admittance by  $\frac{\tanh (\frac{\theta}{2})}{(\frac{\theta}{2})}$ , we have

$$2.1854 \times 10^{-3} / 90^\circ \times 1.27 \sqrt{1^\circ.5} = 2.78 \times 10^{-3} / 88^\circ.5 \text{ mhos} = 359.77 \sqrt{88^\circ.5} \text{ ohms.}$$

If now we apply an artificial condenser leak of  $2.78 \times 10^{-3} / 88^\circ.5$  mhos to each end of a localised impedance coil of  $360.69 / 87^\circ.377$  ohms, we obtain the "equivalent II" of the line at the frequency considered, and such a combination of localised impedance and admittances would behave exactly like the line, at its terminals, or outside them, so as to be capable of replacing the line in any electrical system, at that frequency.

# EXPLANATORY TEXT

## TABLE VI

### POLAR FUNCTIONS OF POLAR SEMI-IMAGINARY QUANTITIES

A semi-imaginary quantity is a complex numerical quantity which, when expressed in rectangular coördinates, has equal real and imaginary components; or, when expressed in polar coördinates, has an argument of  $45^\circ$ . That is  $x/\pm 45^\circ = a \pm ia$ . The interest of the table pertains primarily to the application of hyperbolic functions to uniform alternating-current lines of negligibly small linear inductance and leakance, a case approximated to by cabled lines at low frequencies. The table was first published by the author in the transactions of the International Electrical Congress of St. Louis (1904). The arguments of the results are given in degrees and minutes, and not in degrees and decimals like the rest of the tables.

The table gives the hyperbolic sine, cosine, tangent, cosecant, secant, and cotangent of the vector  $x/45^\circ$  for the range  $x = 0$  to  $x = 20.5$ , by steps in  $x$  of  $0.1$  up to  $x = 6$ , and of  $0.05$  beyond that point. At  $x = 6$ , the values of the hyp. sine and cosine so nearly coincide, that they are taken as equal in the table, thus bringing  $\text{sech } x$  and  $\text{cosech } x$  into equality as well as  $\tanh x = \coth x = 1$ . Graphs of the functions are given in Chart VI as far as  $x = 4$ , approximately.

### INTERPOLATION BY SIMPLE PROPORTION

In general, interpolation may be quickly effected by simple proportional parts of modulus since the argument is constant at  $45^\circ$ . This procedure is sufficiently evident to require no exemplification.

### INTERPOLATION OF TAYLOR'S THEOREM

When precise interpolation is necessary, we have the following expansions for  $f(\theta + \Delta\theta)$

$$\sinh \left\{ (x + \Delta x)/45^\circ \right\} = \sinh (x/45^\circ) + (\Delta x)/45^\circ \cdot \cosh (x/45^\circ) + \frac{(\Delta x)^2}{2!}/90^\circ \cdot \sinh (x/45^\circ) + \frac{(\Delta x)^3}{3!}/135^\circ \cdot \cosh (x/45^\circ) + \dots \quad (49)$$

$$\cosh \left\{ (x + \Delta x)/45^\circ \right\} = \cosh (x/45^\circ) + (\Delta x)/45^\circ \cdot \sinh (x/45^\circ) + \frac{(\Delta x)^2}{2!}/90^\circ \cdot \cosh (x/45^\circ) + \frac{(\Delta x)^3}{3!}/135^\circ \cdot \sinh (x/45^\circ) + \dots \quad (50)$$

$$\tanh \left\{ (x + \Delta x)/45^\circ \right\} = \tanh (x/45^\circ) + (\Delta x)/45^\circ \cdot \text{sech}^2(x/45^\circ) - \frac{(\Delta x)^2}{2!}/90^\circ \cdot 2 \text{sech}^2(x/45^\circ) \cdot \tanh (x/45^\circ) - \frac{(\Delta x)^3}{3!}/135^\circ \cdot 2 \text{sech}^2(x/45^\circ) \cdot \{ \text{sech}^2(x/45^\circ) - 2 \tanh^2(x/45^\circ) \} + \dots \quad (51)$$

Example: Required  $\cosh (3.1/45^\circ)$ , having given in Table VI

$$\sinh (3.0/45^\circ) = 4.1986/120^\circ.48'$$

$$\cosh (3.0/45^\circ) = 4.1443/122^\circ.16'$$

[187]

# EXPLANATORY TEXT

$$\begin{aligned}
 \text{Here } \cosh (3.1 / 45^{\circ}) &= 4.1443 / 122^{\circ}.16' + 0.1 / 45^{\circ} \times 4.1986 / 120^{\circ}.48' \\
 &+ \frac{0.01 / 90^{\circ}}{2} \times 4.1443 / 122^{\circ}.16' + \frac{0.001}{6} / 135^{\circ} \times 4.1986 / 120^{\circ}.48' \\
 &= 4.1443 / 122^{\circ}.16' + 4.1986 / 120^{\circ}.48 (0.1 / 45^{\circ} + 0.00017 / 135^{\circ}) \\
 &+ 4.1443 / 122^{\circ}.16' (0.005 / 90^{\circ}) \\
 &= 4.1443 / 122^{\circ}.16' (1 + i 0.005) + 4.1986 / 165^{\circ}.48' (0.1 + i 0.00017) \\
 &= 4.1443 / 122^{\circ}.16' (1.0000 / 0^{\circ}.17') + 4.1986 / 165^{\circ}.48' (0.1 / 0^{\circ}.01') \\
 &= 4.1443 / 122^{\circ}.33' + 0.4199 / 165^{\circ}.49' \\
 &= 4.1443 / 122^{\circ}.33' (1 + 0.10132 / 43^{\circ}.16') \\
 &= 4.1443 / 122^{\circ}.33' (1 + 0.07378 + i 0.06944) \\
 &= 4.1443 / 122^{\circ}.33' (1.07378 + i 0.06944) \\
 &= 4.1443 / 122^{\circ}.33' \times 1.0760 / 3^{\circ}.42' \\
 &= 4.4590 / 126^{\circ}.15'.
 \end{aligned}$$

which is in substantial agreement with the tabulated value of  $\cosh 3.1$ .

Beyond  $x = 6$ , the value of either  $\sinh (x / 45^{\circ})$  or  $\cosh (x / 45^{\circ})$  was computed from the formula:—

$$\sinh (x / 45^{\circ}) = \cosh (x / 45^{\circ}) = \frac{\epsilon^{\frac{x}{\sqrt{2}}}}{2} \left/ \frac{x}{\sqrt{2}} \text{ radians.} \right. \quad (52)$$

where  $\epsilon = 2.71828 \dots$

Thus, with  $x = 7$ ,  $\frac{x}{\sqrt{2}} = 4.9498$ ,  $\frac{\epsilon^{\frac{x}{\sqrt{2}}}}{2} = \frac{141.14}{2} = 70.57$  at the argument of 4.9498 circular radians =  $283^{\circ}.36'$ ; so that:—

$$\sinh (7 / 45^{\circ}) = \cosh (7 / 45^{\circ}) = 70.57 / 283^{\circ}.36'$$

which coincides with the tabulated value in Table VI.

## TABLE VII

$$\sinh (x + iy) = u + iv$$

### RECTANGULAR HYPERBOLIC SINES OF A RECTANGULAR VARIABLE

Tables I to VI contain certain restrictions in range which limit their general application. They are primarily designed to cover particular applications of hyperbolic functions to electrical engineering. Tables VII to XIV, however, are free from such restrictions, and are intended to furnish the circular as well as the hyperbolic sine, cosine, and tangent of a complex angle, and to furnish this result either in the rectangular or polar form. That is, they furnish:—

$$\begin{aligned}
 &\left. \begin{array}{l} \sinh (x + iy) \\ \text{or } \sin (x + iy) \end{array} \right\} \text{ in the form } u + iv; \text{ also in the form } r / \gamma \\
 &\left. \begin{array}{l} \cosh (x + iy) \\ \text{or } \cos (x + iy) \end{array} \right\} \text{ in the form } u + iv; \text{ also in the form } r / \gamma \\
 &\left. \begin{array}{l} \tanh (x + iy) \\ \text{or } \tan (x + iy) \end{array} \right\} \text{ in the form } u + iv; \text{ also in the form } r / \gamma
 \end{aligned}$$

## EXPLANATORY TEXT

between the limits, for the hyperbolic functions,  $x = 0$  and  $x = \pm 10$ , by steps of  $0.05$  and between the limits  $y = 0$  and  $y = \pm \infty$ , by steps of  $0.07854 = \pi/40$ .

### PERIODIC PROPERTIES OF THE RECTANGULAR COMPLEX HYPERBOLIC SINES AND COSINES

It is well known that  $\sinh \{x + i(y + 2n\pi)\} = \sinh \{x + iy\}$  (53)

and  $\cosh \{x + i(y + 2n\pi)\} = \cosh \{x + iy\}$  (54)

where  $n$  is any integer.

This means that, keeping  $x$  constant, the values of the hyp. sine and hyp. cosine repeat themselves as  $iy$  passes through increments of  $i.2\pi$ ; or they are periodic functions of  $iy$ , having the period  $2\pi i$ .

The matter may be visualised more clearly from geometrical reasoning. Considering the exponential form of the hyperbolic cosine,

$$\cosh (x + iy) = \frac{\epsilon^{x + iy} + \epsilon^{-(x + iy)}}{2}. \quad (55)$$

This may be written in the form:  $\frac{\epsilon^x}{2} \cdot \epsilon^{iy} + \frac{\epsilon^{-x}}{2} \epsilon^{-iy}$ . If  $x$  be kept constant, we require to study the changes produced in this form of the hyp. cosine by varying  $y$ .

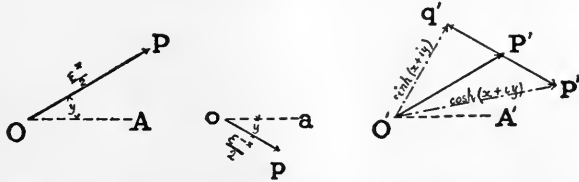


FIG. 29. — Geometrical constructions for  $\cosh (x + iy)$  and  $\sinh (x + iy)$ .

In Fig. 29,  $OA$  is an initial line and  $OP$  a radius vector of length or modulus  $\epsilon^x/2$ , multiplied by  $\epsilon^{iy}$ ; that is rotated positively about  $O$ , from  $OA$  through a circular angle of  $y$  radians. Similarly,  $op$  is a vector of length or modulus  $\epsilon^{-x}/2$  rotated negatively through a circular angle of  $y$  radians from the initial line  $Oa$ . The equation (55) states that the hyperbolic cosine is the plane vector sum of  $OP$  and  $op$ , or  $O'p'$  in the Figure. If now we steadily increase the value of  $y$ , leaving  $x$  constant, we cause  $OP$  to rotate steadily counterclockwise, and also  $op$  to rotate steadily clockwise, through  $\Delta y$  circular radians. When  $\Delta y = 2\pi$ , both  $OP$  and  $op$  will have made one complete revolution and will have returned to their initial positions indicated. Consequently, the value of  $\cosh \{x + i(y + 2\pi)\}$  repeats that of  $\cosh \{x + iy\}$ .

Since 
$$\sinh (x + iy) = \frac{\epsilon^{x + iy} - \epsilon^{-(x + iy)}}{2} \quad (56)$$

the same reasoning applies; but the vector  $op$  is added in the negative or reversed direction; so that  $O'q'$  is the hyperbolic sine of  $x + iy$ .

The above mentioned periodic property of the hyp. sine and cosine has been utilized for shortening the tables of those functions by reducing the circular angle  $y$  of Fig. 29

## EXPLANATORY TEXT

from radians to quadrants. That is, any complex angle  $x + iy$  represented by a point  $P$ , and radius vector  $OP$ , in the complex plane  $XY$ , Fig. 30, is first transferred to a new complex plane  $XQ$ , Fig. 31, at the point  $p = x, q$ , by keeping  $x$  the same in both planes, but making the points  $\frac{\pi}{2}, \frac{2\pi}{2}, \frac{3\pi}{2}, \frac{4\pi}{2}, \dots$  etc., on the  $Y$  axis of the  $XY$  diagram,

become the points 1, 2, 3, 4 . . . etc., on the  $Q$  axis of the  $XQ$  diagram. Thus if

$$x + iy = 2.5 + i6.2832,$$

$$x + iq = 2.5 + i\text{4.00}$$

where 4.00 is underscored to indicate quadrant measure, instead of the ordinary radian measure.

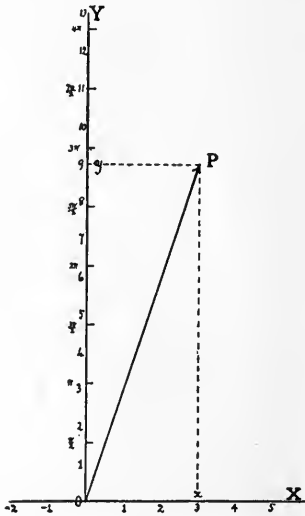


FIG. 30.

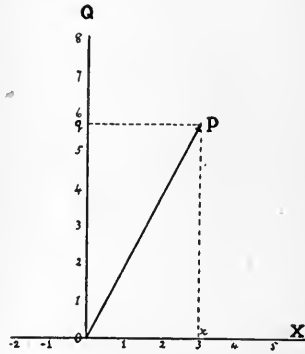


FIG. 31.

Transference of a Complex Quantity from the  $XY$  to the  $XQ$  Plane.

In the case indicated by Fig. 30,  $x + iy = 3 + i9$  and  $x + iq = 3 + i\text{5.74}$  in Fig. 31.

Consequently, after a complex angle has been transferred from the complex plane  $XY$  to the complex plane  $XQ$ , the values of either  $\sinh(x + iq)$  or  $\cosh(x + iq)$  exactly repeat themselves for each 4 units of increase in  $q$ ; or with reference to Fig. 31 for each 4 quadrants of increase in the circular angle instead of 6.2832 . . . radians. The operation of transferring the complex angle from the  $XY$  to the  $XQ$  plane may therefore be described as *quadranting*  $y$ ; *i.e.*, changing the expression of  $y$  from circular radian units to circular quadrant units.

All of the Tables VII to XIII inclusive require to be entered in terms of  $x + iq$ ; so that the complex entering value has to be quadrantated by dividing its imaginary or  $y$ -component by the numeric  $\pi/2 = 1.57079$ . . . . This preliminary step occupies a certain extra time and effort; but it actually economises the total time and effort involved. If the tables were computed for  $x + iy$ , they would have to be repeated in bulk for each  $\pi$  radians, or 2 quadrants, increase in  $y$ . In electrical engineering appli-

## EXPLANATORY TEXT

cations,  $y$  frequently rises to 100 radians, and might easily be much greater than 100. In order to go up to 100 radians, the bulk of the Tables VII to XIII would have to be increased about thirty fold. Altogether, aside from the greatly increased bulk and expense of such tables, the extra time and effort consumed in turning over the numerous pages would be comparable with that saved by eliminating the preliminary step of quadranting the imaginary component or dividing it by  $\pi/2$ .

### RULES FOR THE USE OF TABLE VII

Express the "angle" whose hyperbolic sine is required in the form of an ordinary rectangular complex quantity  $x + iy$ .

Quadrant the imaginary component  $y$  through the process of dividing it by  $\pi/2$ ; *i.e.*, transfer the quantity from the  $XY$  to  $XQ$  plane; so that the new expression of the complex quantity is  $x + iq$ ; where  $q = y/1.57079$ . . . .

If  $q$  is greater than 4.0, divide by 4 and retain only the remainder. If the remainder exceeds 2, subtract 2 therefrom, and apply a negative sign to the result found in the table. A change of 2 quadrants simply reverses the sign of the result. If the remainder on the other hand does not exceed 2, enter Table VII with it, and take out the result with unchanged sign.

Example: Required the hyperbolic sine of  $0.65 + i 25.75$ . Here  $x = 0.65$  and  $y = 25.75$ . That is  $y$  is 25.75 circular radians. Reduce this to quadrants through dividing by 1.57079. . . .

$$\begin{array}{rcl} 25.75 & * & \log 25.75 = 1.4107772. \\ 1.57079 & . . . & \log \pi/2 = 0.1961199. \\ & & \log 16.393 = 1.2146573. \end{array}$$

The quadrant value  $x + iq = 0.65 + i 16.393$

NOTE. — It is found convenient to underscore quadrantal quantities to distinguish them from radianal quantities.

Rejecting quadrant multiples of 4, *i.e.*, 16 in this case, we enter Table VII with  $x + iq = 0.65 + i 0.393$ . The nearest entry to this is  $x = 0.65$ ,  $q = 0.4$ , for which the hyperbolic sine is  $0.56368 + i 0.71639$ , an ordinary rectangular complex quantity on the  $UV$  plane. Interpolation should be made in this result to meet the change from  $q = 0.40$  to  $q = 0.393$ , as will be explained later.

Second Example: Required  $\sinh (x + iy) = \sinh (1.15 + i 10.10)$ .

$$\begin{aligned} \text{Quadranting the imaginary, } \sinh (x + iq) &= \sinh \left( 1.15 + i \frac{10.10}{1.5708} \right) \\ &= \sinh (1.15 + i 6.430). \end{aligned}$$

Rejecting 4's from the imaginary  $= \sinh (1.15 + i 2.430)$ .

Deducting 2 from the residual and changing

$$\text{the sign} \dots \dots \dots = - \sinh (1.15 + i 0.430).$$

\* This operation would ordinarily be effected with the slide-rule, when a high degree of precision is not aimed at.



## EXPLANATORY TEXT

We now enter Table VII with  $x = 1.15$  and  $q = 0.43$ . The nearest entry is  $x = 1.15$   $q = 0.45$ , the result for which is  $1.08037 + i 1.12836$ . But we must apply a negative sign to the whole of this result because of the 2 rejected in the quadrantal residuum. Hence,

$$\begin{aligned}\sinh (1.15 + i 10.10) &= - (1.08037 + i 1.12836) \\ &= - 1.08037 - i 1.12836 = u + iv\end{aligned}$$

except for the interpolation from  $q = 0.45$  to  $q = 0.43$ . The operation of interpolation will be discussed later on.

Third Example: Required  $\sinh (x + iy) = \sinh (3.60 + i 18.1)$ .

$$\begin{aligned}\text{Quadranting the imaginary, } \sinh (x + iq) &= \sinh \left( 3.60 + i \frac{18.1}{1.5708} \right) \\ &= \sinh (3.60 + i 11.523).\end{aligned}$$

Rejecting 4's from the quadrants  $= \sinh (3.60 + i 3.523)$ .

Deducting 2 from the residual imaginary

$$\text{and changing the sign} \dots \dots \dots = - \sinh (3.60 + i 1.523).$$

Entering Table VII with  $x = 3.6$  and  $q = 1.523$ , the nearest entry is  $x = 3.6$  and  $q = 1.5$ , for which the result is  $- 12.92978 + i 12.94910$ . But applying the negative sign to this result because of 2 deducted from the quadrantal imaginary, and we have finally:—

$$\begin{aligned}\sinh (3.60 + i 18.1) &= - (- 12.92978 + i 12.94910) \\ &= 12.92978 - i 12.94910 = u + iv\end{aligned}$$

except for the interpolated correction from  $q = 1.500$  to  $q = 1.523$ , to be considered later.

### RANGE OF THE TABLE

Table VII extends by steps of 0.05 in  $x$  up to  $x = 3.95$ , and in Table XIII up to  $x = 4.0$ . In  $y$ , the range is indefinitely great; because after dividing  $y$  by  $\pi/2$  so as to reduce it to quadrant measure, all multiples of 4 are rejected. From 0 to 2, in the remainder, the table gives the result directly and from 2 to 4, by change of sign in the total. Cases of  $x$  greater than 4.0 are dealt with in connection with Table XIV.

### REPETITIONS IN THE TABLE

$$\text{If } \sinh \left\{ x + i \left( \frac{\pi}{2} - a \right) \right\} = u + iv \quad (57)$$

it is easy to show that:

$$\sinh \left\{ x + i \left( \frac{\pi}{2} + a \right) \right\} = - u + iv. \quad (58)$$

It follows that in any column of Table VII, the entry for  $q = (1 - a)$  is the same as that for  $q = 1 + a$  except for a change in the sign of  $u$ . Consequently, the table might have been reduced to half its present size, if the responsibility for making this change of sign had been left to the reader. It was considered, however, that since the reader is already charged with the duty of applying a negative sign to the total result when the  $q$

## EXPLANATORY TEXT

residuum lies between 2 and 4, the retention of the full size of the present table was warranted, especially as the duplication of the text in each column provides a certain check upon the numerical work of tabulation.

### INTERPOLATION BY SIMPLE PROPORTION

As a first approximation, interpolation may be effected by simple proportion, first in regard to  $x$  and second in regard to  $q$ .

Example: Required  $\sinh (0.15 + i 0.25)$ , having given:

|                                                                                                                                                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\sinh (0.2 + i 0.2) = 0.19148 + i 0.31522.$<br>$\sinh (0.1 + i 0.2) = 0.09526 + i 0.31056.$<br><hr style="width: 100%;"/> Diff. for $0.1 x = 0.09622 + i 0.00466.$<br>Diff. for $0.05x = 0.04811 + i 0.00233.$<br><hr style="width: 100%;"/> $\sinh (0.15 + i 0.2) = 0.14337 + i 0.31289.$ | $\sinh (0.2 + i 0.3) = 0.17939 + i 0.46310.$<br>$\sinh (0.1 + i 0.3) = 0.08925 + i 0.45626.$<br><hr style="width: 100%;"/> Diff. for $0.1x = 0.09014 + i 0.00684.$<br>Diff. for $0.05x = 0.04507 + i 0.00342.$<br><hr style="width: 100%;"/> $\sinh (0.15 + i 0.3) = 0.13432 + i 0.45968.$<br>$\sinh (0.15 + i 0.2) = 0.14337 + i 0.31289.$<br><hr style="width: 100%;"/> Diff. for $q 0.10 = -0.00905 + i 0.14679.$<br>Diff. for $q 0.05 = -0.00453 + i 0.07340.$<br><hr style="width: 100%;"/> $\sinh (0.15 + i 0.25) = 0.13885 + i 0.38629.$ |
| Correct value                                                                                                                                                                                                                                                                               | <hr style="width: 100%;"/> $\underline{\underline{= 0.13910 + i 0.38700.}}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |

### INTERPOLATION BY TAYLOR'S THEOREM

When a higher degree of precision is desired than that which can be expected by simple proportion, we may use Taylor's theorem in the following form:—

$$\sinh (\theta + \Delta \theta) = \sinh \theta + \Delta \theta \cosh \theta + \frac{(\Delta \theta)^2}{2!} \sinh \theta + \frac{(\Delta \theta)^3}{3!} \cosh \theta + \dots \quad (59)$$

$$\begin{aligned} \sinh \{ (x + iy) + (\Delta x + i \Delta y) \} &= \sinh (x + iy) + (\Delta x + i \Delta y) \cosh (x + iy) \\ &+ \frac{(\Delta x + i \Delta y)^2}{2!} \sinh (x + iy) + \frac{(\Delta x + i \Delta y)^3}{3!} \cosh (x + iy) + \dots \quad (60) \end{aligned}$$

Quadranting imaginaries on both sides; or transferring to the  $XQ$  plane,

$$\begin{aligned} \sinh \{ (x + iq) + (\Delta x + i \Delta q) \} &= \sinh (x + iq) + \{ \Delta x + i (\pi/2) \Delta q \} \cosh (x + iq) \\ &+ \frac{\{ \Delta x + i (\pi/2) \Delta q \}^2}{2} \sinh (x + iq) + \frac{\{ \Delta x + i (\pi/2) \Delta q \}^3}{3!} \cosh (x + iq) + \dots \quad (61) \\ &= \sinh (x + iq) + \Delta' \theta \cosh (x + iq) + \frac{(\Delta' \theta)^2}{2!} \sinh (x + iq) \\ &+ \frac{(\Delta' \theta)^3}{3!} \cosh (x + iq) + \dots \end{aligned}$$

$$\text{where } \Delta' \theta = \Delta x + i \Delta y = \Delta x + i (\pi/2) \Delta q. \quad (62)$$

## EXPLANATORY TEXT

Example (1): With  $\Delta q = 0$ .

Required  $\sinh (0.15 + i \underline{0.2})$ , having given in Table VII and in Table VIII:

$$\sinh (0.1 + i \underline{0.2}) = 0.09526 + i 0.31056.$$

$$\cosh (0.1 + i \underline{0.2}) = 0.95582 + i 0.03095. \quad \text{Then by (60);}$$

$$\begin{aligned} \sinh (0.15 + i \underline{0.2}) &= \sinh (0.1 + i \underline{0.2}) + 0.05 \cosh (0.1 + i \underline{0.2}) + \frac{0.0025}{2} \\ &\quad \sinh (0.1 + i \underline{0.2}) + \frac{0.00013}{6} \cosh (0.1 + i \underline{0.2}) \\ &= 0.09526 + i 0.31056 + 0.05 (0.95582 + i 0.03095) \\ &\quad + 0.00125 (0.09526 + i 0.31056) + 0.00002 (0.95582 + \\ &\quad i 0.03095) \\ &= 1.00125 (0.09526 + i 0.31056) + 0.05002 (0.95582 + i 0.03095) \\ &= (0.09538 + i 0.31095) + (0.04781 + i 0.00155) \\ &= \underline{0.14319 + i 0.31250} \end{aligned}$$

which is the correct tabular value of  $\sinh (0.15 + i \underline{0.2})$  in Table VII.

Example (2): With  $\Delta x = 0$ .

Required  $\sinh (0.1 + i \underline{0.25})$ , having given in Table VII and in Table VIII:

$$\sinh (0.1 + i \underline{0.2}) = 0.09526 + i 0.31056.$$

$$\cosh (0.1 + i \underline{0.2}) = 0.95582 + i 0.03095. \quad \text{Then by (62);}$$

$$\begin{aligned} \sinh (0.1 + i \underline{0.25}) &= \sinh (0.1 + i \underline{0.2}) + i 0.05 \times 1.5708 \times \cosh (0.1 + i \underline{0.2}) \\ &\quad + i^2 \frac{(0.05 \times 1.5708)^2}{2!} \sinh (1.0 + i \underline{0.2}) \\ &\quad + i^3 \frac{(0.05 \times 1.5708)^3}{3!} \cosh (1.0 + i \underline{0.2}) \\ &= 0.09526 + i 0.31056 + i \times 0.07854 (0.95582 + i 0.03095) \\ &\quad - \frac{0.00617}{2} (0.09526 + i 0.31056) \\ &\quad - i \frac{0.00048}{6} (0.95582 + i 0.03095) \\ &= (0.09526 + i 0.31056) (1 - 0.00309) \\ &\quad + i (0.95582 + i 0.03095) (0.07854 - 0.00006) \\ &= 0.99691 (0.09526 + i 0.31056) + 0.07848 (-0.03095 + i 0.95582) \\ &= 0.09497 + i 0.30960 - 0.00243 + i 0.07501 \\ &= 0.09254 + i 0.38461. \end{aligned}$$

The tabular value is 0.09254 + i 0.38460.

Example (3): Interpolation for both  $\Delta x$  and  $\Delta q$ .

Required  $\sinh (0.15 + i \underline{0.25})$ , having given

$$\sinh (0.1 + i \underline{0.2}) = 0.09526 + i 0.31056 \text{ by Table VII}$$

$$\text{and } \cosh (0.1 + i \underline{0.2}) = 0.95582 + i 0.03095 \text{ by Table VIII.}$$

Here  $\Delta \theta$  in formula (59) =  $(0.05 + i \underline{0.05})$

and  $\Delta' \theta$  in formula (62) =  $(0.05 + i (\pi/2) \times \underline{0.05}) = (0.05 + i 0.07854).$

## EXPLANATORY TEXT

Thus: —

$$\sinh (0.15 + i \underline{0.25}) = \sinh (0.1 + i \underline{0.2}) + \Delta'\theta \cosh (0.1 + i \underline{0.2}) \\ + \frac{(\Delta'\theta)^2}{2!} \sinh (0.1 + i \underline{0.2}) + \dots$$

$$\Delta'\theta = 0.05 + i 0.07854.$$

$$(\Delta'\theta)^2 = + 0.0025 - 0.00617 + i 0.00785$$

$$= - 0.00367 + i 0.00785.$$

$$\frac{(\Delta'\theta)^2}{2} = - 0.00184 + i 0.00393.$$

$$(\Delta'\theta)^3 = (0.05 + i 0.07854)^3 = - 0.00080 + i 0.00016.$$

$$\frac{(\Delta'\theta)^3}{6} = - 0.00013 + i 0.00002.$$

$$\sinh (0.15 + i \underline{0.25}) = \sinh (0.1 + i \underline{0.2}) \left\{ 1 + \frac{(\Delta'\theta)^2}{2!} + \dots \right\} \\ + \cosh (0.1 + i \underline{0.2}) \left\{ \Delta'\theta + \frac{(\Delta'\theta)^3}{3!} + \dots \right\} \\ = (0.09526 + i 0.31056) (0.99816 + i 0.00393) = 0.09386 + i 0.31036 \\ + (0.95582 + i 0.03095) (0.04087 + i 0.07856) = 0.04524 + i 0.07664 \\ = 0.13910 + i 0.38700.$$

The correct tabulated value is = 0.13910 + i 0.38700.

### EFFECTS OF CHANGES OF SIGN IN THE ENTERING QUANTITY

Table VII expresses the relation

$$\sinh (x + iq) = u + iv. \quad (63)$$

(a) If  $x$  be taken with negative sign, we have

$$\sinh (-x + iq) = -u + iv \quad (64)$$

so that changing the sign of the real component entering the table changes the sign of the real component in the result; but leaves the sign of the imaginary component unchanged.

(b) If  $q$  be taken with negative sign, we have

$$\sinh (x - iq) = u - iv \quad (65)$$

so that changing the sign of the imaginary component in the entering quantity changes the sign of the imaginary component in the result, leaving the sign of the real component unchanged.

(c) If both  $x$  and  $q$  be taken with negative sign, we have

$$\sinh (-x - iq) = \sinh \{ -(x + iq) \} = -u - iv = -(u + iv) \quad (66)$$

so that changing the sign of the total entering quantity changes the sign of the total result.

The facts may be summed up by saying that changes in the sign of the entering quantity produce corresponding changes of sign in the result.

## EXPLANATORY TEXT

### CIRCULAR SINES OF COMPLEX " ANGLES "

Since, as is well known: —

$$\sin \theta = -i \sinh (i\theta) \quad (67)$$

we have

$$\sin (x + iy) = -i \sinh (ix - y) \quad (68)$$

$$= i \sinh (y - ix). \quad (69)$$

Consequently, in order to find the circular sine of the complex quantity  $(x + iy)$ , enter Table VII for  $\sinh (y + ix)$ , which on being quadranted, becomes  $\sinh \{ y + ix/(\pi/2) \}$  and let the result be  $(u + iv)$ . Then  $\sinh (y - ix) = u - iv$  and  $\sin (x + iy) = v + iu$ . In other words, invert the entering components, and then invert the components of the result.

Example: Required  $\sin (1 + i2)$  from Table VII. Here  $\theta = (1 + i2)$ .

Enter the Table with  $-\sinh (i\theta) = \sinh (-i\theta) = \sinh (2 - i1)$ .

Quadranting the imaginary, we enter the table with  $(x - iq) = (2 - i0.6366)$ . The nearest entry is  $(2 - i0.65)$ , for which the hyp. sine is given as  $1.89503 - i3.20780$ . Consequently,  $\sin (1 + i2) = 3.2078 + i1.89503$ , except in so far as interpolation is needed to reduce  $\sinh (2 - i0.6366)$  from  $\sinh (2 - i0.65)$ . In this way any circular sine of a complex quantity can always be obtained from the table of hyperbolic sines, between the limits of 0 and  $\pm 4$  in  $y$ , and of 0 and  $\pm \infty$  in  $x$ .

### GRAPHIC INTERPOLATION BY MEANS OF CHARTS VIIA, VIIB, VIIC

Charts VII-VIII A, B, and C, serve for the evaluation of either  $\sinh (x + iq)$  or  $\cosh (x + iq)$ , according to the axis of reference selected. Thus, taking Chart VII-VIIIB, if this is held with the line  $SS$  as the axis of reference or initial line; then by comparison with the entries in Table VII, it will be found that  $\sinh (x + iq)$  can be read from it directly over the range  $q = 0$  to  $q = 4$ , beyond which the values repeat themselves indefinitely. On the other hand, if the chart be turned through  $90^\circ$ , so as to bring the line  $CC$  as the axis of reference, it will be found by comparison with the entries of Table VIII, that  $\cosh (x + iq)$  can be read from it directly over the range  $q = 0$  to  $q = 4$ .

Chart VII-VIIIA gives  $\sinh (x + iq)$  and  $\cosh (x + iq)$  for values of  $x$  up to about 0.9. Chart VII-VIIIB gives the corresponding results for values of  $x$  up to about  $x = 2$ . Finally, Chart VII-VIIIC provides for values of  $x$  up to  $x = 4$ . In all of these charts, interpolation can be made for both  $x$  and  $q$  to 0.01, by direct inspection. The graphs on these charts are undistorted, since they give complex functions as results, in rectangular coördinates. The curves therefore always intersect orthogonally, and they represent a confocal system of ellipses and hyperbolas, the common foci being at two points at unit distances from the center, along one of the reference axes. The curvilinear rectangles into which the charts are divided have pairs of sides the ratio of whose lengths tends to the value  $\pi/2$ .

If the preliminary process of quadranting the imaginary of the entering quantity were not adopted; that is, if the graphs were entered in terms of  $(x + iy)$ , instead of

## EXPLANATORY TEXT

$(x + iq)$ ; then it would be necessary\* to have a new chart for each range of  $2\pi$  units in  $y$ ; or some 16 sets of Charts A, B, and C, in order to reach  $y = 100$ . That is, 48 charts would have to be computed, prepared, drawn, lithographed, bound, sold and operated instead of the 3 charts actually presented. Moreover, if  $y$  were needed greater than 100, the set of 48 would fail; whereas, working with quadrant imaginaries, the three charts serve up to indefinitely great values of  $q$  and  $y$ .

### GRAPHIC CHART VII-VIIIa

This chart corresponds to Tables VII and VIII at least as far as  $x = 0.9$ , or for pages 42 to 45, and 58 to 61 of this book. To find hyperbolic sines from the chart, place it facing the observer with the axis  $OO$  vertical. This is the major axis of all the ellipses shown. Starting from this central axis towards the right hand, the successive ellipses marked 0.1, 0.2, 0.3, etc., represent values of  $x$ ; while the successively rising hyperbolas 0.1, 0.2, represent values of  $q$ . These values of  $q$  will be found to extend over two quadrants. Enter the chart on the curvilinear coördinates for  $x$  and  $q$ . At the proper intersection read off the  $u$  and  $v$  coördinates of the rectilinear ruling,  $u$  being the abscissas and  $v$  the ordinates.

Conversely, to find  $\sinh^{-1}(u + iv)$  within the limits  $u = 0$  and  $u = \pm 1$ ,  $v = 0$  to  $v = 2.0$ , enter the chart with the same aspect on the rectilinearly ruled coördinates and read off at the proper intersection the curvilinear values taking  $x$  on the ellipses and  $q$  on the hyperbolas.

To find hyperbolic cosines from the chart, rotate it clockwise  $90^\circ$ ; so as to have the axis  $OO$  horizontal. Then enter on the curvilinear coördinates with  $x$  on the ellipses and  $q$  on the hyperbolas. The first and fourth quadrants only will be presented to the observer; but from the symmetry of the diagram, it will be easy to reverse the chart, so as to present the second and third quadrants. Read off the result on the rectilinear background using  $u$  for abscissas and  $v$  for ordinates.

Conversely, to find  $\cosh^{-1}(u + iv)$  from the chart with the axis  $OO$  horizontal, enter on the rectilinear background and read off at the proper intersection from the curvilinear coördinates in  $x$  and  $q$ , taking the ellipses as parts of the  $x$ -system and the hyperbolas as part of the  $q$ -system.

### GRAPHIC CHART VII-VIIIb

This chart gives the graph of the functions  $\sinh(x + iq)$  and  $\cosh(x + iq)$  from  $x = 0.8$  at least as far as  $x = 2.05$  along the ellipses and from  $q = 0$  to  $q = \infty$  by virtue of successive rotations. In this and the following charts, the numerical values of  $q$  are all underscored, an indication which may serve readily to distinguish the imaginaries  $q$ , from the reals,  $x$ .

\* A single set of charts entered in terms of  $(x + iy)$  could be used up to  $y = 6.2832$  in one revolution, and could be used for all larger values by throwing out multiples of  $2\pi$ . This operation of dividing  $y$  by  $2\pi$  would, however, take as much time as the operation of quadranting, and would also lead to a dissymmetrical chart in the hyperbolas.

## EXPLANATORY TEXT

In all of the Charts VII to IX inclusive, the curvilinear rectangles all tend to have sides in the ratio  $\pi : 2$ ; that is the long side approximates to being 1.57 times the short side. In IXA exceptions are found; because extra curvilinear coördinates are supplied.

To find  $\sinh (x + iq)$  from Chart VII-VIII<sub>B</sub>, hold the minor axis *SS* horizontal. Enter on the curvilinear coördinates with *x* on ellipses and *q* on hyperbolas. At the proper intersection read off on the rectilinear background in *u* and *v*. Proceed inversely to find the inverse function  $\sinh^{-1} (u + iv)$ .

To find  $\cosh (x + iq)$  from the same chart, hold the major axis *CC* horizontal. Enter on the curvilinear coördinates with *x* on ellipses and *q* on hyperbolas. Read off on the rectilinear background.

All four quadrants appear in this and the following chart, so that it is not necessary to limit the value of *q* to less than 2 quadrants.

### GRAPHIC CHART VII-VIII<sub>C</sub>

This Chart gives the graph of  $\sinh (x + iq)$  and  $\cosh (x + iq)$  from  $x = 2.0$ , at least as far as  $x = 3.90$ . The procedure is precisely the same as that for VII-VIII<sub>B</sub> already described.

### TABLE VIII

$$\cosh (x + iq) = u + iv$$

#### RECTANGULAR HYPERBOLIC COSINES OF A RECTANGULAR VARIABLE

Table VIII may be regarded as an inversion of Table VII; because:

$$\cosh \theta = -i \sinh (\theta + i\pi/2) \quad (70)$$

or in quadrant imaginaries,

$$\cosh \theta = -i \sinh (\theta + i\underline{1}). \quad (71)$$

That is the hyp. cosine of any complex quantity  $(x + iq)$  is  $-i$  times the hyp. sine of that quantity with an additional quadrant in the imaginary. Thus

$$\begin{aligned} \cosh (0.5 + i0.6) &= -i \sinh (0.5 + i1.6) \\ &= -i (-0.42158 + i0.66280) \\ &= + \underline{0.66280} + i \underline{0.42158}. \end{aligned}$$

All of the entries in Table VII thus reproduce themselves by inversion in corresponding parts of Table VIII, a fact which serves as a numerical check upon both.

In order to find the value of  $\cosh (x + iy)$ , quadrant the imaginary quantity *y*, by dividing it with  $\pi/2$ , as in entering Table VII. The complex quantity  $(x + iy)$  will now be expressed as  $(x + iq)$ ; or will in effect have been transferred from the *XY* to the *XQ* plane. Next throw out multiples of 4 from *q*, so as to leave a remainder less than 4. If this remainder exceeds 2, deduct 2 from it, but change the sign of the total result thereupon deduced. • If the remainder, however, is not greater than 2, then, the result is taken directly from the table.

Example: To find  $\cosh (1 + i5) = \cosh (x + iy)$ .

Quadranting, we have  $\cosh (1 + i3.183) = \cosh (x + iq)$ .

Deducting 2 from *q*,  $\cosh (1 + i\underline{1.183})$ .

## EXPLANATORY TEXT

With this we enter Table VIII. The nearest entry is  $x + iq = 1 + i \underline{1.2}$ , the result for which is  $-0.47684 + i \underline{1.11768}$ . This has to be corrected by interpolation from  $q = \underline{1.2}$  to  $q = \underline{1.183}$ . Reverse the sign of the result to  $0.47684 - i \underline{1.11768}$  for the deduction of 2 quadrants.

Example 2: Required  $\cosh (0.25 + i \underline{30}) = \cosh (x + iy)$ .  
 Quadranting, this becomes  $\cosh (0.25 + i \underline{19.099}) = \cosh (x + iq)$ .  
 Rejecting imaginary quadruples  $= \cosh (0.25 + i \underline{3.099})$   
 $= \cosh (x + iq)$ .  
 Deducting 2 quadrants  $= \cosh (0.25 + i \underline{1.099})$   
 $= -\cosh (x + iq)$ .

The nearest entry is  $0.25 + i \underline{1.1}$  for which the result is  $(-0.16135 + i \underline{0.24950})$ . Applying the negative sign on account of the two deducted quadrants, the final result is, neglecting interpolation,

$$\cosh (0.25 + i \underline{30}) = 0.16135 - i \underline{0.24950} = u - iv.$$

## INTERPOLATION BY SIMPLE PROPORTION

A first approximation can be obtained by interpolating according to simple proportion,

Example: Required  $\cosh (0.55 + i \underline{0.55}) = \cosh (x + iq)$

having given

$$\cosh (0.6 + i \underline{0.5}) = 0.83825 + i \underline{0.45018}.$$

$$\cosh (0.5 + i \underline{0.5}) = 0.79735 + i \underline{0.36847}.$$

$$\text{Diff. for } x \underline{0.1} = 0.04090 + i \underline{0.08171}.$$

$$\text{Diff. for } x \underline{0.05} = 0.02045 + i \underline{0.04086}.$$

$$\cosh (0.55 + i \underline{0.5}) = 0.81780 + i \underline{0.40933}.$$

$$\cosh (0.6 + i \underline{0.6}) = 0.69680 + i \underline{0.51506}.$$

$$\cosh (0.5 + i \underline{0.6}) = 0.66280 + i \underline{0.42158}.$$

$$\text{Diff. for } x \underline{0.1} = 0.03400 + i \underline{0.09348}.$$

$$\text{Diff. for } x \underline{0.05} = 0.01700 + i \underline{0.04674}.$$

$$\cosh (0.55 + i \underline{0.6}) = 0.67980 + i \underline{0.46832}.$$

$$\cosh (0.55 + i \underline{0.5}) = 0.81780 + i \underline{0.40933}.$$

$$\text{Diff. for } q \underline{0.1} = -0.13800 + i \underline{0.05899}.$$

$$\text{Diff. for } q \underline{0.05} = -0.06900 + i \underline{0.02950}.$$

$$\cosh (0.55 + i \underline{0.55}) = 0.74880 + i \underline{0.43883}.$$

$$\text{Correct value} = \underline{\underline{0.75018 + i \underline{0.43963}}}.$$

## INTERPOLATION BY TAYLOR'S THEOREM

For a higher degree of precision than simple proportion affords, reference may be had to Taylor's theorem in the following form:—

$$\cosh (\theta + \Delta\theta) = \cosh \theta + \Delta\theta \sinh \theta + \frac{(\Delta\theta)^2}{2!} \cosh \theta + \frac{(\Delta\theta)^3}{3!} \sinh \theta + \dots$$

$$\begin{aligned} \cosh \{ (x + iy) + (\Delta x + i \Delta y) \} &= \cosh (x + iy) + (\Delta x + i \Delta y) \sinh (x + iy) \\ &+ \frac{(\Delta x + i \Delta y)^2}{2!} \cosh (x + iy) + \frac{(\Delta x + i \Delta y)^3}{3!} \sinh (x + iy) + \dots \quad (71a) \end{aligned}$$



## EXPLANATORY TEXT

Quadranting imaginaries on both sides, or transferring to the  $XQ$  plane,

$$\begin{aligned} \cosh \{ (x + iq) + (\Delta x + i \Delta q) \} &= \cosh (x + iq) + (\Delta x + i \Delta q \pi/2) \sinh (x + iq) \\ &\quad + \frac{(\Delta x + i \Delta q \pi/2)^2}{2!} \cosh (x + iq) \\ &\quad + \frac{(\Delta x + i \Delta q \pi/2)^3}{3!} \sinh (x + iq) + \dots \\ &= \cosh (x + iq) + \Delta' \theta \sinh (x + iq) \\ &\quad + \frac{(\Delta' \theta)^2}{2!} \cosh (x + iq) + \dots \end{aligned}$$

(71b)

Example: Required  $\cosh (0.5 + i \underline{0.55}) = \cosh (x + iq)$

having given  $\cosh (0.5 + i \underline{0.5}) = 0.79735 + i 0.36847$  in Table VIII

and  $\sinh (0.5 + i \underline{0.5}) = 0.36847 + i 0.79735$  in Table VII.

Here  $\Delta x = 0$ ,  $\Delta q = i \underline{0.05}$ ,  $\Delta' \theta = i \underline{0.05} \times 1.5708 = i 0.07854$ .

$$\begin{aligned} \cosh (0.5 + i \underline{0.55}) &= \cosh (0.5 + i \underline{0.5}) \left\{ 1 + \frac{(\Delta' \theta)^2}{2!} + \frac{(\Delta' \theta)^4}{4!} + \dots \right\} \\ &\quad + \sinh (0.5 + i \underline{0.5}) \left\{ \Delta' \theta + \frac{(\Delta' \theta)^3}{3!} + \dots \right\}. \end{aligned}$$

$$\Delta' \theta = i 0.07854.$$

$$(\Delta' \theta)^2 = -0.00617. \quad \frac{(\Delta' \theta)^2}{2!} = -0.00309.$$

$$(\Delta' \theta)^3 = -i 0.00048. \quad \frac{(\Delta' \theta)^3}{3!} = -i 0.00008.$$

$$(\Delta' \theta)^4 = +0.00004. \quad \frac{(\Delta' \theta)^4}{4!} = 0.00000.$$

$$\begin{aligned} \cosh (0.5 + i \underline{0.55}) &= (0.79735 + i 0.36847) (1 - 0.00309) \\ &\quad + (0.36847 + i 0.79735) (i 0.07854 - i 0.00008) \\ &= (0.79735 + i 0.36847) 0.99691 \\ &\quad + (0.36847 + i 0.79735) i 0.07846 \\ &= 0.79489 + i 0.36733 + i 0.02891 - 0.06256 \\ &= 0.73233 + i 0.39624. \end{aligned}$$

The tabulated value = 0.73233 + i 0.39624.

In view of the similarity of the interpolation operations by Taylor's theorem to those already discussed in relation to Table VII, further examples are probably not needed.

### EFFECTS OF CHANGES OF SIGN IN THE ENTERING QUANTITY

$$\begin{aligned} \text{Since if } \cosh (x + iy) &= u + iv & (72) \\ \cosh (-x + iy) &= u - iv & (73) \\ \cosh (x - iy) &= u - iv & (74) \\ \cosh (-x - iy) &= u + iv & (75) \end{aligned}$$

changing the sign of either the real or imaginary entering component only changes the sign of the imaginary component in the result; while changing the sign of the entering quantity as a whole, has no effect on the sign of the result.

## EXPLANATORY TEXT

### CIRCULAR COSINES OF COMPLEX "ANGLES"

It is well known that if  $\theta$  be any angle, real or complex,

$$\cos \theta = \cosh (i \theta). \quad (76)$$

Consequently,

$$\cos (x + iy) = \cosh (-y + ix) \quad (77)$$

or, quadranting the imaginary component,

$$\cos (x + iy) = \cosh (-y + i 2x/\pi) = u + iv. \quad (78)$$

To find the circular cosine of any complex quantity  $x + iy$ , we enter Table VIII with  $(-y + ix/1.5708)$ . The result is the desired cosine.

Example: Required  $\cos (0.4 + i 1.2)$ .

Thus we require  $\cosh (-1.2 + i 0.2546)$ .

We now enter Table VIII with  $x = -1.2$  and  $q = 0.2546$  the nearest entry being  $x = -1.2$  and  $q = 0.25$ , for which the result is  $1.67283 - i 0.57765$ .

Hence  $\cos (0.4 + i 1.2) = 1.67283 - i 0.57765$  neglecting interpolation from  $q = 0.25$  to  $q = 0.2546$ .

### GRAPHIC CHART INTERPOLATIONS

The use of the Graphic Charts VII-VIII A, B, C, for hyperbolic cosines has already been described in connection with sines, on pages 197-198.

### TABLE IX

$$\tanh (x + iq) = u + iv$$

#### RECTANGULAR HYPERBOLIC TANGENTS OF A RECTANGULAR VARIABLE

##### *Entering Process*

Let  $\tanh (x + iy)$  be the required function. Quadrant the imaginary component, as described under Tables VII and VIII; that is, divide  $y$  by  $\pi/2$ ; so that  $y/(\pi/2) = q$ . The required function is now expressed in the form  $\tanh (x + iq)$ . Throw out multiples of 2 from  $q$  and retain only the remainder as  $q$ . Enter Table IX with  $(x + iq)$ , and find the result directly as  $u \pm iv$ . It is a well-known property of  $\tanh (x + iy)$ , that it is periodic in  $iy$ , and that the period is  $i\pi$  circular radians; or, in quadrants,  $i.2$ . That is

$$\tanh \{x + i(y + n\pi)\} = \tanh (x + iy). \quad (79)$$

where  $n$  is any integer; or, in quadrant measure of the imaginary,

$$\tanh \{x + i(q + 2n)\} = \tanh (x + iq). \quad (80)$$

Example: Required  $\tanh (0.25 + i 30) = \tanh (x + iy)$ .

Quadranting,  $\tanh (0.25 + i 19.099) = \tanh (x + iq)$ .

Rejecting multiples of 2,  $\tanh (0.25 + i 1.099) = \tanh (x + iq)$ .

We now enter Table IX with  $x = 0.25$  and  $q = 1.099$ , the nearest entry to which  $x = 0.25$ ,  $q = 1.1$ . The result is  $2.95122 - i 1.75011$ .

INTERPOLATION

Interpolation may be approximately effected by simple proportion, first in  $x$  and then in  $q$ , as indicated in connection with Tables VII and VIII; or, when a higher degree of precision is required, recourse may be had to Taylor's theorem in the following form:—

$$\begin{aligned} \tanh (\theta+\Delta \theta) &= \tanh \theta+\Delta \theta \operatorname{sech}^2 \theta-\frac{(\Delta \theta)^2}{2 !} 2 \operatorname{sech}^2 \theta \tanh \theta \\ &+ \frac{(\Delta \theta)^3}{3 !} 2 \operatorname{sech}^2 \theta\left(2 \tanh ^2 \theta-\operatorname{sech}^2 \theta\right)+\ldots \end{aligned} \quad (81)$$

or

$$\begin{aligned} \tanh \{(x+i y)+(\Delta x+i \Delta y)\} &= \tanh (x+i y)+\frac{(\Delta x+i \Delta y)}{\cosh ^2(x+i y)} \\ &- \frac{(\Delta x+i \Delta y)^2}{\cosh ^2(x+i y)} \tanh (x+i y)+\ldots \end{aligned} \quad (82)$$

and quadranting,

$$\begin{aligned} \tanh \{(x+i q)+(\Delta x+i \Delta q)\} &= \tanh (x+i q)+\frac{\{\Delta x+i \Delta q(\pi / 2)\}}{\cosh ^2(x+i q)} \\ &- \frac{\{\Delta x+i \Delta q(\pi / 2)\}^2}{\cosh ^2(x+i q)} \tanh (x+i q)+\ldots \end{aligned} \quad (83)$$

so that as far as the second correction term:—

$$\begin{aligned} \tanh \{(x+i q)+(\Delta x+i \Delta q)\} &= \tanh (x+i q)+\frac{\Delta' \theta}{\cosh ^2(x+i q)} \\ &- \frac{(\Delta' \theta)^2 \tanh (x+i q)}{\cosh ^2(x+i q)}+\ldots \end{aligned} \quad (84)$$

$$\text { where } \Delta' \theta=(\Delta x+i \Delta y)=\{\Delta x+i \Delta q(\pi / 2)\} \quad (85)$$

Example: Required  $\tanh (0.5+i 0.55)=\tanh (x+i q)$

having given  $\cosh (0.5+i 0.5)=0.79735+i 0.36847$  by Table VIII

$=0.87837 / 24^{\circ} .803$  by Table XI

and  $\tanh (0.5+i 0.5)=0.76159+i 0.64805$  by Table IX

$=1.0 / 40^{\circ} .395$  by Table XII.

Here  $\Delta' \theta=(0+i 0.05 \times 1.5708)=(0+i 0.07854)$ .

$$\begin{aligned} \tanh (0.5+i 0.55) &= 0.76159+i 0.64805+\frac{i 0.07854}{(0.87837)^2 / 49^{\circ} .606}+\frac{0.00617 \times 1 / 40^{\circ} .395}{(0.87837)^2 / 49^{\circ} .606} \\ &= 0.76159+i 0.64805+\frac{0.07854 / 40^{\circ} .394}{0.77153}+\frac{0.00617 \sqrt{9^{\circ} .211}}{0.77153} \\ &= 0.76159+i 0.64805+0.10180 / 40^{\circ} .394+0.00800 \sqrt{9^{\circ} .211} \\ &= 0.76159+i 0.64805+0.07753+i 0.06597+0.00790-i 0.00128 \\ &= 0.84702+i 0.71274. \end{aligned}$$

The correct value is 0.84752 + i 0.71229.

As the third correction term is inconvenient for computation, it is often preferable to obtain a precise interpolation of  $\tanh (x+i y)$  in working out the correct interpolations of  $\sinh (x+i y)$  and  $\cosh (x+i y)$  by the methods already illustrated, and then to take their ratio.

# EXPLANATORY TEXT

## EFFECTS OF CHANGES OF SIGN IN THE ENTERING QUANTITY

|      |                     |              |      |
|------|---------------------|--------------|------|
| If   | $\tanh (x + iy)$    | $u + iv$     | (86) |
| then | $\tanh (x - iy) =$  | $u - iv$     | (87) |
|      | $\tanh (-x + iy) =$ | $-u + iv$    | (88) |
| and  | $\tanh (-x - iy) =$ | $-(u + iv).$ | (89) |

Consequently, changes in the sign of the entering quantity produce corresponding changes of sign in the result.

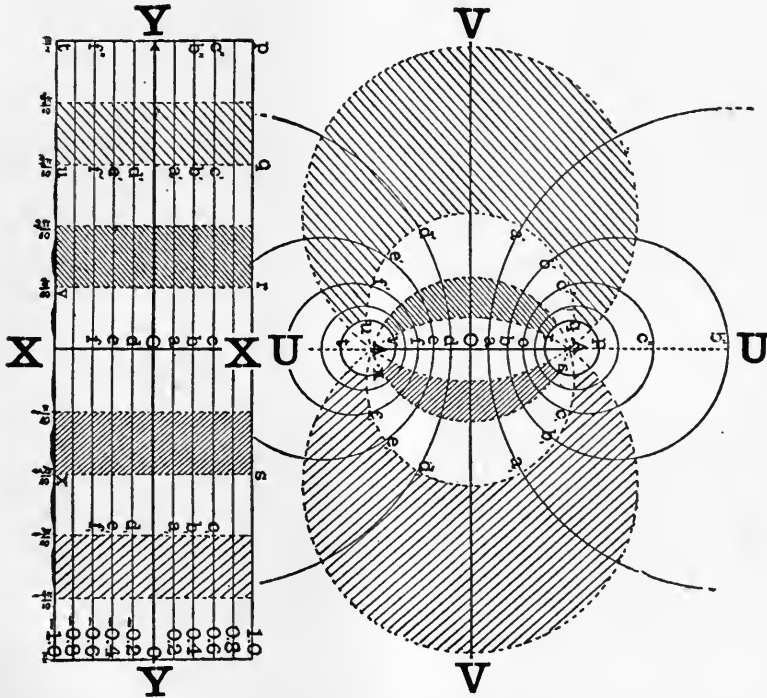


FIG. 32. — Graphs of  $x + iy$  and  $\tanh (x + iy)$  in the  $XY$  and  $UV$  planes respectively.

## CIRCULAR TANGENTS OF COMPLEX “ ANGLES ”

|                 |                                      |      |
|-----------------|--------------------------------------|------|
| Since           | $\tan \theta = -i \tanh (i\theta).$  | (90) |
| It follows that | $\tan (x + iy) = -i \tanh (-y + ix)$ | (91) |

or, quadranting the imaginary: —

$$\begin{aligned} \tan (x + iy) &= -i \tanh (-y + i2x/\pi) = -i \tanh (-y + iq) \\ &= -i(-u + iv) \\ &= v + iu. \end{aligned} \quad (92)$$

Consequently, to find  $\tan (x - iy)$  from Table IX, enter it with  $y$  as  $x$  and with  $x/(\pi/2)$  as  $q$ . Invert the components of the result and the required function is obtained. Thus

## EXPLANATORY TEXT

required  $\tan (1 + i 2)$ . We enter with  $x = 2$  and  $q = \underline{0.6366}$ . The nearest entry is  $x = 2.0$  and  $q = \underline{0.65}$  for which  $u + iv = 1.01623 + i 0.03318$ . Therefore, inverting,

$$\tan (1 + i 2) = 0.03318 + i 1.01623$$

neglecting the interpolation from  $q = \underline{0.65}$  to  $q = \underline{0.6366}$ .

### GRAPHIC INTERPOLATION BY MEANS OF CHARTS IXA, IXB, AND IXC

These charts contain all of the entries in Table IX, and also a certain number of additional results. They present circles intersecting circles orthotomically; *i.e.*, by rectangular intersection. It is clear that for values of  $x$  less than 0.10, the curves run off Chart IXA. In fact the first curve shown of  $x = 0.01$  extends as far as  $u = 100$ . By taking  $x$  small enough, the corresponding values of  $u$  and  $v$  may become indefinitely great. The entire  $UV$  plane is covered to infinity once between  $x = 0$  and  $x = \infty$ ,  $q = 0$  and  $q = \underline{2}$ . It is covered once more for each 2 quadrants increase in  $q$ .

When entering for  $\tanh (-x \pm iq)$ ; or for the inverse operation  $\tanh^{-1} (-u \pm v)$ , it must be remembered that the confocal conic-section diagrams VII and VIII are complete for negative as well as for positive values of  $x$  and  $q$ ; but that only half of the  $UV$  plane is presented in Charts IX. The full graph is indicated in Fig. 32, by the aid of which the functions corresponding to negative real values are readily apprehended.

### TABLE X

$$\sinh (x + iq) = r / \gamma$$

#### POLAR HYPERBOLIC SINES OF A RECTANGULAR VARIABLE

This table corresponds completely to Table VII, already considered, except that it offers results in polar instead of rectangular coördinates.

To find  $\sinh (x + iy)$  expressed in polar coördinates, quadrant the imaginary, and express the entering variable as  $(x + iq)$ . Reject multiples of 4 in  $q$ , and if the remainder exceeds 2, reject 2 but change the sign of the total result.

#### INTERPOLATION BY SIMPLE PROPORTION

Required  $\sinh (0.15 + i \underline{0.25})$  having given

|                                                                            |                                                                             |
|----------------------------------------------------------------------------|-----------------------------------------------------------------------------|
| $\sinh (0.2 + i \underline{0.2}) = 0.36882 / \underline{58^{\circ}.723.}$  | $\sinh (0.2 + i \underline{0.3}) = 0.49663 / \underline{68^{\circ}.825.}$   |
| $\sinh (0.1 + i \underline{0.2}) = 0.32485 / \underline{72^{\circ}.947.}$  | $\sinh (0.1 + i \underline{0.3}) = 0.46491 / \underline{78^{\circ}.932.}$   |
| Diff. for 0.1 $x = +0.04397 / \underline{-14^{\circ}.224.}$                | Diff. for 0.1 $x = +0.03172 / \underline{-10^{\circ}.107.}$                 |
| Diff. for 0.05 $x = 0.02199 / \underline{-7^{\circ}.112.}$                 | Diff. for 0.05 $x = 0.01586 / \underline{-5^{\circ}.054.}$                  |
| $\sinh (0.15 + i \underline{0.2}) = 0.34684 / \underline{65^{\circ}.835.}$ | $\sinh (0.15 + i \underline{0.3}) = 0.48077 / \underline{73^{\circ}.878.}$  |
|                                                                            | $\sinh (0.15 + i \underline{0.2}) = 0.34684 / \underline{65^{\circ}.835.}$  |
|                                                                            | Diff. for $i 0.1 = 0.13393 / \underline{8^{\circ}.043.}$                    |
|                                                                            | Diff. for $i 0.05 = 0.06697 / \underline{4^{\circ}.022.}$                   |
|                                                                            | $\sinh (0.15 + i \underline{0.25}) = 0.41381 / \underline{69^{\circ}.857.}$ |
| Correct Value                                                              | $0.41124 / \underline{70^{\circ}.229.}$                                     |

## EXPLANATORY TEXT

### INTERPOLATION BY TAYLOR'S THEOREM

For a higher degree of precision than is obtainable by simple proportion, it is convenient to use rectangular coördinates and apply formula (62). Thus, required  $\sinh(0.15 + i0.25)$ . Here referring to Table VII and to the work on page 195, we find for the result  $u + iv = 0.13911 + i0.38701$ .

|                             |                                |                                     |                                     |
|-----------------------------|--------------------------------|-------------------------------------|-------------------------------------|
| Here                        | log 0.38701 = 1.5877110        | 70°.13'. $\frac{70}{100}$ = 70°.229 | log 0.13911 = 1.1433584             |
|                             | log 0.13911 = <u>1.1433584</u> |                                     | log sec. 70°.229 = <u>0.4707400</u> |
|                             | 0.4443526                      |                                     | log 0.41124 = 1.6140984             |
| log tan 70°.13' = 0.4440674 | Result                         | 0.41124 / 70°.229.                  |                                     |
| 0.72 = <u>2853</u>          | Correct Value                  | <u>0.41124 / 70°.229.</u>           |                                     |
| 3968                        |                                |                                     |                                     |

### INTERPOLATION BY CHARTS X-XIA AND X-XIB

These charts present the polar coördinate results on rectangular coördinate sheets, so that they are not true graphs, but are merely to be regarded as interpolation diagrams.

To find  $\sinh(x + iy)$ , proceed as in the use of the tables and quadrant the imaginary so as to obtain the entering quantity in the form  $(x + iq)$ . Enter with the curvilinear coördinates, taking the more nearly vertical wavy lines for  $x$  and the more nearly horizontal lines for  $q$ , starting from the line  $SS$  as the zero of  $q$ . Read off the result on the rectangular background to the left-hand scale of ordinates.

When we leave X-XIA and enter X-XIB, it is noticeable that the curves of constant  $x$  approach vertical straight lines and the curves of constant  $q$  approach horizontal straight lines. At and beyond  $x = 3.0$ , we may approximate to the modulus  $r$  at any required  $q$ , by taking the value of  $r$  at  $q = 0.5$  and simple proportional parts between this and  $r$  at  $q = 0$  or  $r$  at  $q = 1.0$ . The change in modulus  $r$  between  $q = 0.5$ , and either of the above limits is very nearly  $\epsilon^{-x/2}$ . Thus at  $x = 3.5$  and  $q = 0.5$ ,  $r$  by the tables is 16.55774. At  $q = 0$ ,  $r = 16.54263$ , a change of  $-0.01511$ , and at  $q = 1.0$   $r = 16.57282$ , a change of  $+0.01508$ . The value of  $\epsilon^{-3.5/2}$  will be found to be 0.01510, and over the entire range of  $q$  from 0 to 1.0, the change in  $r$  follows in nearly simple proportion.

Beyond  $x = 3.2$ , the limit of Chart X-XIB, the values of  $r$  can be obtained by the above rule applied to Table X and with the aid of Chart VII-VIIIc. The values of the amplitude  $\gamma$  beyond  $x = 3$  closely approximate to  $q$  quadrants. That is  $\sinh(x + iq)$  approximates to  $\frac{\epsilon^x}{2} / q$ , with  $q$  in quadrant measure.

TABLE XI

$$\cosh(x + iq) = r / \gamma$$

#### POLAR HYPERBOLIC COSINES OF A RECTANGULAR VARIABLE

Table XI corresponds completely with Table VIII, except that it gives results expressed in polar instead of rectangular coördinates. It is entered with  $(x + iq)$  just as in Table VIII.

## EXPLANATORY TEXT

Interpolation may be effected by simple proportion, as in the case of Table X, or when a higher degree of precision is required, it may be carried on by Taylor's theorem. In the latter case, it is more convenient to refer to the corresponding entries in Table VIII, interpolating according to formula (71b). The rectangular coördinates duly interpolated are then transformed into polar coördinates, as in the last example on page 225.

### INTERPOLATION BY CHARTS X-XIA AND X-XIB

When Charts X-XI are used to find  $\cosh(x + iy)$ , the imaginary is first quadranted by dividing with  $\pi/2$ , so as to obtain the entering variable in the form  $\cosh(x + iq)$ . Starting then from  $q = 0$  at the horizontal line CC, near the middle of the chart, the underscored figures correspond to  $q$  for a little more than the first quadrant. The manifest repetition of the curves enables the lower half of the sheet, however, to be used for the second quadrant. The result is read off on the rectangular background to the right-hand scale of argument.

Beyond  $x = 3.2$ , the limit of Chart X-XIB reference may be had to Chart VII-VIIIb; or the approximate formula may be used:

$$\cosh(x + iq) = \frac{e^x}{2} / q. \quad (93)$$

the argument  $q$  of the result being interpreted in quadrant measure and converted into degrees.

### TABLE XII

$$\tanh(x + iq) = r / \gamma$$

#### POLAR HYPERBOLIC TANGENTS OF A RECTANGULAR VARIABLE

Table XII corresponds completely with Table IX, except that it gives results expressed in polar instead of rectangular coördinates.

If we desire to find  $\tanh(x + iy)$ , we must first divide  $y$  by  $\pi/2$  so as to obtain the entering quantity in the form  $(x + iq)$ . Multiples of 2 are then rejected in  $q$  leaving a remainder less than 2. With this remainder the table is entered.

Interpolation may be made by simple proportion to a moderate degree of precision.

#### GRAPHIC INTERPOLATION BY MEANS OF CHARTS XIIA, B, C, D

These charts cover between them the full range of Table XII. To find  $\tanh(x + iq)$  from them with  $q$  less than 2, find the proper chart, and enter on the curvilinear coördinates keeping the underscored number for  $q$ . Read off the result on the rectilinear background.

For  $\tan(x + iy)$  and also for the effects of changes of sign, see directions in the discussion on Table IX.

To find  $\tanh^{-1}(r / \gamma)$ , enter immediately on the rectangular background of  $r$  and  $\gamma$  in the proper chart, and read off at the correct intersection the corresponding values on the curvilinear coördinates. The result will appear in terms of  $(x + iq)$ . The imaginary  $q$  must be dequadranted, or multiplied by  $\pi/2$ , in order to be expressed in terms of  $(x + iy)$ .

TABLE XIII

$$f(4 + iq) = u + iv \text{ or } r/\gamma$$

RECTANGULAR AND POLAR FUNCTIONS OF THE RECTANGULAR VARIABLE  $(4 + iq)$

In this table the hyperbolic sine, cosine and tangent of  $(4 + iq)$  are collected from  $q = 0$  to  $q = 2.0$ . The results are expressed both in rectangular coördinates  $(u + iv)$ , and in polar coördinates  $r/\gamma$ .

It will be seen that the moduli of the tangents vary between 0.99933 and 1.00067, or differ from unity by two thirds of one per mil, at most. The arguments also differ from  $0^\circ$  by less than  $0.04^\circ$ , or about  $2'.17''$  of arc.

Beyond  $x = 4$ , it is evident that the hyp. sine and cosine differ by so small a percentage, that no tabulation of these differences would ordinarily be required.

TABLE XIV

$$e^x/2 \text{ and } \log_{10}(e^x/2)$$

SEMI-EXPONENTIALS

This table enables the hyp. sine or cosine of any rectangular variable  $(x + iq)$  to be found for values of  $x$  greater than 4 and less than 10. It is shown in the preceding table that when  $x$  reaches 4, the ratio of the sine to the cosine never differs from unity by more than two-thirds of 1 per mil. This deviation from unity rapidly diminishes as  $x$  is further increased. Consequently, the sine and cosine may each be computed from the formula.

$$\sinh(x + iq) = \cosh(x + iq) = \frac{e^x}{2} / q. \quad (94)$$

Example: Required the value of  $\sinh(8.51 + i 25.75)$ . The first step is to quadrant the imaginary by dividing with  $\pi/2$ , as on page 191. This gives the required function in the form  $\sinh(8.51 + i 16.393)$ . Rejecting multiples of 4.0 in  $q$ , we may then write it  $\sinh(8.51 + i 0.393)$ . Turning to the top of page 143, we find  $e^x/2 = 2482.082$  for  $x = 8.51$ ; so that the result is  $2482.082 / 0.393$  quadrant. Expressing the argument in degrees by multiplying with 90 and we have  $0.393 \times 90 = 35.37^\circ$ . Thus

$$\sinh(8.51 + i 16.393) = \cosh(8.51 + i 16.393) = 2482.082 / 35.37.$$

#### INTERPOLATION IN $x$

$$\text{Since } \frac{e^{x+\Delta x}}{2} = \frac{e^x}{2} e^{\Delta x} = \frac{e^x}{2} \left\{ 1 + \Delta x + \frac{(\Delta x)^2}{2!} + \frac{(\Delta x)^3}{3!} + \dots \right\} \quad (95)$$

it follows that when  $\Delta x$  is a small quantity, it suffices to multiply the tabular value of  $e^x/2$  by  $(1 + \Delta x)$  in order to arrive at the interpolated result unless  $(\Delta x)^2/2!$  the second correction term, is of sufficient magnitude to need consideration.



## EXPLANATORY TEXT

Example: To find  $\sinh (8.51 + iq)$ , having given that  $\sinh (8.50 + iq) = 2457.383 /q$ . Here  $\Delta x = 0.01$ .

|                   |                                    |                                 |
|-------------------|------------------------------------|---------------------------------|
| $2457.383 \times$ | $1 = 1$                            | $2457.383$                      |
| $2457.383 \times$ | $\Delta x = 0.01$                  | $24.574$                        |
| $2457.383 \times$ | $\frac{(\Delta x)^2}{2} = 0.00005$ | $.123$                          |
|                   |                                    | $2482.080$                      |
| Result            |                                    | $2482.080 /q$                   |
| Tabulated value   |                                    | <u><math>2482.082 /q</math></u> |

### TABLE XV

$$f(x + i0)$$

#### REAL HYPERBOLIC FUNCTIONS

This is a short table of real, as distinguished from complex hyperbolic functions for convenience of reference. It was prepared and published by the author in 1903 in relation to continuous-current electric circuit applications, taking the sines, cosines, and tangents from Ligowski's tables, and adding the corresponding computed reciprocals for the cosecants, secants, and cotangents. Much more extensive tables of real hyperbolic functions are, however, available. See Bibliography, page 211.

### TABLE XVI

#### SUBDIVISIONS OF A DEGREE

This is a short table for convenience in changing the expression of a circular angle from decimals of a degree to minutes and seconds, or inversely. By its aid, three-decimal subdivisions of a degree may be converted into minutes and seconds of arc, by direct inspection; or minutes and seconds may be read off as decimals of a degree to three-digit accuracy.

#### METHODS EMPLOYED IN COMPUTATION

Tables I to V, inclusive, were computed as one group, and Tables VII to XIII, inclusive, as a separate group.

Tables I to V were computed, at first, by using the formulas:—

$$\sinh(x + iy) = \sqrt{\sinh^2 x + \sin^2 y} / \tan^{-1}(\tan y / \tanh x) = r_1 / \gamma_1. \quad (96)$$

$$\cosh(x + iy) = \sqrt{\cosh^2 x - \sin^2 y} / \tan^{-1}(\tan y \cdot \tanh x) = r_2 / \gamma_2. \quad (97)$$

$$\tanh(x + iy) = (r_1/r_2) / \gamma_1 - \gamma_2. \quad (98)$$

At a later stage of the work, the following formulas, kindly suggested by Professor Bouton, were substituted:—

$$\sinh(x + iy) = \sqrt{\cosh 2x} \cdot \sin z / \tan^{-1}(\tan y / \tanh x) = r_1 / \gamma_1. \quad (99)$$

$$\cosh(x + iy) = \sqrt{\cosh 2x} \cdot \cos z / \tan^{-1}(\tan y \cdot \tanh x) = r_2 / \gamma_2. \quad (100)$$

$$\tanh(x + iy) = \tan z / \gamma_1 - \gamma_2. \quad (101)$$

Where the auxiliary circular angle  $z$  is defined by:

$$\frac{\cos 2y}{\cosh 2x} = \cos 2z. \quad (102)$$

The arithmetical work was conducted with the aid of five-place logarithms, and was checked by tabulating successive first and second differences in the tabulated results.

Tables VII to XII were computed by means of the following formulas:—

$$\sinh (x + iy) = \sinh x \cos y + i \cosh x \sin y. \quad (103)$$

$$\cosh (x + iy) = \cosh x \cos y + i \sinh x \sin y. \quad (104)$$

$$\tanh (x + iy) = \frac{\sinh 2x}{\cosh 2x + \cos 2y} + i \frac{\sin 2y}{\cosh 2x + \cos 2y}. \quad (105)$$

A standard schedule was prepared and seven-place logarithms used in the computation. The value of  $\tanh (x + iy)$  was arrived at in two ways, first by dividing (103) by (104), and second by the independent formula (105). If these two methods did not give identical results for  $\tanh (x + iy)$  to five decimal places, when expressed both in rectangular and polar coördinates, the steps of the computation were gone over afresh.\* Complete agreement being secured, leads to the inference that the values of  $\sinh$ ,  $\cosh$ , and  $\tanh (x + iy)$  are correct, at least as far as their logarithms.

Finally, all of the tables have been reduced to graphic form in the Atlas, each entry of the tables being marked off on its proper chart with a sharp needle, and the ruling pen drawn through the successive punctures. In this process a certain number of errors were discovered and rectified. The tables were then set up in type from the MSS. used in making the charts, and were proofread three times. By this procedure it is hoped that the outstanding errors are neither large nor numerous.

#### BIBLIOGRAPHY AND APPLICATIONS OF HYPERBOLIC FUNCTIONS

Hyperbolic functions of a real variable are employed extensively in mathematics generally. In particular, they are used in the solution of cubic equations.

In navigation, real hyperbolic functions enter in connection with Mercator sailing.

In cartography, real hyperbolic functions are used in preparing maps on certain projections, especially on Mercator's projection, which appears to have been the first application of hyperbolic functions.

In statics, real hyperbolic functions naturally present themselves in relation to the properties of the catenary and of the funicular polygon; also in the discussion of the forms and stresses of elastic bodies.

In dynamics, the same functions present themselves in the theory of vibrations, and in the motion of bodies through a resisting medium.

\* The author desires to express his acknowledgement of the care and painstaking effort of his assistants engaged in computation, namely,

Miss Ethel Smith, A.B. Radcliffe, 1911.

Miss A. F. Daniell, A.B. Radcliffe, 1911.

Miss Mary M. Devlin, A.B. Radcliffe, 1912.

Miss Hope M. Hearn, A.B. Radcliffe, 1912.

## EXPLANATORY TEXT

A good summary of the historical development of real hyperbolic functions is given in Becker and van Orstrand's "Hyperbolic Functions," Smithsonian Mathematical Tables, 1909, together with a fine compendium of formulas involving these functions.\*

In electrical engineering, the earliest published application of real hyperbolic functions is perhaps in T. H. Blakesley's "Alternating Currents of Electricity," London, 1889, which also appends a short table of these (real) functions. The real functions were also introduced by Sir J. J. Thomson, in "The Electrician," Vol. XXVIII, page 599, 1891. "On the Heat Produced by Eddy Currents in an Iron Plate Exposed to an Alternating Magnetic Field."

The fundamental differential equation of the alternating potential-current, steady-state distribution along a uniform conductor, involving hyperbolic functions, nominally real, seems to have been first published by O. Heaviside in 1893, "Electromagnetic Theory," Vol. I, page 450.

The first published application of complex hyperbolic functions to the last-named problem was by the author, "On the Fall of Pressure in Long-Distance Alternating-Current Conductors," *Electrical World*, N. Y., Vol. XXIII, page 17, January, 1894, and "The Electrician," London (abstract), Vol. XXXII, page 239, January 5, 1894.

Complex hyperbolic functions also present themselves in the discussion of Hertzian-wave reflections, and in other branches of electrical engineering. They naturally enter the subject of confocal ellipses and hyperbolas, such as Captain Weir's Azimuth diagram of these confocals, for indicating the azimuth of a celestial object in terms of the hour-angle, latitude and declination. (Godfray's "Astronomy," § 222.)

The mathematical discussion of hyperbolic functions is found in Greenhill's "Differential and Integral Calculus," Macmillan and Co., 1896; Ligowski's "Tafeln der Hyperbelfunctionen und der Kreisfunctionen," Berlin, Ernst & Korn, 1890; McMahon's "Hyperbolic Functions," Wiley and Sons, N. Y., 1896; Becker and van Orstrand's "Hyperbolic Functions," Smithsonian Institution, 1909; Vassall's "Nouvelles Tables des Logarithmes," Paris, Gauthier-Villars, 1872; as well as other text-books.

Works dealing with the applications of hyperbolic functions to electrical engineering are: "The Application of Hyperbolic Functions to Electrical Engineering Problems," by the author, The University of London Press, 1911, and Fleming's "The Propagation of Electrical Currents in Telephone and Telegraph Conductors," Constable & Co., London, 1911.

A three-dimensional complex-angle geometrical model,† from which the hyperbolic sines and cosines of complex angles can be presented projectively, has been constructed and described.

### BRIEF BIBLIOGRAPHY OF TABLES OF HYPERBOLIC FUNCTIONS

(1) "Tafeln der Hyperbelfunctionen und der Kreisfunctionen," by Dr. W. Ligowski, Berlin, 1890, Ernst and Korn, 104 pages, giving five-figure logarithms of  $\sinh \theta$ ,  $\cosh \theta$ , and  $\tanh \theta$  up to  $\theta = 9$ , by steps of 0.001 up to  $\theta = 2$  and from 2.0 to 9.0 by steps of

\* This compendium has, by permission, been included in this book at its second edition, as Table XXIII.

† "A new geometrical model for the orthogonal projection of the cosines and sines of complex angles" by A. E. Kennelly, Proc. Am. Ac. of Arts and Sciences, Vol. 54, April, 1919, pp. 371-378.

0.01; also the Gudermannian angle to two or more decimals of a second of arc, and other tables.

(2) Smithsonian Mathematical Tables, "Hyperbolic Functions," by George F. Becker, and C. E. van Orstrand, Smithsonian Institute, Washington, D.C., 1909, 321 pages, giving five-figure logarithms of  $\sinh \theta$ ,  $\cosh \theta$ , and  $\tanh \theta$ , by steps of 0.0001 up to 0.1, by steps of 0.001 from 0.1 to 3.0, and by steps of 0.01 from 3.0 to 6.0; also similar five-figure tables of natural real hyperbolic functions, and various other tables.

(3) "Alternating-Current Phenomena in Parallel Conductors," Vol. I by F. E. Pernot, John Wiley, New York, 1918, containing a Table of six-decimal logarithms of hyperbolic functions, up to 2.0 by steps of 0.001. These present a higher order of precision by one unit, than have been previously available for real hyperbolic functions.

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The following is a list of all the tables of Complex Hyperbolic Functions known to the present writer, in the order of date of publication:—

(4) Chrystal's "Algebra," Edinburgh, 1889, briefly discusses the theory of  $\sinh \theta$ ,  $\cosh \theta$ , and  $\tanh \theta$  where  $\theta$  is complex; or of the form  $x + iy$ . Graphs are given in outline for these functions, from which a few numerical values may be read.

(5) The paper on "Resonance in Alternating-Current Lines," by E. J. Houston and A. E. Kennelly, *Transactions A. I. E. E.*, April, 1895, Vol. XII, pages 133-169, contains a Plate for the graphical evaluation of  $\sinh \theta$  and of  $\cosh \theta$ ,  $\theta$  being a complex variable  $x + iq$ , between the limits of  $x = 0$  and  $x = 1.25$ ;  $q = 0$  and  $q = \alpha$ , by steps of 0.05 in  $x$  and  $q$ . The Plate is 40 cm.  $\times$  34 cm. and corresponds to Plates VII—VIII A of the Atlas prepared from tables in this book, except that it gives the result in regular polar coördinates instead of regular rectangular coördinates. It was produced, by a graphical process, for a precision of the 2.5th order.

(6) The first tables of complex hyperbolic functions were a short set published by Dr. James McMahon in his Chapter IV, entitled "Hyperbolic Functions," of a book by Merriam and Woodward on "Higher Mathematics," pages 107-168. The tables gave  $\sinh (x + iy)$  and  $\cosh (x + iy)$  from  $x = 0$  to  $x = 1.5$ , by steps of 0.1, and also from  $y = 0$  to  $y = 1.5$ , by steps of 0.1, Wiley & Sons, New York, 1896. The chapter has since been issued as a separate volume by the same publishers.

(7) A table of hyperbolic functions of semi-imaginaries or  $\sinh$ ,  $\cosh$ ,  $\tanh$ ,  $\coth$ ,  $\operatorname{sech}$  and  $\operatorname{cosech}$  of  $x/45^\circ$ , by steps of 0.1 in  $x$  up to  $x = 20.5$ , was published by the present writer in a paper on "The Alternating-Current Theory of Transmission Speed over Submarine Telegraph Cables," in the *Proceedings of The International Electrical Congress of St. Louis*, Section A, Vol. I, pages 68-105, 1904. This table is reproduced in Table VI of this volume.

(8) Some short tables of  $\sinh$ ,  $\cosh$ ,  $\tanh$ ,  $\coth$ ,  $\operatorname{sech}$ , and  $\operatorname{cosech} \rho/\delta$  by steps of 0.1 in  $\rho$ , up to  $\rho = 1.5$ , for five particular values of  $\delta$ , published by the present writer in an article on "The Distribution of Pressure and Current over Alternating-Current Circuits," in the *Harvard Engineering Journal*, 1905-06.

## EXPLANATORY TEXT

(9) Short three-digit tables of  $\sinh$  and  $\cosh (x + iy)$  up to  $x = 1$ , and  $y = 1$ , by W. E. Miller, in a paper "Formulae, Constants, and Hyperbolic Functions for Transmission-Line Problems" in the *General Electrical Review*, Schenectady, N. Y., May, 1910. Supplement.

(10) "Tables of Hyperbolic Functions in Reference to Long Alternating-Current Transmission Lines," published by the present writer in the *Transactions of the American Institute of Electrical Engineers*, December 1911, pages 2495-2506. These give  $\sinh$ ,  $\cosh$ , and  $\tanh \rho/\delta$  from  $\rho = 0$  to  $\rho = 0.5$ , by steps of 0.1, and from  $\delta = 60^\circ$  to  $\delta = 90^\circ$  by steps of  $1^\circ$ . These tables are incorporated in Tables I, II, and III of this volume.

(11) "Tables of Sines, Cosines, Tangents, Cosecants, Secants, and Cotangents of Real and Complex Hyperbolic Angles," published by the present writer in the *Harvard Engineering Journal*, 1912. These gave  $\sinh$ ,  $\cosh$ , and  $\tanh \rho/\delta$  from  $\rho = 0$  to  $\rho = 1$  by steps of 0.1, and from  $\delta = 45^\circ$  to  $\delta = 90^\circ$  by steps of  $1^\circ$ ; also corresponding tables of  $(\sinh \theta)/\theta$  and of  $(\tanh \theta)/\theta$ . These tables are published in separate form by the *Harvard Engineering Journal*. They are incorporated in tables I, II, III, IV, and V of this volume.

### NEW TABLES INTRODUCED IN THE SECOND EDITION

Tables I to V in this volume were computed for the range of  $45^\circ$  to  $90^\circ$  in the slope or argument  $\delta$  of the entering vector quantity; because at that time it did not appear that there would be any need for the range from  $0^\circ$  to  $45^\circ$ . Alternating-current lines used for the transmission or distribution of power have linear hyperbolic angles  $\alpha$ , the slope of which is commonly between  $80^\circ$  and  $90^\circ$ , rarely falling as low as  $45^\circ$ . It has been shown during recent years, however, that railway-signal engineers employ track-signaling circuits, formed of the rails. These are metallic circuits of low frequency, small linear capacitance and large distributed linear leakance. The linear hyperbolic angles  $\alpha$  of such circuits develop slopes lying within the range  $\delta = 0^\circ$  to  $45^\circ$ . It has therefore become desirable to cover this range, at least as far as  $\rho = 1$ . For that purpose, Tables XVII to XXI have been inserted. They run by steps of 0.05 in  $\rho$ , from 0 to 1.0, and by steps of  $5^\circ$  in  $\delta$ , from  $0^\circ$  to  $45^\circ$ . This new tabulated material is available for use in track-signaling and similar computations. It is hoped to incorporate it graphically into the associated Chart Atlas at the first opportunity.

Table XVII presents  $\sinh \rho/\delta$  as a polar planevector. It corresponds to and may be regarded as an extension of Table I. Similarly Tables XVIII, XIX, XX and XXI correspond respectively to Tables II, III, IV and V. Whereas, however, Tables I to V are carried to five decimal places in the sizes and three decimal places in the slopes of the evaluated quantities, the new tables are carried to six decimal places in sizes and four decimal places in slopes. They thus aim at one higher order of precision.\*

Table XXII is similar to Table XI and expands the region covered by the first six entries of the latter into a correspondingly magnified field. It has been found that in

\* The author desires to express his acknowledgment of the painstaking assistance, on these tables, by Miss Lillian L. Hodgdon, of the Harvard Observatory computing staff.

## EXPLANATORY TEXT

dealing with short lengths of alternating-current line, having negligible linear inductance as well as negligible linear leakance, and therefore having a semi-imaginary linear angle, there is frequent need for a magnified table of this kind. It may be noted that whereas Table VI expresses slopes in degrees and minutes, Table XXII expresses them as degrees, and four-place decimals of a degree.

Table XXIII is a useful collection of 238 formulas, with a few insertions, taken from Becker and van Orstrand's book of Tables of real hyperbolic functions, referred to in the footnote on page 210.

TABLE XVII. HYPERBOLIC SINES.  $\sinh(\rho/\delta) = r/\gamma$ .

|    | 0        |         | 0.05     |         | 0.10     |         |
|----|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       |
| 0  | 0.000000 | 0.0000  | 0.050021 | 0.0000  | 0.100167 | 0.0000  |
| 5  | 0.000000 | 5.0000  | 0.050020 | 5.0042  | 0.100164 | 5.0167  |
| 10 | 0.000000 | 10.0000 | 0.050020 | 10.0081 | 0.100157 | 10.0325 |
| 15 | 0.000000 | 15.0000 | 0.050019 | 15.0119 | 0.100144 | 15.0478 |
| 20 | 0.000000 | 20.0000 | 0.050016 | 20.0153 | 0.100128 | 20.0614 |
| 25 | 0.000000 | 25.0000 | 0.050014 | 25.0183 | 0.100107 | 25.0730 |
| 30 | 0.000000 | 30.0000 | 0.050011 | 30.0208 | 0.100083 | 30.0828 |
| 35 | 0.000000 | 35.0000 | 0.050006 | 35.0222 | 0.100057 | 35.0897 |
| 40 | 0.000000 | 40.0000 | 0.050003 | 40.0236 | 0.100029 | 40.0942 |
| 45 | 0.000000 | 45.0000 | 0.050000 | 45.0236 | 0.100000 | 45.0952 |

|    | 0.35     |         | 0.40     |         | 0.45     |         |
|----|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       |
| 0  | 0.357190 | 0.0000  | 0.410752 | 0.0000  | 0.465342 | 0.0000  |
| 5  | 0.357081 | 5.2014  | 0.410589 | 5.2625  | 0.465109 | 5.3314  |
| 10 | 0.356757 | 10.3969 | 0.410105 | 10.5172 | 0.464420 | 10.6531 |
| 15 | 0.356228 | 15.5808 | 0.409316 | 15.7569 | 0.463294 | 15.9558 |
| 20 | 0.355512 | 20.7472 | 0.408245 | 20.9742 | 0.461767 | 21.2303 |
| 25 | 0.354628 | 25.8908 | 0.406925 | 26.1625 | 0.459886 | 26.4686 |
| 30 | 0.353606 | 31.0089 | 0.405397 | 31.3161 | 0.457709 | 31.6630 |
| 35 | 0.352475 | 36.0961 | 0.403709 | 36.4305 | 0.455393 | 36.8086 |
| 40 | 0.351270 | 41.1503 | 0.401910 | 41.5017 | 0.452741 | 41.8997 |
| 45 | 0.35003  | 46.1694 | 0.40006  | 46.5278 | 0.45010  | 46.9338 |

|    | 0.70     |         | 0.75     |         | 0.80     |         |
|----|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       |
| 0  | 0.758584 | 0.0000  | 0.822317 | 0.0000  | 0.888106 | 0.0000  |
| 5  | 0.757700 | 5.7875  | 0.821227 | 5.9000  | 0.886780 | 6.0192  |
| 10 | 0.755078 | 11.5533 | 0.817993 | 11.7753 | 0.882843 | 12.0108 |
| 15 | 0.750800 | 17.2755 | 0.812719 | 17.6019 | 0.876425 | 17.9481 |
| 20 | 0.745004 | 22.9344 | 0.805576 | 23.3567 | 0.867734 | 23.8044 |
| 25 | 0.737873 | 28.5103 | 0.796787 | 29.0175 | 0.857048 | 29.5561 |
| 30 | 0.729632 | 33.9861 | 0.786636 | 34.5647 | 0.844710 | 35.1800 |
| 35 | 0.720538 | 39.3467 | 0.775439 | 39.9811 | 0.831105 | 40.6567 |
| 40 | 0.710872 | 44.5800 | 0.763544 | 45.2525 | 0.816660 | 45.9697 |
| 45 | 0.700934 | 49.6767 | 0.751317 | 50.3678 | 0.801819 | 51.1064 |

Examples.  $\sinh(0.35/35^\circ) = 0.352475/36^\circ.0961$ . $\sinh(0.80/5^\circ) = 0.886780/6^\circ.0192$ .

TABLE XVII. HYPERBOLIC SINES.  $\sinh(\rho/\delta) = r/\gamma$ . CONTINUED

|    | 0.15     |         | 0.20     |         | 0.25     |         | 0.30     |         |
|----|----------|---------|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       | °        | °       |
| 0  | 0.150563 | 0.0000  | 0.201336 | 0.0000  | 0.252612 | 0.0000  | 0.304520 | 0.0000  |
| 5  | 0.150554 | 5.0372  | 0.201316 | 5.0661  | 0.252573 | 5.1033  | 0.304452 | 5.1483  |
| 10 | 0.150529 | 10.0733 | 0.201256 | 10.1303 | 0.252455 | 10.2033 | 0.304248 | 10.2922 |
| 15 | 0.150488 | 15.0172 | 0.201157 | 15.1906 | 0.252263 | 15.2972 | 0.303915 | 15.4275 |
| 20 | 0.150432 | 20.1380 | 0.201024 | 20.2450 | 0.252002 | 20.3825 | 0.303465 | 20.5500 |
| 25 | 0.150362 | 25.1644 | 0.200859 | 25.2919 | 0.251680 | 25.4561 | 0.302909 | 25.6558 |
| 30 | 0.150282 | 30.1858 | 0.200660 | 30.3303 | 0.251308 | 30.5158 | 0.302265 | 30.7419 |
| 35 | 0.150193 | 35.2017 | 0.200458 | 35.3586 | 0.250896 | 35.5600 | 0.301553 | 35.8058 |
| 40 | 0.150098 | 40.2114 | 0.200233 | 40.3761 | 0.250457 | 40.5872 | 0.300795 | 40.8456 |
| 45 | 0.150000 | 45.2146 | 0.200000 | 45.3817 | 0.250001 | 45.5965 | 0.300001 | 45.8592 |

|    | 0.50     |         | 0.55     |         | 0.60     |         | 0.65     |         |
|----|----------|---------|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       | °        | °       |
| 0  | 0.521095 | 0.0000  | 0.578152 | 0.0000  | 0.636654 | 0.0000  | 0.696748 | 0.0000  |
| 5  | 0.520776 | 5.4078  | 0.577726 | 5.4919  | 0.636100 | 5.5833  | 0.696042 | 5.6819  |
| 10 | 0.519829 | 10.8039 | 0.576463 | 10.9697 | 0.634456 | 11.1500 | 0.693947 | 11.3477 |
| 15 | 0.518282 | 16.1767 | 0.574400 | 16.4197 | 0.631774 | 16.6842 | 0.690530 | 16.9694 |
| 20 | 0.516184 | 21.5153 | 0.571604 | 21.8286 | 0.628138 | 22.1700 | 0.685898 | 22.5386 |
| 25 | 0.513601 | 26.8094 | 0.568162 | 27.1844 | 0.623662 | 27.5933 | 0.680199 | 28.0355 |
| 30 | 0.510612 | 32.0503 | 0.564179 | 32.4764 | 0.618485 | 32.9414 | 0.673609 | 33.4450 |
| 35 | 0.507309 | 37.2305 | 0.559779 | 37.6956 | 0.612769 | 38.2036 | 0.666335 | 38.7539 |
| 40 | 0.503794 | 42.3439 | 0.555099 | 42.8344 | 0.606689 | 43.3705 | 0.658601 | 43.9525 |
| 45 | 0.500017 | 47.3872 | 0.550279 | 47.8880 | 0.600432 | 48.4369 | 0.650644 | 49.0330 |

|    | 0.85     |         | 0.90     |         | 0.95     |         | 1.00     |         |
|----|----------|---------|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       | °        | °       |
| 0  | 0.956116 | 0.0000  | 1.026517 | 0.0000  | 1.099484 | 0.0000  | 1.175201 | 0.0000  |
| 5  | 0.954520 | 6.1447  | 1.024615 | 6.2767  | 1.097239 | 6.4147  | 1.172573 | 6.5592  |
| 10 | 0.949784 | 12.2592 | 1.018975 | 12.5200 | 1.090583 | 12.7928 | 1.164779 | 13.0775 |
| 15 | 0.942064 | 18.3131 | 1.009783 | 18.6967 | 1.079736 | 19.0986 | 1.152083 | 19.5180 |
| 20 | 0.931612 | 24.2775 | 0.997344 | 24.7753 | 1.065062 | 25.2969 | 1.134913 | 25.8417 |
| 25 | 0.918768 | 30.1255 | 0.982060 | 30.7253 | 1.047043 | 31.3544 | 1.113841 | 32.0122 |
| 30 | 0.903942 | 35.8314 | 0.964429 | 36.5183 | 1.026266 | 37.2400 | 1.089558 | 37.9955 |
| 35 | 0.887604 | 41.3730 | 0.945011 | 42.1294 | 1.003398 | 42.9253 | 1.062847 | 43.7603 |
| 40 | 0.870267 | 46.7314 | 0.924414 | 47.5372 | 0.979156 | 48.3864 | 1.034550 | 49.2789 |
| 45 | 0.852463 | 51.8917 | 0.903276 | 52.7242 | 0.954292 | 53.6033 | 1.005545 | 54.5292 |

Examples.  $\sinh(0.90/20^\circ) = 0.997344/24.7753$ .  
 $\sinh(1.0/0^\circ) = 1.175201/0^\circ$ .



TABLE XVIII. HYPERBOLIC COSINES.  $\cosh(\rho/\delta) = r/\gamma$ .

|    | 0        |        | 0.05     |        | 0.10     |        |
|----|----------|--------|----------|--------|----------|--------|
| °  | °        | °      | °        | °      | °        | °      |
| 0  | 1.000000 | 0.0000 | 1.001250 | 0.0000 | 1.005004 | 0.0000 |
| 5  | 1.000000 | 0.0000 | 1.001231 | 0.0125 | 1.004929 | 0.0497 |
| 10 | 1.000000 | 0.0000 | 1.001175 | 0.0244 | 1.004703 | 0.0978 |
| 15 | 1.000000 | 0.0000 | 1.001083 | 0.0358 | 1.004335 | 0.1428 |
| 20 | 1.000000 | 0.0000 | 1.000958 | 0.0461 | 1.003836 | 0.1836 |
| 25 | 1.000000 | 0.0000 | 1.000804 | 0.0547 | 1.003221 | 0.2189 |
| 30 | 1.000000 | 0.0000 | 1.000626 | 0.0619 | 1.002507 | 0.2478 |
| 35 | 1.000000 | 0.0000 | 1.000428 | 0.0672 | 1.001718 | 0.2689 |
| 40 | 1.000000 | 0.0000 | 1.000218 | 0.0705 | 1.000876 | 0.2820 |
| 45 | 1.000000 | 0.0000 | 1.000000 | 0.0716 | 1.000001 | 0.2864 |

|    | 0.35     |        | 0.40     |        | 0.45     |        |
|----|----------|--------|----------|--------|----------|--------|
| °  | °        | °      | °        | °      | °        | °      |
| 0  | 1.061878 | 0.0000 | 1.081072 | 0.0000 | 1.102970 | 0.0000 |
| 5  | 1.060965 | 0.5861 | 1.079886 | 0.7567 | 1.101477 | 0.9453 |
| 10 | 1.058252 | 1.1561 | 1.076361 | 1.4933 | 1.097041 | 1.8669 |
| 15 | 1.053819 | 1.6950 | 1.070598 | 2.1911 | 1.089786 | 2.7411 |
| 20 | 1.047791 | 2.1872 | 1.062762 | 2.8303 | 1.079919 | 3.5450 |
| 25 | 1.040347 | 2.6189 | 1.053079 | 3.3936 | 1.067721 | 4.2567 |
| 30 | 1.031702 | 2.9772 | 1.041829 | 3.8639 | 1.053539 | 4.8550 |
| 35 | 1.022112 | 3.2506 | 1.029339 | 4.2264 | 1.037782 | 5.3208 |
| 40 | 1.011859 | 3.4295 | 1.015974 | 4.4678 | 1.020908 | 5.6375 |
| 45 | 1.00125  | 3.5071 | 1.00213  | 4.5784 | 1.00341  | 5.7906 |

|    | 0.70     |         | 0.75     |         | 0.80     |         |
|----|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       |
| 0  | 1.255169 | 0.0000  | 1.294683 | 0.0000  | 1.337435 | 0.0000  |
| 5  | 1.251669 | 2.1083  | 1.290690 | 2.3744  | 1.332918 | 2.6483  |
| 10 | 1.241264 | 4.1750  | 1.278820 | 4.7044  | 1.319493 | 5.2500  |
| 15 | 1.224242 | 6.1581  | 1.259402 | 6.9456  | 1.297535 | 7.7578  |
| 20 | 1.201069 | 8.0155  | 1.232970 | 9.0520  | 1.267648 | 10.1236 |
| 25 | 1.172378 | 9.7039  | 1.200241 | 10.9775 | 1.230648 | 12.2978 |
| 30 | 1.138949 | 11.1803 | 1.162104 | 12.6747 | 1.187536 | 14.2294 |
| 35 | 1.101690 | 12.4000 | 1.119583 | 14.0942 | 1.139459 | 15.8644 |
| 40 | 1.061615 | 13.3186 | 1.073820 | 15.1850 | 1.087691 | 17.1458 |
| 45 | 1.019823 | 13.8911 | 1.026048 | 15.8947 | 1.033602 | 18.0136 |

Examples.  $\cosh(0.10/25^\circ) = 1.003221/0.2189$ . $\cosh(0.75/40^\circ) = 1.073820/15.1850$ .

TABLE XVIII. HYPERBOLIC COSINES.  $\cosh (\rho/\delta) = r/\gamma$ . CONTINUED

| 0.15 |          | 0.20    |          | 0.25    |          | 0.30    |          |         |
|------|----------|---------|----------|---------|----------|---------|----------|---------|
| °    | °        | °       | °        | °       | °        | °       | °        |         |
| 0    | 1.011271 | 0.0000  | 1.020067 | 0.0000  | 1.031413 | 0.0000  | 1.045339 | 0.0000  |
| 5    | 1.011101 | 0.1111  | 1.019765 | 0.1964  | 1.030943 | 0.3047  | 1.044665 | 0.4350  |
| 10   | 1.010595 | 0.2189  | 1.018868 | 0.3872  | 1.029547 | 0.6006  | 1.042663 | 0.8578  |
| 15   | 1.009769 | 0.3203  | 1.017404 | 0.5664  | 1.027265 | 0.8794  | 1.039390 | 1.2567  |
| 20   | 1.008648 | 0.4119  | 1.015415 | 0.7292  | 1.024167 | 1.1328  | 1.034944 | 1.6200  |
| 25   | 1.007265 | 0.4914  | 1.012961 | 0.8703  | 1.020343 | 1.3533  | 1.029455 | 1.9375  |
| 30   | 1.005662 | 0.5561  | 1.010116 | 0.9858  | 1.015908 | 1.5344  | 1.023084 | 2.1994  |
| 35   | 1.003887 | 0.6042  | 1.006966 | 1.0719  | 1.010992 | 1.6703  | 1.016021 | 2.3975  |
| 40   | 1.001995 | 0.6339  | 1.003604 | 1.1258  | 1.005746 | 1.7567  | 1.008476 | 2.5253  |
| 45   | 1.00005  | 0.6445  | 1.00013  | 1.1458  | 1.00033  | 1.7901  | 1.00068  | 2.5773  |
|      |          |         |          |         |          |         |          |         |
| 0.50 |          | 0.55    |          | 0.60    |          | 0.65    |          |         |
| °    | °        | °       | °        | °       | °        | °       | °        |         |
| 0    | 1.127626 | 0.0000  | 1.155101 | 0.0000  | 1.185465 | 0.0000  | 1.218793 | 0.0000  |
| 5    | 1.125794 | 1.1506  | 1.152899 | 1.3711  | 1.182861 | 1.6053  | 1.215756 | 1.8514  |
| 10   | 1.120349 | 2.2736  | 1.146351 | 2.7108  | 1.175118 | 3.1753  | 1.206728 | 3.6644  |
| 15   | 1.111445 | 3.3411  | 1.135641 | 3.9872  | 1.162452 | 4.6750  | 1.191956 | 5.4000  |
| 20   | 1.099330 | 4.3264  | 1.121066 | 5.1694  | 1.145211 | 6.0689  | 1.171848 | 7.0194  |
| 25   | 1.084345 | 5.2028  | 1.103034 | 6.2267  | 1.123873 | 7.3222  | 1.146955 | 8.4836  |
| 30   | 1.066914 | 5.9453  | 1.082045 | 7.1292  | 1.099026 | 8.4006  | 1.117959 | 9.7533  |
| 35   | 1.047534 | 6.5297  | 1.058692 | 7.8481  | 1.071363 | 9.2700  | 1.085656 | 10.7894 |
| 40   | 1.026760 | 6.9350  | 1.033637 | 8.3566  | 1.041655 | 9.8980  | 1.050938 | 11.5539 |
| 45   | 1.00520  | 7.1424  | 1.007598 | 8.6311  | 1.010745 | 10.2544 | 1.014773 | 12.0094 |
|      |          |         |          |         |          |         |          |         |
| 0.85 |          | 0.90    |          | 0.95    |          | 1.00    |          |         |
| °    | °        | °       | °        | °       | °        | °       | °        |         |
| 0    | 1.383531 | 0.0000  | 1.433086 | 0.0000  | 1.486225 | 0.0000  | 1.543081 | 0.0000  |
| 5    | 1.378460 | 2.9289  | 1.427430 | 3.2150  | 1.479951 | 3.5053  | 1.536155 | 3.7989  |
| 10   | 1.363390 | 5.8092  | 1.410623 | 6.3794  | 1.461313 | 6.9589  | 1.515588 | 7.5475  |
| 15   | 1.338746 | 8.5914  | 1.383145 | 9.4428  | 1.430851 | 10.3086 | 1.481987 | 11.1858 |
| 20   | 1.305212 | 11.2253 | 1.345771 | 12.3525 | 1.389438 | 13.5005 | 1.436333 | 14.6653 |
| 25   | 1.263709 | 13.6589 | 1.299533 | 15.0545 | 1.338233 | 16.4789 | 1.379926 | 17.9267 |
| 30   | 1.215358 | 15.8375 | 1.245685 | 17.4914 | 1.278634 | 19.1844 | 1.314321 | 20.9097 |
| 35   | 1.161440 | 17.7030 | 1.185653 | 19.6019 | 1.212219 | 21.5531 | 1.241261 | 23.5483 |
| 40   | 1.103367 | 19.1936 | 1.120988 | 21.3192 | 1.140689 | 23.5142 | 1.162611 | 25.7689 |
| 45   | 1.042645 | 20.2411 | 1.053338 | 22.5686 | 1.065840 | 24.9872 | 1.080307 | 27.4870 |

Examples.  $\cosh (0.25 / 30^\circ) = 1.015908 / 1^\circ.5344$ .  
 $\cosh (1.00 / 40^\circ) = 1.162611 / 25^\circ.7689$ .

TABLE XIX. HYPERBOLIC TANGENTS.  $\tanh (\rho / \delta) = r / \gamma$ .

|    | 0        |         | 0.05     |         | 0.10     |         |
|----|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       |
| 0  | 0.000000 | 0.0000  | 0.049958 | 0.0000  | 0.099668 | 0.0000  |
| 5  | 0.000000 | 5.0000  | 0.049959 | 4.9917  | 0.099673 | 4.9670  |
| 10 | 0.000000 | 10.0000 | 0.049961 | 9.9837  | 0.099688 | 9.9347  |
| 15 | 0.000000 | 15.0000 | 0.049965 | 14.9761 | 0.099712 | 14.9050 |
| 20 | 0.000000 | 20.0000 | 0.049968 | 19.9692 | 0.099745 | 19.8778 |
| 25 | 0.000000 | 25.0000 | 0.049974 | 24.9636 | 0.099786 | 24.8541 |
| 30 | 0.000000 | 30.0000 | 0.049980 | 29.9589 | 0.099833 | 29.8350 |
| 35 | 0.000000 | 35.0000 | 0.049985 | 34.9550 | 0.099886 | 34.8208 |
| 40 | 0.000000 | 40.0000 | 0.049992 | 39.9531 | 0.099941 | 39.8122 |
| 45 | 0.000000 | 45.0000 | 0.050000 | 44.9520 | 0.100000 | 44.8087 |

|    | 0.35     |         | 0.40     |         | 0.45     |         |
|----|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       |
| 0  | 0.336375 | 0.0000  | 0.379949 | 0.0000  | 0.421899 | 0.0000  |
| 5  | 0.336562 | 4.6153  | 0.380215 | 4.5058  | 0.422260 | 4.3861  |
| 10 | 0.337119 | 9.2408  | 0.381011 | 9.0239  | 0.423339 | 8.7862  |
| 15 | 0.338036 | 13.8858 | 0.382325 | 13.5658 | 0.425124 | 13.2147 |
| 20 | 0.339296 | 18.5600 | 0.384135 | 18.1439 | 0.427594 | 17.6853 |
| 25 | 0.340875 | 23.2719 | 0.386414 | 22.7689 | 0.430717 | 22.2119 |
| 30 | 0.342740 | 28.0317 | 0.389121 | 27.4522 | 0.434449 | 26.8080 |
| 35 | 0.344849 | 32.8455 | 0.392202 | 32.2041 | 0.438727 | 31.4878 |
| 40 | 0.347153 | 37.7208 | 0.395591 | 37.0339 | 0.443469 | 36.2622 |
| 45 | 0.34959  | 42.6623 | 0.39921  | 41.9494 | 0.44857  | 41.1432 |

|    | 0.70     |         | 0.75     |         | 0.80     |         |
|----|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       |
| 0  | 0.604367 | 0.0000  | 0.635150 | 0.0000  | 0.664037 | 0.0000  |
| 5  | 0.605352 | 3.6792  | 0.636270 | 3.5256  | 0.665292 | 3.3709  |
| 10 | 0.608313 | 7.3783  | 0.639647 | 7.0709  | 0.669078 | 6.7608  |
| 15 | 0.613278 | 11.1174 | 0.645321 | 10.6563 | 0.675454 | 10.1903 |
| 20 | 0.620284 | 14.9189 | 0.653363 | 14.3047 | 0.684523 | 13.6808 |
| 25 | 0.629382 | 18.8064 | 0.663856 | 18.0400 | 0.696420 | 17.2583 |
| 30 | 0.640619 | 22.8058 | 0.676907 | 21.8900 | 0.711313 | 20.9506 |
| 35 | 0.654029 | 26.9467 | 0.692614 | 25.8869 | 0.729386 | 24.7922 |
| 40 | 0.669614 | 31.2614 | 0.711054 | 30.0675 | 0.750820 | 28.8239 |
| 45 | 0.687309 | 35.7856 | 0.732244 | 34.4731 | 0.775752 | 33.0928 |

Examples.  $\tanh (0.75 / 25^\circ) = 0.663856 / 18^\circ.0400$ . $\tanh (0.40 / 20^\circ) = 0.384135 / 18^\circ.1439$ .

TABLE XIX. HYPERBOLIC TANGENTS.  $\tanh (\rho / \delta) = r / \gamma$ . CONTINUED

|    | 0.15     |         | 0.20     |         | 0.25     |         | 0.30     |         |
|----|----------|---------|----------|---------|----------|---------|----------|---------|
| °  | °        | °       | °        | °       | °        | °       | °        | °       |
| 0  | 0.148885 | 0.0000  | 0.197375 | 0.0000  | 0.244919 | 0.0000  | 0.291312 | 0.0000  |
| 5  | 0.148901 | 4.9261  | 0.197414 | 4.8697  | 0.244992 | 4.7986  | 0.291435 | 4.7133  |
| 10 | 0.148951 | 9.8544  | 0.197529 | 9.7431  | 0.245210 | 9.6027  | 0.291799 | 9.4344  |
| 15 | 0.149032 | 14.7869 | 0.197718 | 14.6242 | 0.245567 | 14.4178 | 0.292398 | 14.1708 |
| 20 | 0.149142 | 19.7261 | 0.197972 | 19.5158 | 0.246055 | 19.2497 | 0.293219 | 18.9300 |
| 25 | 0.149277 | 24.6730 | 0.198289 | 24.4216 | 0.246663 | 24.1028 | 0.294242 | 23.7183 |
| 30 | 0.149436 | 29.6297 | 0.198659 | 29.3445 | 0.247373 | 28.9814 | 0.295445 | 28.5425 |
| 35 | 0.149611 | 34.5975 | 0.199071 | 34.2867 | 0.248168 | 33.8897 | 0.296798 | 33.4083 |
| 40 | 0.149799 | 39.5775 | 0.199514 | 39.2503 | 0.249027 | 38.8305 | 0.298267 | 38.3203 |
| 45 | 0.14999  | 44.5701 | 0.19997  | 44.2359 | 0.24993  | 43.8064 | 0.29981  | 43.2819 |
|    |          |         |          |         |          |         |          |         |
|    | 0.50     |         | 0.55     |         | 0.60     |         | 0.65     |         |
| °  | °        | °       | °        | °       | °        | °       | °        | °       |
| 0  | 0.462117 | 0.0000  | 0.500521 | 0.0000  | 0.537049 | 0.0000  | 0.571669 | 0.0000  |
| 5  | 0.462586 | 4.2572  | 0.501107 | 4.1208  | 0.537764 | 3.9780  | 0.572518 | 3.8305  |
| 10 | 0.463988 | 8.5303  | 0.502867 | 8.2859  | 0.539908 | 7.9747  | 0.575065 | 7.6803  |
| 15 | 0.466314 | 12.8356 | 0.505793 | 12.4325 | 0.543484 | 12.0092 | 0.579325 | 11.5694 |
| 20 | 0.469545 | 17.1889 | 0.509875 | 16.6592 | 0.548491 | 16.1011 | 0.585313 | 15.5192 |
| 25 | 0.473651 | 21.6066 | 0.515090 | 20.9577 | 0.554922 | 20.2711 | 0.593047 | 19.5519 |
| 30 | 0.478587 | 26.1050 | 0.521400 | 25.3472 | 0.562757 | 24.5408 | 0.602535 | 23.6917 |
| 35 | 0.484289 | 30.7008 | 0.528746 | 29.8475 | 0.571953 | 28.9336 | 0.613763 | 27.9645 |
| 40 | 0.490664 | 35.4089 | 0.537034 | 34.4778 | 0.582428 | 33.4725 | 0.626679 | 32.3986 |
| 45 | 0.49759  | 40.2448 | 0.546130 | 39.2569 | 0.594049 | 38.1825 | 0.641172 | 37.0236 |
|    |          |         |          |         |          |         |          |         |
|    | 0.85     |         | 0.90     |         | 0.95     |         | 1.00     |         |
| °  | °        | °       | °        | °       | °        | °       | °        | °       |
| 0  | 0.691070 | 0.0000  | 0.716298 | 0.0000  | 0.739782 | 0.0000  | 0.761595 | 0.0000  |
| 5  | 0.692454 | 3.2158  | 0.717804 | 3.0617  | 0.741402 | 2.9094  | 0.763317 | 2.7603  |
| 10 | 0.696634 | 6.4500  | 0.722358 | 6.1406  | 0.746304 | 5.8339  | 0.768533 | 5.5300  |
| 15 | 0.703692 | 9.7217  | 0.730063 | 9.2539  | 0.754611 | 8.7900  | 0.777391 | 8.3322  |
| 20 | 0.713763 | 13.0522 | 0.741095 | 12.4228 | 0.766541 | 11.7964 | 0.790146 | 11.1764 |
| 25 | 0.727041 | 16.4666 | 0.755702 | 15.6708 | 0.782407 | 14.8755 | 0.807174 | 14.0855 |
| 30 | 0.743766 | 19.9939 | 0.774215 | 19.0269 | 0.802627 | 18.0556 | 0.828989 | 17.0858 |
| 35 | 0.764227 | 23.6700 | 0.797038 | 22.5275 | 0.827737 | 21.3722 | 0.856264 | 20.2120 |
| 40 | 0.788737 | 27.5378 | 0.824642 | 26.2180 | 0.858389 | 24.8722 | 0.889851 | 23.5100 |
| 45 | 0.817596 | 31.6506 | 0.857537 | 30.1556 | 0.895343 | 28.6161 | 0.930795 | 27.0422 |

Examples.  $\tanh (0.25 / 25^{\circ}) = 0.246663 / 24^{\circ}.1028$ .

$\tanh (0.90 / 30^{\circ}) = 0.774215 / 19^{\circ}.0269$ .

TABLE XX. CORRECTING FACTOR.  $\frac{\sinh \theta}{\theta} = r/\gamma.$

|    | 0        |        | 0.05     |        | 0.10     |        |
|----|----------|--------|----------|--------|----------|--------|
| °  | °        | °      | °        | °      | °        | °      |
| 0  | 1.000000 | 0.0000 | 1.000420 | 0.0000 | 1.001670 | 0.0000 |
| 5  | 1.000000 | 0.0000 | 1.000404 | 0.0042 | 1.001638 | 0.0167 |
| 10 | 1.000000 | 0.0000 | 1.000398 | 0.0081 | 1.001570 | 0.0325 |
| 15 | 1.000000 | 0.0000 | 1.000373 | 0.0119 | 1.001442 | 0.0478 |
| 20 | 1.000000 | 0.0000 | 1.000317 | 0.0153 | 1.001279 | 0.0614 |
| 25 | 1.000000 | 0.0000 | 1.000279 | 0.0183 | 1.001071 | 0.0730 |
| 30 | 1.000000 | 0.0000 | 1.000216 | 0.0208 | 1.000834 | 0.0828 |
| 35 | 1.000000 | 0.0000 | 1.000134 | 0.0222 | 1.000571 | 0.0897 |
| 40 | 1.000000 | 0.0000 | 1.000061 | 0.0236 | 1.000290 | 0.0942 |
| 45 | 1.000000 | 0.0000 | 1.000000 | 0.0236 | 1.000000 | 0.0952 |

|    | 0.35     |        | 0.40     |        | 0.45     |        |
|----|----------|--------|----------|--------|----------|--------|
| °  | °        | °      | °        | °      | °        | °      |
| 0  | 1.020543 | 0.0000 | 1.026880 | 0.0000 | 1.034093 | 0.0000 |
| 5  | 1.020230 | 0.2014 | 1.026473 | 0.2625 | 1.033577 | 0.3314 |
| 10 | 1.019305 | 0.3969 | 1.025264 | 0.5172 | 1.032044 | 0.6531 |
| 15 | 1.017796 | 0.5808 | 1.023290 | 0.7569 | 1.029542 | 0.9558 |
| 20 | 1.015748 | 0.7472 | 1.020612 | 0.9742 | 1.026148 | 1.2303 |
| 25 | 1.013224 | 0.8908 | 1.017312 | 1.1625 | 1.021969 | 1.4686 |
| 30 | 1.010302 | 1.0089 | 1.013493 | 1.3161 | 1.017131 | 1.6630 |
| 35 | 1.007070 | 1.0961 | 1.009272 | 1.4305 | 1.011784 | 1.8086 |
| 40 | 1.003630 | 1.1503 | 1.004775 | 1.5017 | 1.006092 | 1.8997 |
| 45 | 1.00008  | 1.1694 | 1.00014  | 1.5278 | 1.00023  | 1.9338 |

|    | 0.70     |        | 0.75     |        | 0.80     |        |
|----|----------|--------|----------|--------|----------|--------|
| °  | °        | °      | °        | °      | °        | °      |
| 0  | 1.083691 | 0.0000 | 1.096423 | 0.0000 | 1.110132 | 0.0000 |
| 5  | 1.082429 | 0.7875 | 1.094969 | 0.9000 | 1.108475 | 1.0192 |
| 10 | 1.078682 | 1.5533 | 1.090658 | 1.7753 | 1.103554 | 2.0108 |
| 15 | 1.072572 | 2.2755 | 1.083625 | 2.6019 | 1.095532 | 2.9481 |
| 20 | 1.064292 | 2.9344 | 1.074102 | 3.3567 | 1.084668 | 3.8044 |
| 25 | 1.054105 | 3.5103 | 1.062383 | 4.0175 | 1.071311 | 4.5561 |
| 30 | 1.042331 | 3.9861 | 1.048848 | 4.5647 | 1.055881 | 5.1800 |
| 35 | 1.029340 | 4.3467 | 1.033919 | 4.9811 | 1.038881 | 5.6567 |
| 40 | 1.015532 | 4.5800 | 1.018059 | 5.2525 | 1.020825 | 5.9697 |
| 45 | 1.001334 | 4.6767 | 1.001756 | 5.3678 | 1.002274 | 6.1064 |

Example.  $\frac{\sinh (0.40 / 25^{\circ})}{0.40 / 25^{\circ}} = 1.017312 / 1^{\circ}.1625.$

TABLE XX. CORRECTING FACTOR.  $\frac{\sinh \theta}{\theta} = r/\gamma$ . CONTINUED

|    | 0.15     |        | 0.20     |        | 0.25     |        | 0.30     |        |
|----|----------|--------|----------|--------|----------|--------|----------|--------|
|    | 0        | 0      | 0        | 0      | 0        | 0      | 0        | 0      |
| 0  | 1.003753 | 0.0000 | 1.006680 | 0.0000 | 1.010448 | 0.0000 | 1.015067 | 0.0000 |
| 5  | 1.003696 | 0.0372 | 1.006579 | 0.0661 | 1.010291 | 0.1033 | 1.014839 | 0.1483 |
| 10 | 1.003527 | 0.0733 | 1.006278 | 0.1303 | 1.009820 | 0.2033 | 1.014161 | 0.2922 |
| 15 | 1.003253 | 0.0172 | 1.005784 | 0.1906 | 1.009051 | 0.2972 | 1.013051 | 0.4275 |
| 20 | 1.002878 | 0.1380 | 1.005119 | 0.2450 | 1.008007 | 0.3825 | 1.011549 | 0.5500 |
| 25 | 1.002412 | 0.1644 | 1.004296 | 0.2919 | 1.006722 | 0.4561 | 1.009696 | 0.6558 |
| 30 | 1.001878 | 0.1858 | 1.003343 | 0.3303 | 1.005233 | 0.5158 | 1.007550 | 0.7419 |
| 35 | 1.001287 | 0.2017 | 1.002290 | 0.3586 | 1.003584 | 0.5600 | 1.005177 | 0.8058 |
| 40 | 1.000653 | 0.2114 | 1.001167 | 0.3761 | 1.001830 | 0.5872 | 1.002651 | 0.8456 |
| 45 | 1.000000 | 0.2146 | 1.000001 | 0.3817 | 1.000003 | 0.5965 | 1.000005 | 0.8592 |
|    | 0.50     |        | 0.55     |        | 0.60     |        | 0.65     |        |
|    | 0        | 0      | 0        | 0      | 0        | 0      | 0        | 0      |
| 0  | 1.042190 | 0.0000 | 1.051185 | 0.0000 | 1.061090 | 0.0000 | 1.071920 | 0.0000 |
| 5  | 1.041552 | 0.4078 | 1.050411 | 0.4919 | 1.060167 | 0.5833 | 1.070833 | 0.6819 |
| 10 | 1.039657 | 0.8039 | 1.048114 | 0.9697 | 1.057426 | 1.1500 | 1.067611 | 1.3477 |
| 15 | 1.036564 | 1.1767 | 1.044363 | 1.4197 | 1.052956 | 1.6842 | 1.062354 | 1.9694 |
| 20 | 1.032369 | 1.5153 | 1.039280 | 1.8286 | 1.046896 | 2.1700 | 1.055228 | 2.5386 |
| 25 | 1.027202 | 1.8094 | 1.033021 | 2.1844 | 1.039436 | 2.5933 | 1.046459 | 3.0355 |
| 30 | 1.021223 | 2.0503 | 1.025779 | 2.4764 | 1.030808 | 2.9414 | 1.036322 | 3.4450 |
| 35 | 1.014618 | 2.2305 | 1.017780 | 2.6956 | 1.021281 | 3.2036 | 1.025131 | 3.7539 |
| 40 | 1.007587 | 2.3439 | 1.009270 | 2.8344 | 1.011148 | 3.3705 | 1.013232 | 3.9525 |
| 45 | 1.00035  | 2.3872 | 1.000508 | 2.8880 | 1.000720 | 3.4369 | 1.000991 | 4.0330 |
|    | 0.85     |        | 0.90     |        | 0.95     |        | 1.00     |        |
|    | 0        | 0      | 0        | 0      | 0        | 0      | 0        | 0      |
| 0  | 1.124842 | 0.0000 | 1.140574 | 0.0000 | 1.157352 | 0.0000 | 1.175201 | 0.0000 |
| 5  | 1.122964 | 1.1447 | 1.138461 | 1.2767 | 1.154989 | 1.4147 | 1.172573 | 1.5592 |
| 10 | 1.117393 | 2.2592 | 1.132194 | 2.5200 | 1.147982 | 2.7928 | 1.164779 | 3.0775 |
| 15 | 1.108311 | 3.3131 | 1.121981 | 3.6967 | 1.136564 | 4.0986 | 1.152083 | 4.5180 |
| 20 | 1.096014 | 4.2775 | 1.108160 | 4.7753 | 1.121117 | 5.2969 | 1.134913 | 5.8417 |
| 25 | 1.080904 | 5.1255 | 1.091177 | 5.7253 | 1.102151 | 6.3544 | 1.113841 | 7.0122 |
| 30 | 1.063461 | 5.8314 | 1.071587 | 6.5183 | 1.080280 | 7.2400 | 1.089558 | 7.9955 |
| 35 | 1.044241 | 6.3730 | 1.050012 | 7.1294 | 1.056208 | 7.9253 | 1.062847 | 8.7603 |
| 40 | 1.023843 | 6.7314 | 1.027127 | 7.5372 | 1.030691 | 8.3864 | 1.034550 | 9.2789 |
| 45 | 1.002897 | 6.8917 | 1.003640 | 7.7242 | 1.004518 | 8.6033 | 1.005545 | 9.5292 |

Example.  $\frac{\sinh (0.95 / 25^{\circ})}{0.95 / 25^{\circ}} = 1.102151 / 6^{\circ}.3544$ .

TABLE XXI. CORRECTING FACTOR.  $\frac{\tanh \theta}{\theta} = r/\gamma$ .

|    | 0        |        | 0.05     |        | 0.10     |        |
|----|----------|--------|----------|--------|----------|--------|
| °  | °        | °      | °        | °      | °        | °      |
| 0  | 1.000000 | 0.0000 | 0.999160 | 0.0000 | 0.996680 | 0.0000 |
| 5  | 1.000000 | 0.0000 | 0.999177 | 0.0083 | 0.996732 | 0.0330 |
| 10 | 1.000000 | 0.0000 | 0.999223 | 0.0163 | 0.996882 | 0.0653 |
| 15 | 1.000000 | 0.0000 | 0.999291 | 0.0239 | 0.997119 | 0.0950 |
| 20 | 1.000000 | 0.0000 | 0.999359 | 0.0308 | 0.997448 | 0.1222 |
| 25 | 1.000000 | 0.0000 | 0.999475 | 0.0364 | 0.997858 | 0.1459 |
| 30 | 1.000000 | 0.0000 | 0.999591 | 0.0411 | 0.998331 | 0.1650 |
| 35 | 1.000000 | 0.0000 | 0.999707 | 0.0450 | 0.998856 | 0.1792 |
| 40 | 1.000000 | 0.0000 | 0.999844 | 0.0469 | 0.999414 | 0.1878 |
| 45 | 1.000000 | 0.0000 | 1.000000 | 0.0480 | 0.999999 | 0.1913 |

|    | 0.35     |        | 0.40     |        | 0.45     |        |
|----|----------|--------|----------|--------|----------|--------|
| °  | °        | °      | °        | °      | °        | °      |
| 0  | 0.961071 | 0.0000 | 0.949872 | 0.0000 | 0.937553 | 0.0000 |
| 5  | 0.961606 | 0.3847 | 0.950538 | 0.4942 | 0.938355 | 0.6139 |
| 10 | 0.963197 | 0.7592 | 0.952528 | 0.9761 | 0.940753 | 1.2138 |
| 15 | 0.965817 | 1.1142 | 0.955812 | 1.4342 | 0.944719 | 1.7853 |
| 20 | 0.969419 | 1.4400 | 0.960338 | 1.8561 | 0.950209 | 2.3147 |
| 25 | 0.973929 | 1.7281 | 0.966036 | 2.2311 | 0.957150 | 2.7881 |
| 30 | 0.979258 | 1.9683 | 0.972801 | 2.5478 | 0.965442 | 3.1920 |
| 35 | 0.985284 | 2.1545 | 0.980505 | 2.7959 | 0.974949 | 3.5122 |
| 40 | 0.991867 | 2.2792 | 0.988977 | 2.9661 | 0.985487 | 3.7378 |
| 45 | 0.99883  | 2.3377 | 0.99800  | 3.0506 | 0.99684  | 3.8568 |

|    | 0.70     |        | 0.75     |         | 0.80     |         |
|----|----------|--------|----------|---------|----------|---------|
| °  | °        | °      | °        | °       | °        | °       |
| 0  | 0.863381 | 0.0000 | 0.846867 | 0.0000  | 0.830046 | 0.0000  |
| 5  | 0.864789 | 1.3208 | 0.848360 | 1.4744  | 0.831615 | 1.6291  |
| 10 | 0.869019 | 2.6217 | 0.852862 | 2.9291  | 0.836347 | 3.2392  |
| 15 | 0.876111 | 3.8826 | 0.860428 | 4.3437  | 0.844318 | 4.8097  |
| 20 | 0.886121 | 5.0811 | 0.871150 | 5.6953  | 0.855654 | 6.3192  |
| 25 | 0.899117 | 6.1936 | 0.885142 | 6.9600  | 0.870525 | 7.7417  |
| 30 | 0.915160 | 7.1942 | 0.902542 | 8.1100  | 0.889141 | 9.0494  |
| 35 | 0.934328 | 8.0533 | 0.923485 | 9.1131  | 0.911732 | 10.2078 |
| 40 | 0.956592 | 8.7386 | 0.948072 | 9.9325  | 0.938525 | 11.1761 |
| 45 | 0.981870 | 9.2144 | 0.976325 | 10.5269 | 0.969690 | 11.9072 |

Note. Negative quantities are in heavy type.

Example.  $\frac{\tanh (0.75 / 25^{\circ})}{0.75 / 25^{\circ}} = 0.885142 \sqrt{6^{\circ}.9600}.$

TABLE XXI. CORRECTING FACTOR.  $\frac{\tanh \theta}{\theta} = r/\gamma$ . CONTINUED

| 0.15 |          | 0.20    |          | 0.25    |          | 0.30    |          |         |
|------|----------|---------|----------|---------|----------|---------|----------|---------|
| °    | °        | °       | °        | °       | °        | °       | °        |         |
| 0    | 0.992567 | 0.0000  | 0.986875 | 0.0000  | 0.979676 | 0.0000  | 0.971040 | 0.0000  |
| 5    | 0.992676 | 0.0739  | 0.987069 | 0.1303  | 0.979968 | 0.2014  | 0.971449 | 0.2867  |
| 10   | 0.993006 | 0.1456  | 0.987642 | 0.2569  | 0.980839 | 0.3973  | 0.972665 | 0.5656  |
| 15   | 0.993547 | 0.2131  | 0.988588 | 0.3758  | 0.982269 | 0.5822  | 0.974660 | 0.8292  |
| 20   | 0.994280 | 0.2739  | 0.989860 | 0.4842  | 0.984222 | 0.7503  | 0.977396 | 1.0700  |
| 25   | 0.995182 | 0.3270  | 0.991446 | 0.5784  | 0.986650 | 0.8972  | 0.980807 | 1.2817  |
| 30   | 0.996237 | 0.3703  | 0.993295 | 0.6555  | 0.989493 | 1.0186  | 0.984817 | 1.4575  |
| 35   | 0.997410 | 0.4025  | 0.995357 | 0.7133  | 0.992673 | 1.1103  | 0.989327 | 1.5917  |
| 40   | 0.998660 | 0.4225  | 0.997571 | 0.7497  | 0.996106 | 1.1695  | 0.994224 | 1.6797  |
| 45   | 0.99995  | 0.4299  | 0.99987  | 0.7641  | 0.99971  | 1.1936  | 0.99937  | 1.7181  |
| 0.50 |          | 0.55    |          | 0.60    |          | 0.65    |          |         |
| °    | °        | °       | °        | °       | °        | °       | °        |         |
| 0    | 0.924234 | 0.0000  | 0.910038 | 0.0000  | 0.895082 | 0.0000  | 0.879491 | 0.0000  |
| 5    | 0.925172 | 0.7428  | 0.911104 | 0.8792  | 0.896274 | 1.0220  | 0.880796 | 1.1695  |
| 10   | 0.927976 | 1.4697  | 0.914304 | 1.7411  | 0.899847 | 2.0253  | 0.884716 | 2.3197  |
| 15   | 0.932627 | 2.1644  | 0.919624 | 2.5675  | 0.905806 | 2.9908  | 0.891269 | 3.4306  |
| 20   | 0.939089 | 2.8111  | 0.927046 | 3.3408  | 0.914151 | 3.8989  | 0.900482 | 4.4808  |
| 25   | 0.947302 | 3.3934  | 0.936527 | 4.0423  | 0.924870 | 4.2289  | 0.912380 | 5.4481  |
| 30   | 0.957174 | 3.8950  | 0.948001 | 4.6528  | 0.937928 | 5.4592  | 0.926976 | 6.3083  |
| 35   | 0.968577 | 4.2992  | 0.961356 | 5.1525  | 0.953254 | 6.0664  | 0.944250 | 7.0355  |
| 40   | 0.981327 | 4.5911  | 0.976426 | 5.5222  | 0.970713 | 6.5275  | 0.964122 | 7.6014  |
| 45   | 0.99516  | 4.7552  | 0.992963 | 5.7431  | 0.990081 | 6.8175  | 0.986419 | 7.9764  |
| 0.85 |          | 0.90    |          | 0.95    |          | 1.00    |          |         |
| °    | °        | °       | °        | °       | °        | °       | °        |         |
| 0    | 0.813024 | 0.0000  | 0.795887 | 0.0000  | 0.778718 | 0.0000  | 0.761595 | 0.0000  |
| 5    | 0.814651 | 1.7842  | 0.797560 | 1.9383  | 0.780424 | 2.0906  | 0.763317 | 2.2397  |
| 10   | 0.819569 | 3.5500  | 0.802620 | 3.8594  | 0.785583 | 4.1661  | 0.768533 | 4.4700  |
| 15   | 0.827873 | 5.2783  | 0.811181 | 5.7461  | 0.794327 | 6.2100  | 0.777391 | 6.6678  |
| 20   | 0.839722 | 6.9478  | 0.823438 | 7.5772  | 0.806886 | 8.2036  | 0.790146 | 8.8236  |
| 25   | 0.855342 | 8.5334  | 0.839660 | 9.3292  | 0.823586 | 10.1245 | 0.807174 | 10.9145 |
| 30   | 0.875019 | 10.0061 | 0.860239 | 10.9731 | 0.844870 | 11.9444 | 0.828989 | 12.9142 |
| 35   | 0.899091 | 11.3300 | 0.885598 | 12.4725 | 0.871302 | 13.6278 | 0.856264 | 14.7880 |
| 40   | 0.927926 | 12.4622 | 0.916269 | 13.7820 | 0.903567 | 15.1278 | 0.889851 | 16.4900 |
| 45   | 0.961877 | 13.3494 | 0.952819 | 14.8444 | 0.942466 | 16.3839 | 0.930795 | 17.9578 |

Note. Negative quantities are in heavy type.

Example.  $\frac{\tanh (1.0 / 10^{\circ})}{1.0 / 10^{\circ}} = 0.768533 \sqrt{4^{\circ}.4700}.$



TABLE XXII. FUNCTIONS OF SEMI-IMAGINARIES

COMPLEX VARIABLE  $\theta/45^\circ$  (slope constant).

| $\theta$<br>hyp.<br>rads. | Sinh $\theta/45^\circ$ |                  | Cosh $\theta/45^\circ$ |                  | tanh $\theta/45^\circ$ |                  | Cosech $\theta/45^\circ$ |                  |
|---------------------------|------------------------|------------------|------------------------|------------------|------------------------|------------------|--------------------------|------------------|
|                           | Size<br>numeric        | Slope<br>degrees | Size<br>numeric        | Slope<br>degrees | Size<br>numeric        | Slope<br>degrees | Size<br>numeric          | Slope<br>degrees |
| 0.00                      | 0.00000                | 45.0000          | 1.00000                | 0.0000           | 0.00000                | 45.0000          | $\infty$                 | 45.0000          |
| 0.01                      | 0.01000                | 45.0009          | 1.00000                | 0.0028           | 0.01000                | 44.9981          | 100.00000                | 45.0009          |
| 0.02                      | 0.02000                | 45.0037          | 1.00000                | 0.0114           | 0.02000                | 44.9923          | 50.00000                 | 45.0037          |
| 0.03                      | 0.03000                | 45.0084          | 1.00000                | 0.0257           | 0.03000                | 44.9827          | 33.33333                 | 45.0084          |
| 0.04                      | 0.04000                | 45.0151          | 1.00000                | 0.0458           | 0.04000                | 44.9693          | 25.00000                 | 45.0151          |
| 0.05                      | 0.05000                | 45.0236          | 1.00000                | 0.0716           | 0.05000                | 44.9520          | 20.00000                 | 45.0236          |
| 0.06                      | 0.06000                | 45.0342          | 1.00000                | 0.1031           | 0.06000                | 44.9310          | 16.66667                 | 45.0342          |
| 0.07                      | 0.07000                | 45.0465          | 1.00000                | 0.1403           | 0.07000                | 44.9062          | 14.28571                 | 45.0465          |
| 0.08                      | 0.08000                | 45.0608          | 1.00000                | 0.1833           | 0.08000                | 44.8775          | 12.50000                 | 45.0608          |
| 0.09                      | 0.09000                | 45.0770          | 1.00001                | 0.2320           | 0.09000                | 44.8450          | 11.11111                 | 45.0770          |
| 0.10                      | 0.10000                | 45.0952          | 1.00001                | 0.2864           | 0.10000                | 44.8087          | 10.00000                 | 45.0952          |
| 0.11                      | 0.11000                | 45.1152          | 1.00001                | 0.3466           | 0.11000                | 44.7686          | 9.09091                  | 45.1152          |
| 0.12                      | 0.12000                | 45.1372          | 1.00002                | 0.4125           | 0.12000                | 44.7247          | 8.33333                  | 45.1372          |
| 0.13                      | 0.13000                | 45.1611          | 1.00003                | 0.4841           | 0.13000                | 44.6770          | 7.69231                  | 45.1611          |
| 0.14                      | 0.14000                | 45.1869          | 1.00004                | 0.5614           | 0.13999                | 44.6255          | 7.14286                  | 45.1869          |
| 0.15                      | 0.15000                | 45.2146          | 1.00005                | 0.6445           | 0.14999                | 44.5701          | 6.66667                  | 45.2146          |
| 0.16                      | 0.16000                | 45.2442          | 1.00006                | 0.7333           | 0.15999                | 44.5109          | 6.25000                  | 45.2442          |
| 0.17                      | 0.17000                | 45.2757          | 1.00008                | 0.8278           | 0.16999                | 44.4479          | 5.88235                  | 45.2757          |
| 0.18                      | 0.18000                | 45.3092          | 1.00009                | 0.9281           | 0.17998                | 44.3811          | 5.55556                  | 45.3092          |
| 0.19                      | 0.19000                | 45.3445          | 1.00011                | 1.0341           | 0.18998                | 44.3104          | 5.26316                  | 45.3445          |
| 0.20                      | 0.20000                | 45.3817          | 1.00013                | 1.1458           | 0.19997                | 44.2359          | 5.00000                  | 45.3817          |
| 0.21                      | 0.21000                | 45.4208          | 1.00016                | 1.2632           | 0.20997                | 44.1576          | 4.76190                  | 45.4208          |
| 0.22                      | 0.22000                | 45.4619          | 1.00020                | 1.3864           | 0.21996                | 44.0755          | 4.54545                  | 45.4619          |
| 0.23                      | 0.23000                | 45.5048          | 1.00024                | 1.5152           | 0.22995                | 43.9896          | 4.34783                  | 45.5048          |
| 0.24                      | 0.24000                | 45.5497          | 1.00028                | 1.6498           | 0.23994                | 43.8999          | 4.16667                  | 45.5497          |
| 0.25                      | 0.25001                | 45.5965          | 1.00033                | 1.7901           | 0.24993                | 43.8064          | 3.99984                  | 45.5965          |
| 0.26                      | 0.26001                | 45.6453          | 1.00038                | 1.9361           | 0.25991                | 43.7092          | 3.84601                  | 45.6453          |
| 0.27                      | 0.27001                | 45.6959          | 1.00044                | 2.0878           | 0.26989                | 43.6081          | 3.70357                  | 45.6959          |
| 0.28                      | 0.28001                | 45.7485          | 1.00051                | 2.2452           | 0.27987                | 43.5032          | 3.57130                  | 45.7485          |
| 0.29                      | 0.29001                | 45.8029          | 1.00060                | 2.4084           | 0.28984                | 43.3945          | 3.44816                  | 45.8029          |
| 0.30                      | 0.30001                | 45.8592          | 1.00068                | 2.5773           | 0.29981                | 43.2819          | 3.33322                  | 45.8592          |
| 0.31                      | 0.31002                | 45.9174          | 1.00077                | 2.7520           | 0.30978                | 43.1654          | 3.22560                  | 45.9174          |
| 0.32                      | 0.32002                | 45.9775          | 1.00087                | 2.9323           | 0.31974                | 43.0452          | 3.12481                  | 45.9775          |
| 0.33                      | 0.33002                | 46.0396          | 1.00098                | 3.1183           | 0.32970                | 42.9213          | 3.03012                  | 46.0396          |
| 0.34                      | 0.34002                | 46.1036          | 1.00111                | 3.3099           | 0.33965                | 42.7937          | 2.94100                  | 46.1036          |
| 0.35                      | 0.35003                | 46.1694          | 1.00125                | 3.5071           | 0.34959                | 42.6623          | 2.85690                  | 46.1694          |
| 0.36                      | 0.36003                | 46.2372          | 1.00140                | 3.7100           | 0.35953                | 42.5272          | 2.77755                  | 46.2372          |
| 0.37                      | 0.37004                | 46.3069          | 1.00156                | 3.9185           | 0.36946                | 42.3884          | 2.70241                  | 46.3069          |
| 0.38                      | 0.38004                | 46.3786          | 1.00174                | 4.1328           | 0.37939                | 42.2458          | 2.63130                  | 46.3786          |
| 0.39                      | 0.39005                | 46.4522          | 1.00193                | 4.3528           | 0.38930                | 42.0994          | 2.56377                  | 46.4522          |
| 0.40                      | 0.40006                | 46.5278          | 1.00213                | 4.5784           | 0.39921                | 41.9494          | 2.49903                  | 46.5278          |
| 0.41                      | 0.41006                | 46.6053          | 1.00235                | 4.8096           | 0.40910                | 41.7957          | 2.43867                  | 46.6053          |
| 0.42                      | 0.42007                | 46.6846          | 1.00259                | 5.0464           | 0.41899                | 41.6382          | 2.38056                  | 46.6846          |
| 0.43                      | 0.43008                | 46.7658          | 1.00285                | 5.2889           | 0.42886                | 41.4778          | 2.32515                  | 46.7658          |
| 0.44                      | 0.44009                | 46.8489          | 1.00312                | 5.5369           | 0.43872                | 41.3120          | 2.27226                  | 46.8489          |
| 0.45                      | 0.45010                | 46.9338          | 1.00341                | 5.7906           | 0.44857                | 41.1432          | 2.22173                  | 46.9338          |
| 0.46                      | 0.46012                | 47.0206          | 1.00372                | 6.0498           | 0.45841                | 40.9708          | 2.17335                  | 47.0206          |
| 0.47                      | 0.47013                | 47.1093          | 1.00406                | 6.3146           | 0.46823                | 40.7947          | 2.12707                  | 47.1093          |
| 0.48                      | 0.48014                | 47.2000          | 1.00442                | 6.5850           | 0.47803                | 40.6150          | 2.08273                  | 47.2000          |
| 0.49                      | 0.49016                | 47.2926          | 1.00480                | 6.8609           | 0.48782                | 40.4317          | 2.04015                  | 47.2926          |
| 0.50                      | 0.50017                | 47.3872          | 1.00520                | 7.1424           | 0.49759                | 40.2448          | 1.99932                  | 47.3872          |

Note. Negative quantities are in heavy type.

Examples.  $\sinh 0.13/45^\circ = 0.13000/45^\circ.1611.$   
 $\cosh 0.26/45^\circ = 1.00038/1^\circ.9361.$

TABLE XXII. FUNCTIONS OF SEMI-IMAGINARIES. CONTINUED

COMPLEX VARIABLE  $\theta/45^\circ$  (slope constant).

| $\theta$      | Sech $\theta/45^\circ$ |                  | Coth $\theta/45^\circ$ |                  | Sinh $\theta/45^\circ / \theta/45^\circ$ |                  | tanh $\theta/45^\circ / \theta/45^\circ$ |                  |
|---------------|------------------------|------------------|------------------------|------------------|------------------------------------------|------------------|------------------------------------------|------------------|
| hyp.<br>rads. | Size<br>numeric        | Slope<br>degrees | Size<br>numeric        | Slope<br>degrees | Size<br>numeric                          | Slope<br>degrees | Size<br>numeric                          | Slope<br>degrees |
| 0.00          | 1.00000                | 0.0000           | $\infty$               | 45.0000          | 1.00000                                  | 0.0000           | 1.00000                                  | 0.0000           |
| 0.01          | 1.00000                | 0.0028           | 100.00000              | 44.9981          | 1.00000                                  | 0.0009           | 1.00000                                  | 0.0019           |
| 0.02          | 1.00000                | 0.0114           | 50.00000               | 44.9923          | 1.00000                                  | 0.0037           | 1.00000                                  | 0.0077           |
| 0.03          | 1.00000                | 0.0257           | 33.33333               | 44.9827          | 1.00000                                  | 0.0084           | 1.00000                                  | 0.0173           |
| 0.04          | 1.00000                | 0.0458           | 25.00000               | 44.9693          | 1.00000                                  | 0.0151           | 1.00000                                  | 0.0307           |
| 0.05          | 1.00000                | 0.0716           | 20.00000               | 44.9520          | 1.00000                                  | 0.0236           | 1.00000                                  | 0.0480           |
| 0.06          | 1.00000                | 0.1031           | 16.66667               | 44.9310          | 1.00000                                  | 0.0342           | 1.00000                                  | 0.0690           |
| 0.07          | 1.00000                | 0.1403           | 14.28571               | 44.9062          | 1.00000                                  | 0.0465           | 0.99999                                  | 0.0938           |
| 0.08          | 1.00000                | 0.1833           | 12.50000               | 44.8775          | 1.00000                                  | 0.0608           | 0.99999                                  | 0.1225           |
| 0.09          | 0.99999                | 0.2320           | 11.11111               | 44.8450          | 1.00000                                  | 0.0770           | 0.99999                                  | 0.1550           |
| 0.10          | 0.99999                | 0.2864           | 10.00000               | 44.8087          | 1.00000                                  | 0.0952           | 0.99999                                  | 0.1913           |
| 0.11          | 0.99999                | 0.3466           | 9.09091                | 44.7686          | 1.00000                                  | 0.1152           | 0.99998                                  | 0.2314           |
| 0.12          | 0.99998                | 0.4125           | 8.33333                | 44.7247          | 1.00000                                  | 0.1372           | 0.99998                                  | 0.2753           |
| 0.13          | 0.99997                | 0.4841           | 7.69231                | 44.6770          | 1.00000                                  | 0.1611           | 0.99997                                  | 0.3230           |
| 0.14          | 0.99996                | 0.5614           | 7.14337                | 44.6255          | 1.00000                                  | 0.1869           | 0.99996                                  | 0.3745           |
| 0.15          | 0.99995                | 0.6445           | 6.66712                | 44.5701          | 1.00000                                  | 0.2146           | 0.99995                                  | 0.4299           |
| 0.16          | 0.99994                | 0.7333           | 6.25039                | 44.5109          | 1.00000                                  | 0.2442           | 0.99994                                  | 0.4891           |
| 0.17          | 0.99992                | 0.8278           | 5.88271                | 44.4479          | 1.00001                                  | 0.2757           | 0.99992                                  | 0.5521           |
| 0.18          | 0.99991                | 0.9281           | 5.55618                | 44.3811          | 1.00001                                  | 0.3092           | 0.99991                                  | 0.6189           |
| 0.19          | 0.99989                | 1.0341           | 5.26371                | 44.3104          | 1.00001                                  | 0.3445           | 0.99989                                  | 0.6896           |
| 0.20          | 0.99987                | 1.1458           | 5.00050                | 44.2359          | 1.00001                                  | 0.3817           | 0.99987                                  | 0.7641           |
| 0.21          | 0.99984                | 1.2632           | 4.76258                | 44.1576          | 1.00002                                  | 0.4208           | 0.99984                                  | 0.8424           |
| 0.22          | 0.99980                | 1.3864           | 4.54628                | 44.0755          | 1.00002                                  | 0.4619           | 0.99981                                  | 0.9245           |
| 0.23          | 0.99976                | 1.5152           | 4.34878                | 43.9896          | 1.00002                                  | 0.5048           | 0.99978                                  | 1.0104           |
| 0.24          | 0.99972                | 1.6498           | 4.16771                | 43.8999          | 1.00002                                  | 0.5497           | 0.99975                                  | 1.1001           |
| 0.25          | 0.99967                | 1.7901           | 4.00112                | 43.8064          | 1.00003                                  | 0.5965           | 0.99971                                  | 1.1936           |
| 0.26          | 0.99962                | 1.9361           | 3.84748                | 43.7092          | 1.00003                                  | 0.6453           | 0.99966                                  | 1.2908           |
| 0.27          | 0.99956                | 2.0878           | 3.70521                | 43.6081          | 1.00003                                  | 0.6959           | 0.99960                                  | 1.3919           |
| 0.28          | 0.99949                | 2.2452           | 3.57309                | 43.5032          | 1.00004                                  | 0.7485           | 0.99953                                  | 1.4968           |
| 0.29          | 0.99940                | 2.4084           | 3.45018                | 43.3945          | 1.00004                                  | 0.8029           | 0.99945                                  | 1.6055           |
| 0.30          | 0.99932                | 2.5773           | 3.33544                | 43.2819          | 1.00005                                  | 0.8592           | 0.99937                                  | 1.7181           |
| 0.31          | 0.99923                | 2.7520           | 3.22810                | 43.1654          | 1.00005                                  | 0.9174           | 0.99928                                  | 1.8346           |
| 0.32          | 0.99913                | 2.9323           | 3.12754                | 43.0452          | 1.00006                                  | 0.9775           | 0.99919                                  | 1.9548           |
| 0.33          | 0.99902                | 3.1183           | 3.03305                | 42.9213          | 1.00006                                  | 1.0396           | 0.99908                                  | 2.0787           |
| 0.34          | 0.99889                | 3.3099           | 2.94421                | 42.7937          | 1.00007                                  | 1.1036           | 0.99896                                  | 2.2063           |
| 0.35          | 0.99875                | 3.5071           | 2.86049                | 42.6623          | 1.00008                                  | 1.1694           | 0.99883                                  | 2.3377           |
| 0.36          | 0.99860                | 3.7100           | 2.78141                | 42.5272          | 1.00009                                  | 1.2372           | 0.99869                                  | 2.4728           |
| 0.37          | 0.99844                | 3.9185           | 2.70665                | 42.3884          | 1.00010                                  | 1.3069           | 0.99854                                  | 2.6116           |
| 0.38          | 0.99826                | 4.1328           | 2.63580                | 42.2458          | 1.00011                                  | 1.3786           | 0.99837                                  | 2.7542           |
| 0.39          | 0.99807                | 4.3528           | 2.56870                | 42.0994          | 1.00013                                  | 1.4522           | 0.99819                                  | 2.9006           |
| 0.40          | 0.99788                | 4.5784           | 2.50494                | 41.9494          | 1.00014                                  | 1.5278           | 0.99800                                  | 3.0506           |
| 0.41          | 0.99766                | 4.8096           | 2.44438                | 41.7957          | 1.00016                                  | 1.6053           | 0.99780                                  | 3.2043           |
| 0.42          | 0.99742                | 5.0464           | 2.38668                | 41.6382          | 1.00017                                  | 1.6846           | 0.99759                                  | 3.3618           |
| 0.43          | 0.99716                | 5.2889           | 2.33174                | 41.4778          | 1.00019                                  | 1.7658           | 0.99736                                  | 3.5230           |
| 0.44          | 0.99689                | 5.5369           | 2.27935                | 41.3120          | 1.00021                                  | 1.8489           | 0.99711                                  | 3.6880           |
| 0.45          | 0.99660                | 5.7906           | 2.22930                | 41.1432          | 1.00023                                  | 1.9338           | 0.99684                                  | 3.8568           |
| 0.46          | 0.99629                | 6.0498           | 2.18145                | 40.9708          | 1.00025                                  | 2.0206           | 0.99655                                  | 4.0292           |
| 0.47          | 0.99596                | 6.3146           | 2.13571                | 40.7947          | 1.00027                                  | 2.1093           | 0.99624                                  | 4.2053           |
| 0.48          | 0.99560                | 6.5850           | 2.09190                | 40.6150          | 1.00030                                  | 2.2000           | 0.99591                                  | 4.3850           |
| 0.49          | 0.99522                | 6.8609           | 2.04995                | 40.4317          | 1.00032                                  | 2.2926           | 0.99555                                  | 4.5683           |
| 0.50          | 0.99483                | 7.1424           | 2.00969                | 40.2448          | 1.00035                                  | 2.3872           | 0.99516                                  | 4.7552           |

Note.

Negative quantities are in heavy type.

Examples.  $\frac{\tanh 0.39/45^\circ}{0.39/45^\circ} = 0.99819 \sqrt{2^\circ.9006}.$

$\frac{\coth 0.50/45^\circ}{0.50/45^\circ} = 2.00969 \sqrt{40^\circ.2448}.$

# TABLE XXIII. HYPERBOLIC FUNCTION FORMULAS

(from Smithsonian Mathematical Tables No. 1871 of 1909, Becker and van Orstrand's  
"Hyperbolic Functions," by permission.)

## A. RELATIONS BETWEEN HYPERBOLIC AND CIRCULAR FUNCTIONS

1.  $\sinh u = -i \sin iu = \tan gd u.$
2.  $\cosh u = \cos iu = \sec gd u.$
3.  $\tanh u = -i \tan iu = \sin gd u.$
4.  $\tanh \frac{1}{2} u = \tan \frac{1}{2} gd u.$
5.  $e^u = (1 + \sin gd u) \div \cos gd u,$   
 $= [1 - \cos (\frac{1}{2} \pi + gd u)] \div \sin (\frac{1}{2} \pi + gd u),$   
 $= \tan (\frac{1}{4} \pi + \frac{1}{2} gd u).$
6.  $\sinh iu = i \sin u.$
7.  $\cosh iu = \cos u.$
8.  $\tanh iu = i \tan u.$
- 8a.  $\sin u = -i \sinh iu = \tanh (gd^{-1} u).$
- 8b.  $\cos u = \cosh iu = \operatorname{sech} (gd^{-1} u).$
- 8c.  $\tan u = -i \tanh iu = \sinh (gd^{-1} u).$
9.  $\sinh (u \pm iv) = \pm i \sin (v \mp iu),$   
 $= \sinh u \cos v \pm i \cosh u \sin v.$
10.  $\cosh (u \pm iv) = \cos (v \mp iu),$   
 $= \cosh u \cos v \pm i \sinh u \sin v.$
- 10a.  $\sin (u \pm iv) = \pm i \sinh (v \pm iu) = \sin u \cosh v \pm i \cos u \sinh v.$
- 10b.  $\cos (u \pm iv) = \cosh (v \mp iu) = \cos u \cosh v \mp i \sin u \sinh v.$
11.  $\cosh (mi \pi) = \cos m \pi. \quad (m \text{ is an integer.})$
12.  $\sinh (2m + 1) \frac{1}{2} i \pi = i \sin (2m + 1) \frac{1}{2} \pi. \quad (m \text{ is an integer.})$

## B. RELATIONS AMONG THE HYPERBOLIC FUNCTIONS

13.  $\sinh u = \frac{1}{2} (e^u - e^{-u}) = -\sinh (-u) = (\operatorname{csch} u)^{-1},$   
 $= 2 \tanh \frac{1}{2} u \div (1 - \tanh^2 \frac{1}{2} u) = \tanh u \div (1 - \tanh^2 u)^{\frac{1}{2}}.$
14.  $\cosh u = \frac{1}{2} (e^u + e^{-u}) = \cosh (-u) = (\operatorname{sech} u)^{-1},$   
 $= (1 + \tanh^2 \frac{1}{2} u) \div (1 - \tanh^2 \frac{1}{2} u) = 1 \div (1 - \tanh^2 u)^{\frac{1}{2}}.$
15.  $\tanh u = (e^u - e^{-u}) \div (e^u + e^{-u}) = -\tanh (-u),$   
 $= (\coth u)^{-1} = \sinh u \div \cosh u = (1 - \operatorname{sech}^2 u)^{\frac{1}{2}}.$
16.  $\operatorname{sech} u = \operatorname{sech} (-u) = (1 - \tanh^2 u)^{\frac{1}{2}}.$

# FORMULAS

17.  $\operatorname{csch} u = -\operatorname{csch} (-u) = (\coth^2 u - 1)^{\frac{1}{2}}.$
18.  $\coth u = -\coth (-u) = (\operatorname{csch}^2 u + 1)^{\frac{1}{2}}.$
19.  $\cosh^2 u - \sinh^2 u = 1.$
20.  $\sinh \frac{1}{2} u = \sqrt{\frac{1}{2} (\cosh u - 1)}.$
21.  $\cosh \frac{1}{2} u = \sqrt{\frac{1}{2} (\cosh u + 1)}.$
22.  $\tanh \frac{1}{2} u = (\cosh u - 1) \div \sinh u,$   
 $= \sinh u \div (\cosh u + 1) = \sqrt{(\cosh u - 1) \div (\cosh u + 1)}.$
23.  $\sinh 2u = 2 \sinh u \cosh u = 2 \tanh u \div (1 - \tanh^2 u).$
24.  $\cosh 2u = \cosh^2 u + \sinh^2 u = 2 \cosh^2 u - 1,$   
 $= 1 + 2 \sinh^2 u = (1 + \tanh^2 u) \div (1 - \tanh^2 u).$
25.  $\tanh 2u = 2 \tanh u \div (1 + \tanh^2 u).$
26.  $\sinh 3u = 3 \sinh u + 4 \sinh^3 u.$
27.  $\cosh 3u = 4 \cosh^3 u - 3 \cosh u.$
28.  $\tanh 3u = (3 \tanh u + \tanh^3 u) \div (1 + 3 \tanh^2 u).$
- 28a.  $m \cosh u + n \sinh u = \frac{1}{2} (m + n) e^u + \frac{1}{2} (m - n) e^{-u}.$
- 28b.  $m e^u \pm n e^{-u} = (m \pm n) \cosh u + (m \mp n) \sinh u.$
29.  $\sinh nu =$   
$$n \cosh^{n-1} u \sinh u + \frac{(n)(n-1)(n-2)}{6} \cosh^{n-3} u \sinh^3 u + \dots$$
30.  $\cosh nu = \cosh^n u + \frac{n(n-1)}{2} \cosh^{n-2} u \sinh^2 u + \dots$
31.  $\sinh u + \sinh v = 2 \sinh \frac{1}{2} (u + v) \cosh \frac{1}{2} (u - v).$
32.  $\sinh u - \sinh v = 2 \cosh \frac{1}{2} (u + v) \sinh \frac{1}{2} (u - v).$
33.  $\cosh u + \cosh v = 2 \cosh \frac{1}{2} (u + v) \cosh \frac{1}{2} (u - v).$
34.  $\cosh u - \cosh v = 2 \sinh \frac{1}{2} (u + v) \sinh \frac{1}{2} (u - v).$
35.  $\sinh u + \cosh u = (1 + \tanh \frac{1}{2} u) \div (1 - \tanh \frac{1}{2} u).$
36.  $(\sinh u + \cosh u)^n = \cosh nu + \sinh nu.$
- 36a.  $a \sinh u + b \cosh u = \sqrt{a^2 - b^2} \sinh (u + \tanh^{-1} \frac{b}{a}).$   
 $= \sqrt{b^2 - a^2} \cosh (u + \coth^{-1} \frac{b}{a}).$
- 36b.  $a \cosh u \pm b \sinh u = \sqrt{a^2 - b^2} \cosh (u \pm \tanh^{-1} \frac{b}{a}).$
37.  $\tanh u + \tanh v = \sinh (u + v) \div \cosh u \cosh v.$
38.  $\tanh u - \tanh v = \sinh (u - v) \div \cosh u \cosh v.$
39.  $\coth u + \coth v = \sinh (u + v) \div \sinh u \sinh v.$

# FORMULAS

40.  $\coth u - \coth v = -\sinh(u - v) \div \sinh u \sinh v.$
41.  $\sinh(u \pm v) = \sinh u \cosh v \pm \cosh u \sinh v.$
42.  $\cosh(u \pm v) = \cosh u \cosh v \pm \sinh u \sinh v.$
43.  $\tanh(u \pm v) = (\tanh u \pm \tanh v) \div (1 \pm \tanh u \tanh v).$
44.  $\coth(u \pm v) = (\coth u \coth v \pm 1) \div (\coth v \pm \coth u).$
45.  $\sinh(u + v) + \sinh(u - v) = 2 \sinh u \cosh v.$
46.  $\sinh(u + v) - \sinh(u - v) = 2 \cosh u \sinh v.$
47.  $\cosh(u + v) + \cosh(u - v) = 2 \cosh u \cosh v.$
48.  $\cosh(u + v) - \cosh(u - v) = 2 \sinh u \sinh v.$
49.  $\tanh \frac{1}{2}(u + v) = (\sinh u + \sinh v) \div (\cosh u + \cosh v).$
50.  $\tanh \frac{1}{2}(u - v) = (\sinh u - \sinh v) \div (\cosh u + \cosh v).$
51.  $\coth \frac{1}{2}(u + v) = (\sinh u - \sinh v) \div (\cosh u - \cosh v).$
52.  $\coth \frac{1}{2}(u - v) = (\sinh u + \sinh v) \div (\cosh u - \cosh v).$
53.  $\frac{\tanh u + \tanh v}{\tanh u - \tanh v} = \frac{\sinh(u + v)}{\sinh(u - v)}.$
54.  $\frac{\coth u + \coth v}{\coth u - \coth v} = -\frac{\sinh(u + v)}{\sinh(u - v)}.$
55.  $\sinh(u + v) + \cosh(u + v) = (\cosh u + \sinh u)(\cosh v + \sinh v).$
56.  $\sinh(u + v) \sinh(u - v) = \sinh^2 u - \sinh^2 v,$   
 $\qquad\qquad\qquad = \cosh^2 u - \cosh^2 v.$
57.  $\cosh(u + v) \cosh(u - v) = \cosh^2 u + \sinh^2 v,$   
 $\qquad\qquad\qquad = \sinh^2 u + \cosh^2 v.$
58.  $\sinh(mi\pi) = 0. \quad (m \text{ is an integer.})$
59.  $\cosh(mi\pi) = (-1)^m.$
60.  $\tanh(mi\pi) = 0.$
61.  $\sinh(u + mi\pi) = (-1)^m \sinh u.$
62.  $\cosh(u + mi\pi) = (-1)^m \cosh u.$
63.  $\sinh(2m + 1)\frac{1}{2}i\pi = \pm i.$
64.  $\cosh(2m + 1)\frac{1}{2}i\pi = 0.$
65.  $\sinh\left(\frac{i\pi}{2} \pm u\right) = i \cosh u.$
66.  $\cosh\left(\frac{i\pi}{2} \pm u\right) = \pm i \sinh u.$

# FORMULAS

- 66a.  $\sinh \{ (u + iv) + i \frac{\pi}{2} \} = \sinh \{ (u + iv) + i \underline{1} \} = i \cosh (u + iv).$   
 66b.  $\cosh \{ (u + iv) + i \frac{\pi}{2} \} = \cosh \{ (u + iv) + i \underline{1} \} = i \sinh (u + iv).$   
 66c.  $\tanh \{ (u + iv) + i \frac{\pi}{2} \} = \tanh \{ (u + iv) + i \underline{1} \} = \coth (u + iv).$   
 66d.  $\sinh \{ (u + iv) + i \pi \} = \sinh \{ (u + iv) + i \underline{2} \} = -\sinh (u + iv).$   
 66e.  $\cosh \{ (u + iv) + i \pi \} = \cosh \{ (u + iv) + i \underline{2} \} = -\cosh (u + iv).$   
 66f.  $\tanh \{ (u + iv) + i \pi \} = \tanh \{ (u + iv) + i \underline{2} \} = \tanh (u + iv).$   
 67.  $\tanh (u + i\pi) = \tanh u.$   
 67a. If  $\sinh \{ (u + i (\underline{1} - q)) \} = x + iy$ ; then  $\sinh \{ u + i (\underline{1} + q) \} = -x + iy.$   
 67b. If  $\cosh \{ (u + i (\underline{1} - q)) \} = x + iy$ ; then  $\cosh \{ u + i (\underline{1} + q) \} = -x + iy.$   
 67c. If  $\tanh \{ u + i (\underline{1} - q) \} = x + iy$ ; then  $\tanh \{ u + i (\underline{1} + q) \} = x - iy.$

## C. INVERSE HYPERBOLIC FUNCTIONS

68.  $\sinh^{-1} u = \log (u + \sqrt{u^2 + 1}) = \cosh^{-1} \sqrt{u^2 + 1} = \int \frac{du}{(u^2 + 1)^{\frac{1}{2}}}.$   
 69.  $\cosh^{-1} u = \log (u + \sqrt{u^2 - 1}) = \sinh^{-1} \sqrt{u^2 - 1} = \int \frac{du}{(u^2 - 1)^{\frac{1}{2}}}.$   
 70.  $\tanh^{-1} u = \frac{1}{2} \log (1 + u) - \frac{1}{2} \log (1 - u) = \int \frac{du}{1 - u^2}.$   
 71.  $\coth^{-1} u = \frac{1}{2} \log (1 + u) - \frac{1}{2} \log (u - 1) = \int \frac{du}{1 - u^2} = \tanh^{-1} \frac{1}{u}.$   
 72.  $\operatorname{sech}^{-1} u = \log \left( \frac{1}{u} + \sqrt{\frac{1}{u^2} - 1} \right) = - \int \frac{du}{u(1 - u^2)^{\frac{1}{2}}} = \cosh^{-1} \frac{1}{u}.$   
 73.  $\operatorname{csch}^{-1} u = \log \left( \frac{1}{u} + \sqrt{\frac{1}{u^2} + 1} \right) = - \int \frac{du}{u(u^2 + 1)^{\frac{1}{2}}} = \sinh^{-1} \frac{1}{u}.$   
 74.  $\sin^{-1} u = -i \sinh^{-1} iu = -i \log (iu + \sqrt{1 - u^2}).$   
 75.  $\cos^{-1} u = -i \cosh^{-1} u = -i \log (u + i \sqrt{1 - u^2}).$   
 76.  $\tan^{-1} u = -i \tanh^{-1} iu = \frac{1}{2i} \log (1 + iu) - \frac{1}{2i} \log (1 - iu).$   
 77.  $\cot^{-1} u = i \coth^{-1} iu = \frac{1}{2i} \log (iu - 1) - \frac{1}{2i} \log (iu + 1).$   
 78.  $\sin^{-1} iu = i \sinh^{-1} u = i \log (u + \sqrt{1 + u^2}).$   
 79.  $\cos^{-1} iu = -i \cosh^{-1} iu = \frac{\pi}{2} - i \log (u + \sqrt{1 + u^2}).$

# FORMULAS

$$80. \tan^{-1} iu = i \tanh^{-1} u = \frac{i}{2} \log (1 + u) - \frac{i}{2} \log (1 - u).$$

$$81. \cot^{-1} iu = -i \coth^{-1} u = -\frac{i}{2} \log (u + 1) + \frac{i}{2} \log (u - 1).$$

$$82. \cosh^{-1} \frac{1}{2} \left( u + \frac{1}{u} \right) = \sinh^{-1} \frac{1}{2} \left( u - \frac{1}{u} \right) = \tanh^{-1} \frac{u^2 - 1}{u^2 + 1},$$

$$= 2 \tanh^{-1} \frac{u - 1}{u + 1} = \log u.$$

$$83. \tanh^{-1} \tan u = \frac{1}{2} g d^2 u.$$

$$84. \tan^{-1} \tanh u = \frac{1}{2} g d^{-1} 2 u.$$

$$85. \cosh^{-1} \csc 2 u = -\sinh^{-1} \cot 2 u = -\tanh^{-1} \cos 2 u = \log \tan u.$$

$$86. \tanh^{-1} \tan^2 \left( \frac{1}{4} \pi + \frac{1}{2} u \right) = \frac{1}{2} \log \csc u.$$

$$87. \tanh^{-1} \tan^2 \frac{1}{2} u = \frac{1}{2} \log \sec u.$$

$$88. \cosh^{-1} u \pm \cosh^{-1} v = \cosh^{-1} [uv \pm \sqrt{(u^2 - 1)(v^2 - 1)}].$$

$$89. \sinh^{-1} u \pm \sinh^{-1} v = \sinh^{-1} [u \sqrt{1 + v^2} \pm v \sqrt{1 + u^2}].$$

## D. SERIES

$$90. e^u = 1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \frac{u^4}{4!} + \dots \quad (u^2 < \alpha)$$

$$91. \log u = (u - 1) - \frac{1}{2} (u - 1)^2 + \frac{1}{3} (u - 1)^3 - \dots \quad (2 > u > 0)$$

$$92. \log u = \frac{u - 1}{u} + \frac{1}{2} \left( \frac{u - 1}{u} \right)^2 + \frac{1}{3} \left( \frac{u - 1}{u} \right)^3 + \dots \quad (u > \frac{1}{2})$$

$$93. \log u = 2 \left[ \frac{u - 1}{u + 1} + \frac{1}{3} \left( \frac{u - 1}{u + 1} \right)^3 + \frac{1}{5} \left( \frac{u - 1}{u + 1} \right)^5 + \dots \right] \quad (u > 0)$$

$$94. \log (1 + u) = u - \frac{1}{2} u^2 + \frac{1}{3} u^3 - \frac{1}{4} u^4 + \dots \quad (u^2 < 1)$$

$$95. \log \left( \frac{1 + u}{1 - u} \right) = 2 \left[ u + \frac{1}{3} u^3 + \frac{1}{5} u^5 + \frac{1}{7} u^7 + \dots \right] \quad (u^2 < 1)$$

$$96. \log \left( \frac{u + 1}{u - 1} \right) = 2 \left[ \frac{1}{u} + \frac{1}{3} \left( \frac{1}{u} \right)^3 + \frac{1}{5} \left( \frac{1}{u} \right)^5 + \dots \right] \quad (u^2 > 1)$$

$$97. \sinh u = u + \frac{u^3}{3!} + \frac{u^5}{5!} + \frac{u^7}{7!} + \dots \quad (u^2 < \alpha)$$

$$= u \left( 1 + \frac{u^2}{\pi^2} \right) \left( 1 + \frac{u^2}{2^2 \pi^2} \right) \left( 1 + \frac{u^2}{3^2 \pi^2} \right) \dots \quad (u^2 < \alpha)$$

# FORMULAS

$$98. \quad \cosh u = 1 + \frac{u^2}{2!} + \frac{u^4}{4!} + \frac{u^6}{6!} + \dots \quad (u^2 < \infty)$$

$$= \left(1 + \frac{4u^2}{\pi^2}\right) \left(1 + \frac{4u^2}{3^2\pi^2}\right) \left(1 + \frac{4u^2}{5^2\pi^2}\right) \dots \quad (u^2 < \infty)$$

$$99. \quad \tanh u = u - \frac{1}{3}u^3 + \frac{2}{15}u^5 - \frac{17}{315}u^7 + \dots \quad (u^2 < \frac{1}{4}\pi^2)$$

$$100. \quad u \coth u = 1 + \frac{1}{3}u^2 - \frac{1}{45}u^4 + \frac{2}{945}u^6 - \dots \quad (u^2 < \pi^2)$$

$$101. \quad \operatorname{sech} u = 1 - \frac{1}{2}u^2 + \frac{5}{24}u^4 - \frac{61}{720}u^6 + \dots \quad (u^2 < \frac{1}{4}\pi^2)$$

$$102. \quad u \operatorname{csch} u = 1 - \frac{1}{6}u^2 + \frac{7}{360}u^4 - \frac{31}{15120}u^6 + \dots \quad (u^2 < \pi^2)$$

$$103. \quad gd\,u = \phi = u - \frac{1}{6}u^3 + \frac{1}{24}u^5 - \frac{61}{5040}u^7 + \dots \quad (u \text{ small})$$

$$= \frac{\pi}{2} - \operatorname{sech} u - \frac{1}{2} \frac{\operatorname{sech}^3 u}{3} - \frac{1}{2} \frac{3 \operatorname{sech}^5 u}{4 \cdot 5} - \dots \quad (u \text{ large})$$

$$104. \quad u = gd^{-1}\phi = \phi + \frac{1}{6}\phi^3 + \frac{1}{24}\phi^5 + \frac{61}{5040}\phi^7 + \dots \quad \left(\phi < \frac{\pi}{2}\right)$$

$$105. \quad \sinh^{-1} u = u - \frac{1}{2} \frac{u^3}{3} + \frac{1}{2} \frac{3}{4} \frac{u^5}{5} - \frac{1}{2} \frac{3}{4} \frac{5}{6} \frac{u^7}{7} + \dots \quad (u^2 < 1)$$

$$= \log 2u + \frac{1}{2} \frac{1}{2} \frac{1}{u^2} - \frac{1}{2} \frac{3}{4} \frac{1}{4} \frac{1}{u^4} + \frac{1}{2} \frac{3}{4} \frac{5}{6} \frac{1}{6} \frac{1}{u^6} - \dots \quad (u^2 > 1)$$

$$106. \quad \cosh^{-1} u = \log 2u - \frac{1}{2} \frac{1}{2} \frac{1}{u^2} - \frac{1}{2} \frac{3}{4} \frac{1}{4} \frac{1}{u^4} - \frac{1}{2} \frac{3}{4} \frac{5}{6} \frac{1}{6} \frac{1}{u^6} - \dots \quad (u^2 > 1)$$

$$107. \quad \tanh^{-1} u = u + \frac{1}{3}u^3 + \frac{1}{5}u^5 + \frac{1}{7}u^7 + \dots \quad (u^2 < 1)$$

$$108. \quad \coth^{-1} u = \tanh^{-1} \frac{1}{u} = \frac{1}{u} + \frac{1}{3} \frac{1}{u^3} + \frac{1}{5} \frac{1}{u^5} + \frac{1}{7} \frac{1}{u^7} + \dots \quad (u^2 > 1)$$

$$109. \quad \operatorname{sech}^{-1} u = \cosh^{-1} \frac{1}{u} = \log \frac{2}{u} - \frac{1}{2} \frac{u^2}{2} - \frac{1}{2} \frac{3}{4} \frac{u^4}{4} - \frac{1}{2} \frac{3}{4} \frac{5}{6} \frac{u^6}{6} - \dots \quad (u^2 < 1)$$

$$110. \quad \operatorname{csch}^{-1} u = \sinh^{-1} \frac{1}{u} = \frac{1}{u} - \frac{1}{2} \frac{1}{3} \frac{1}{u^3} + \frac{1}{2} \frac{3}{4} \frac{1}{5} \frac{1}{u^5} - \frac{1}{2} \frac{3}{4} \frac{5}{6} \frac{1}{7} \frac{1}{u^7} + \dots \quad (u^2 > 1)$$

$$= \log \frac{2}{u} + \frac{1}{2} \frac{u^2}{2} - \frac{1}{2} \frac{3}{4} \frac{u^4}{4} + \frac{1}{2} \frac{3}{4} \frac{5}{6} \frac{u^6}{6} - \dots \quad (u^2 < 1)$$



# FORMULAS

## E. DERIVATIVES

$$111. \frac{d e^u}{du} = e^u.$$

$$112. \frac{d \log_e u}{du} = \frac{1}{u}.$$

$$113. \frac{d a^v}{du} = a^v \cdot \frac{dv}{du} \cdot \log_e a.$$

$$114. \frac{d u^u}{du} = u^u (1 + \log_e u).$$

$$115. \frac{d \sinh u}{du} = \cosh u.$$

$$116. \frac{d \cosh u}{du} = \sinh u.$$

$$117. \frac{d \tanh u}{du} = \operatorname{sech}^2 u.$$

$$118. \frac{d \coth u}{du} = -\operatorname{csch}^2 u.$$

$$119. \frac{d \operatorname{sech} u}{du} = -\operatorname{sech} u \cdot \tanh u.$$

$$120. \frac{d \operatorname{csch} u}{du} = -\operatorname{csch} u \cdot \coth u.$$

$$121. \frac{d \sinh^{-1} u}{du} = \frac{1}{\sqrt{u^2 + 1}}.$$

$$122. \frac{d \cosh^{-1} u}{du} = \frac{1}{\sqrt{u^2 - 1}}.$$

$$123. \frac{d \tanh^{-1} u}{du} = \frac{1}{1 - u^2}.$$

$$124. \frac{d \coth^{-1} u}{du} = \frac{1}{1 - u^2}.$$

$$125. \frac{d \operatorname{sech}^{-1} u}{du} = \frac{-1}{u \sqrt{1 - u^2}}.$$

$$126. \frac{d \operatorname{csch}^{-1} u}{du} = \frac{-1}{u \sqrt{u^2 + 1}}.$$

$$127. \frac{d \operatorname{gd} u}{du} = \operatorname{sech} u.$$

$$128. \frac{d \operatorname{gd}^{-1} u}{du} = \sec u.$$

# FORMULAS

## F. INTEGRALS. (INTEGRATION CONSTANTS ARE OMITTED.)

$$129. \int \sinh u \, du = \cosh u.$$

$$130. \int \cosh u \, du = \sinh u.$$

$$131. \int \tanh u \, du = \log \cosh u.$$

$$132. \int \coth u \, du = \log \sinh u.$$

$$133. \int \operatorname{sech} u \, du = 2 \tan^{-1} e^u = \operatorname{gd} u.$$

$$134. \int \operatorname{csch} u \, du = \log \tanh \frac{u}{2}.$$

$$\begin{aligned} 135. \int \sinh^n u \, du &= \frac{1}{n} \sinh^{n-1} u \cdot \cosh u - \frac{n-1}{n} \int \sinh^{n-2} u \, du, \\ &= \frac{1}{n+1} \sinh^{n+1} u \cosh u - \frac{n+2}{n+1} \int \sinh^{n+2} u \, du. \end{aligned}$$

$$\begin{aligned} 136. \int \cosh^n u \, du &= \frac{1}{n} \sinh u \cdot \cosh^{n-1} u + \frac{n-1}{n} \int \cosh^{n-2} u \, du, \\ &= -\frac{1}{n+1} \sinh u \cosh^{n+1} u + \frac{n+2}{n+1} \int \cosh^{n+2} u \, du. \end{aligned}$$

$$137. \int u \sinh u \, du = u \cosh u - \sinh u.$$

$$138. \int u \cosh u \, du = u \sinh u - \cosh u.$$

$$139. \int u^2 \sinh u \, du = (u^2 + 2) \cosh u - 2u \sinh u.$$

$$140. \int u^n \sinh u \, du = u^n \cosh u - nu^{n-1} \sinh u + n(n-1) \int u^{n-2} \sinh u \, du.$$

$$141. \int \sinh^2 u \, du = \frac{1}{2} (\sinh u \cosh u - u).$$

$$142. \int \sinh u \cdot \cosh u \, du = \frac{1}{4} \cosh (2u).$$

$$143. \int \cosh^2 u \, du = \frac{1}{2} (\sinh u \cosh u + u).$$

# FORMULAS

$$144. \int \tanh^2 u \, du = u - \tanh u.$$

$$145. \int \coth^2 u \, du = u - \coth u.$$

$$146. \int \operatorname{sech}^2 u \, du = \tanh u.$$

$$147. \int \operatorname{sech}^3 u \, du = \frac{1}{2} \operatorname{sech} u \tanh u + \frac{1}{2} \operatorname{gd} u.$$

$$148. \int \operatorname{csch}^2 u \, du = -\coth u.$$

$$149. \int \sinh^{-1} u \, du = u \sinh^{-1} u - (1 + u^2)^{\frac{1}{2}}.$$

$$150. \int \cosh^{-1} u \, du = u \cosh^{-1} u - (u^2 - 1)^{\frac{1}{2}}.$$

$$151. \int \tanh^{-1} u \, du = u \tanh^{-1} u + \frac{1}{2} \log (1 - u^2).$$

$$152. \int u \sinh^{-1} u \, du = \frac{1}{4} \left[ (2u^2 + 1) \sinh^{-1} u - u(1 + u^2)^{\frac{1}{2}} \right].$$

$$153. \int u \cosh^{-1} u \, du = \frac{1}{4} \left[ (2u^2 - 1) \cosh^{-1} u - u(u^2 - 1)^{\frac{1}{2}} \right].$$

$$154. \int (\cosh a + \cosh u)^{-1} du = 2 \operatorname{csch} a \cdot \tanh^{-1} (\tanh \frac{1}{2} u \cdot \tanh \frac{1}{2} a),$$

$$= \operatorname{csch} a \left[ \log \cosh \frac{1}{2} (u + a) - \log \cosh \frac{1}{2} (u - a) \right].$$

$$155. \int (\cos a + \cosh u)^{-1} du = 2 \csc a \cdot \tan^{-1} (\tanh \frac{1}{2} u \cdot \tan \frac{1}{2} a).$$

$$156. \int (1 + \cos a \cdot \cosh u)^{-1} du = 2 \csc a \cdot \tanh^{-1} (\tanh \frac{1}{2} u \cdot \tanh \frac{1}{2} a).$$

$$157. \int \sinh u \cos u \, du = \frac{1}{2} (\cosh u \cdot \cos u + \sinh u \cdot \sin u).$$

$$158. \int \cosh u \cdot \cos u \, du = \frac{1}{2} (\sinh u \cdot \cos u + \cosh u \cdot \sin u).$$

$$159. \int \sinh u \cdot \sin u \, du = \frac{1}{2} (\cosh u \cdot \sin u - \sinh u \cdot \cos u).$$

$$160. \int \cosh u \cdot \sin u \, du = \frac{1}{2} (\sinh u \cdot \sin u - \cosh u \cdot \cos u).$$

# FORMULAS

$$\begin{aligned} 161. \quad & \int \sinh (mu) \sinh (nu) du \\ &= \frac{1}{m^2 - n^2} \left[ m \sinh (nu) \cosh (mu) - n \cosh (nu) \sinh (mu) \right]. \end{aligned}$$

$$\begin{aligned} 162. \quad & \int \cosh (mu) \sinh (nu) du \\ &= \frac{1}{m^2 - n^2} \left[ m \sinh (nu) \sinh (mu) - n \cosh (nu) \cosh (mu) \right]. \end{aligned}$$

$$\begin{aligned} 163. \quad & \int \cosh (mu) \cosh (nu) du \\ &= \frac{1}{m^2 - n^2} \left[ m \sinh (mu) \cosh (nu) - n \sinh (nu) \cosh (mu) \right]. \end{aligned}$$

$$164. \quad \int \sinh u \tanh u du = \sinh u - gd u.$$

$$165. \quad \int \cosh u \coth u du = \cosh u + \log \tanh \frac{u}{2}.$$

$$166. \quad \int \sec u du = gd^{-1} u.$$

$$\begin{aligned} 167. \quad & \int \sec^3 \phi d\phi = \int (1 + \tan^2 \phi)^{\frac{1}{2}} d \tan \phi = \frac{1}{2} \sec \phi \tan \phi + \frac{1}{2} gd^{-1} \phi, \\ &= \frac{1}{2} \tan \phi (1 + \tan^2 \phi)^{\frac{1}{2}} + \frac{1}{2} \sinh^{-1} (\tan \phi). \quad \text{Here } \phi = gd u. \end{aligned}$$

$$168. \quad \int \frac{du}{(u^2 + a^2)^{\frac{1}{2}}} = \sinh^{-1} \frac{u}{a}. \quad \int \frac{du}{(a^2 - u^2)^{\frac{1}{2}}} = \sin^{-1} \frac{u}{a}.$$

$$169. \quad \int \frac{du}{(u^2 - a^2)^{\frac{1}{2}}} = \cosh^{-1} \frac{u}{a}. \quad \int \frac{-du}{(a^2 - u^2)^{\frac{1}{2}}} = \cos^{-1} \frac{u}{a}.$$

$$170. \quad \int \frac{du}{(a^2 - u^2)_{u < a}} = \frac{1}{a} \tanh^{-1} \frac{u}{a}. \quad \int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a}.$$

$$171. \quad \int \frac{-du}{(u^2 - a^2)_{u > a}} = \frac{1}{a} \coth^{-1} \frac{u}{a}. \quad \int \frac{-du}{a^2 + u^2} = \frac{1}{a} \cot^{-1} \frac{u}{a}.$$

$$172. \quad \int \frac{-du}{u(a^2 - u^2)^{\frac{1}{2}}} = \frac{1}{a} \operatorname{sech}^{-1} \frac{u}{a}. \quad \int \frac{du}{u(u^2 - a^2)^{\frac{1}{2}}} = \frac{1}{a} \operatorname{sec}^{-1} \frac{u}{a}.$$

$$173. \quad \int \frac{-du}{u(a^2 + u^2)^{\frac{1}{2}}} = \frac{1}{a} \operatorname{csch}^{-1} \frac{u}{a}. \quad \int \frac{-du}{u(u^2 - a^2)^{\frac{1}{2}}} = \frac{1}{a} \operatorname{csc}^{-1} \frac{u}{a}.$$

$$\begin{aligned} 174. \quad & \int \frac{du}{(au^2 + 2bu + c)^{\frac{1}{2}}} = \frac{1}{\sqrt{a}} \sinh^{-1} \frac{au + b}{(ac - b^2)^{\frac{1}{2}}}, & a \text{ positive, } ac > b^2; \\ &= \frac{1}{\sqrt{a}} \cosh^{-1} \frac{au + b}{(b^2 - ac)^{\frac{1}{2}}}, & a \text{ positive, } ac < b^2; \\ &= \frac{1}{\sqrt{-a}} \cos^{-1} \frac{au + b}{(b^2 - ac)^{\frac{1}{2}}}, & a \text{ negative.} \end{aligned}$$

# FORMULAS

$$\begin{aligned}
 175. \quad \int \frac{du}{(au^2 + 2bu + c)} &= \frac{1}{(ac - b^2)^{\frac{1}{2}}} \tan^{-1} \frac{au + b}{(ac - b^2)^{\frac{1}{2}}}, & ac > b^2; \\
 &= \frac{-1}{(b^2 - ac)^{\frac{1}{2}}} \tanh^{-1} \frac{au + b}{(b^2 - ac)^{\frac{1}{2}}}, & ac < b^2, \\
 &= \frac{-1}{(b^2 - ac)^{\frac{1}{2}}} \coth^{-1} \frac{au + b}{(b^2 - ac)^{\frac{1}{2}}}, & au + b < (b^2 - ac)^{\frac{1}{2}}. \\
 & & ac < b^2, \\
 & & au + b > (b^2 - ac)^{\frac{1}{2}}.
 \end{aligned}$$

$$\begin{aligned}
 176. \quad \int \frac{du}{(a-u)(u-b)^{\frac{1}{2}}} &= \frac{2}{(a-b)^{\frac{1}{2}}} \tanh^{-1} \sqrt{\frac{u-b}{a-b}}, \\
 &\text{or } \frac{-2}{(b-a)^{\frac{1}{2}}} \tan^{-1} \sqrt{\frac{u-b}{b-a}}, \\
 &\text{or } \frac{2}{(a-b)^{\frac{1}{2}}} \coth^{-1} \sqrt{\frac{u-b}{a-b}}. \quad (\text{The real form is to be taken.})
 \end{aligned}$$

$$\begin{aligned}
 177. \quad \int \frac{du}{(a-u)(b-u)^{\frac{1}{2}}} &= \frac{2}{(b-a)^{\frac{1}{2}}} \tanh^{-1} \sqrt{\frac{b-u}{b-a}}, \\
 &\text{or } \frac{2}{(b-a)^{\frac{1}{2}}} \coth^{-1} \sqrt{\frac{b-u}{b-a}}, \\
 &\text{or } \frac{-2}{(a-b)^{\frac{1}{2}}} \tan^{-1} \sqrt{\frac{b-u}{a-b}}. \quad (\text{The real form is to be taken.})
 \end{aligned}$$

$$178. \quad \int (u^2 - a^2)^{\frac{1}{2}} du = \frac{1}{2} u (u^2 - a^2)^{\frac{1}{2}} - \frac{1}{2} a^2 \cosh^{-1} \frac{u}{a}.$$

$$179. \quad \int (a^2 - u^2)^{\frac{1}{2}} du = \frac{1}{2} u (a^2 - u^2)^{\frac{1}{2}} + \frac{1}{2} a^2 \sin^{-1} \frac{u}{a}.$$

$$180. \quad \int (u^2 + a^2)^{\frac{1}{2}} du = \frac{1}{2} u (u^2 + a^2)^{\frac{1}{2}} + \frac{1}{2} a^2 \sinh^{-1} \frac{u}{a}.$$

$$181. \quad \int e^{au} du = \frac{e^{au}}{a}.$$

$$182. \quad \int u e^{au} du = \frac{e^{au}}{a^2} (au - 1).$$

$$183. \quad \int u^m e^{au} du = \frac{u^m e^{au}}{a} - \frac{m}{a} \int u^{m-1} e^{au} du.$$

$$184. \quad \int \frac{e^{au} du}{u^m} = \frac{1}{m-1} \left[ -\frac{e^{au}}{u^{m-1}} + a \int \frac{e^{au} du}{u^{m-1}} \right].$$

$$185. \quad \int a^{bu} du = \frac{a^{bu}}{b \log a}.$$

# FORMULAS

$$186. \int u^n a^u du = \frac{a^u u^n}{\log a} - \frac{na^u u^{n-1}}{(\log a)^2} + \frac{n(n-1)a^u u^{n-2}}{(\log a)^3} \dots$$

$$\pm \frac{n(n-1)(n-2) \dots 2.1 a^u}{(\log a)^{n+1}}.$$

$$187. \int \frac{a^u du}{u^n} = \frac{a^u}{n-1} \left[ -\frac{1}{u^{n-1}} - \frac{\log a}{(n-2)u^{n-2}} - \frac{(\log a)^2}{(n-2)(n-3)u^{n-3}} \right.$$

$$\left. - \dots + \frac{(\log a)^{n-1}}{(n-2)(n-3) \dots 2.1} \int \frac{a^u du}{u} \right].$$

$$188. \int \frac{a^u du}{u} = \log u + u \log a + \frac{(u \log a)^2}{2.2!} + \frac{(u \log a)^3}{3.3!} + \dots$$

$$189. \int \frac{du}{1+e^u} = \log \frac{e^u}{1+e^u}.$$

$$190. \int \frac{du}{a+be^{mu}} = \frac{1}{am} \left[ mu - \log(a+be^{mu}) \right].$$

$$191. \int \frac{du}{ae^{mu}+be^{-mu}} = \frac{1}{m(ab)^{\frac{1}{2}}} \tan^{-1} \left( e^{mu} \sqrt{\frac{a}{b}} \right).$$

$$192. \int \frac{du}{(a+be^{mu})^{\frac{1}{2}}} = \frac{1}{m\sqrt{a}} \left[ \log(\sqrt{a+be^{mu}} - \sqrt{a}) \right.$$

$$\left. - \log(\sqrt{a+be^{mu}} + \sqrt{a}) \right].$$

$$193. \int \frac{ue^u du}{(1+u)^2} = \frac{e^u}{1+u}.$$

$$194. \int e^{au} \log u du = \frac{e^{au} \log u}{a} - \frac{1}{a} \int \frac{e^{au} du}{u}.$$

$$195. \int \log u du = u \log u - u.$$

$$196. \int u^m \log u du = u^{m+1} \left[ \frac{\log u}{m+1} - \frac{1}{(m+1)^2} \right].$$

$$197. \int (\log u)^n du = u (\log u)^n - n \int (\log u)^{n-1} du.$$

$$198. \int u^m (\log u)^n du = \frac{u^{m+1} (\log u)^n}{m+1} - \frac{n}{m+1} \int u^m (\log u)^{n-1} du.$$

$$199. \int \frac{(\log u)^n du}{u} = \frac{(\log u)^{n+1}}{n+1}.$$

$$200. \int \frac{du}{\log u} = \log(\log u) + \log u + \frac{(\log u)^2}{2.2!} + \frac{(\log u)^3}{3.3!} + \dots$$

# FORMULAS

$$201. \int \frac{du}{(\log u)^n} = -\frac{u}{(n-1)(\log u)^{n-1}} + \frac{1}{n-1} \int \frac{du}{(\log u)^{n-1}}.$$

$$202. \int \frac{u^m du}{(\log u)^n} = -\frac{u^{m+1}}{(n-1)(\log u)^{n-1}} + \frac{m+1}{n-1} \int \frac{u^m du}{(\log u)^{n-1}}.$$

$$203. \int \frac{u^m du}{\log u} = \int \frac{e^{-y}}{y} dy, \text{ where } y = -(m+1) \log u.$$

$$204. \int \frac{du}{u \log u} = \log (\log u).$$

$$205. \int \frac{du}{u (\log u)^n} = -\frac{1}{(n-1)(\log u)^{n-1}}.$$

$$206. \int (a+bu)^m \log u \, du = \frac{1}{b(m+1)} \left[ (a+bu)^{m+1} \log u - \int \frac{(a+bu)^{m+1} du}{u} \right].$$

$$207. \int u^m \log (a+bu) \, du = \frac{1}{m+1} \left[ u^{m+1} \log (a+bu) - b \int \frac{u^{m+1} du}{a+bu} \right].$$

$$208. \int \frac{\log (a+bu) \, du}{u} = \log a \cdot \log u + \frac{bu}{a} - \frac{1}{2^2} \left( \frac{bu}{a} \right)^2 + \frac{1}{3^2} \left( \frac{bu}{a} \right)^3 - \dots, \\ = \frac{1}{2} (\log bu)^2 - \frac{a}{bu} + \frac{1}{2^2} \left( \frac{a}{bu} \right)^2 - \frac{1}{3^2} \left( \frac{a}{bu} \right)^3 + \dots$$

$$209. \int \frac{\log u \, du}{(a+bu)^m} = \frac{1}{b(m-1)} \left[ -\frac{\log u}{(a+bu)^{m-1}} + \int \frac{du}{u(a+bu)^{m-1}} \right].$$

$$210. \int \frac{\log u \, du}{a+bu} = \frac{1}{b} \log u \cdot \log (a+bu) - \frac{1}{b} \int \frac{\log (a+bu)}{u} du.$$

$$211. \int (a+bu) \log u \, du = \frac{(a+bu)^2}{2b} \log u - \frac{a^2 \log u}{2b} - au - \frac{1}{4} bu^2.$$

$$212. \int \frac{\log u \, du}{(a+bu)^{\frac{1}{2}}} = \frac{2}{b} \left[ (\log u - 2) \sqrt{(a+bu)} + \sqrt{a} \log (\sqrt{a+bu} + \sqrt{a}) \right. \\ \left. - \sqrt{a} \log (\sqrt{a+bu} - \sqrt{a}) \right], \text{ if } a > 0, \\ = \frac{2}{b} \left[ (\log u - 2) \sqrt{(a+bu)} + 2 \sqrt{-a} \tan^{-1} \sqrt{\frac{a+bu}{-a}} \right], \text{ if } a < 0.$$

# FORMULAS

$$213. \int_0^{\infty} e^{-a^2 u^2} du = \frac{\sqrt{\pi}}{2a} = \frac{1}{2a} \Gamma\left(\frac{1}{2}\right).$$

$$214. \int_0^{\infty} u^n e^{-au} du = \Gamma\left(\frac{n+1}{a}\right) = \frac{n!}{a^{n+1}}.$$

$$215. \int_0^{\infty} u^{2n} e^{-au^2} du = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}.$$

$$216. \int_0^{\infty} e^{-u^2 - \frac{a^2}{u^2}} du = \frac{e^{-2a}}{2} \sqrt{\pi}.$$

$a > 0.$

$$217. \int_0^{\infty} e^{-nu} \sqrt{u} du = \frac{1}{2n} \sqrt{\frac{\pi}{n}}.$$

$$218. \int_0^{\infty} \frac{e^{-nu}}{\sqrt{u}} du = \sqrt{\frac{\pi}{n}}.$$

$a > 0.$

$$219. \int_0^{\infty} \frac{du}{\sinh(nu)} = \frac{\pi}{2n}.$$

$$220. \int_0^{\infty} \frac{u du}{\sinh(nu)} = \frac{\pi^2}{4n^2}.$$

$$221. \int_0^{i\pi} \sinh(mu) \cdot \sinh(nu) du = \int_0^{i\pi} \cosh(mu) \cdot \cosh(nu) du \\ = 0, \text{ if } m \text{ is different from } n.$$

$$222. \int_0^{i\pi} \cosh^2(mu) du = - \int_0^{i\pi} \sinh^2(mu) du = \frac{i\pi}{2}.$$

$$223. \int_{-i\pi}^{+i\pi} \sinh(mu) du = 0.$$

$$224. \int_0^{i\pi} \cosh(mu) du = 0.$$

$$225. \int_{-i\pi}^{i\pi} \sinh(mu) \cosh(nu) du = 0.$$

$$226. \int_0^{i\pi} \sinh(mu) \cosh(mu) du = 0.$$

$$227. \int_0^1 \frac{\log u}{1-u} du = -\frac{\pi^2}{6}.$$

$$228. \int_0^1 \frac{\log u}{1+u} du = -\frac{\pi^2}{12}.$$

$$229. \int_0^1 \frac{\log u}{1-u^2} du = -\frac{\pi^2}{8}.$$



$$230. \int_0^1 \log \left( \frac{1+u}{1-u} \right) \cdot \frac{du}{u} = \frac{\pi^2}{4}.$$

$$231. \int_0^1 \frac{\log u \, du}{(1-u^2)^{\frac{1}{2}}} = -\frac{\pi}{2} \log 2.$$

$$232. \int_0^1 \frac{(u^p - u^q) \, du}{\log u} = \log \frac{p+1}{q+1}, \text{ if } p+1 > 0, q+1 > 0.$$

$$233. \int_0^1 (\log u)^n \, du = (-1)^n \cdot n!.$$

$$234. \int_0^1 \left( \log \frac{1}{u} \right)^{\frac{1}{2}} \, du = \frac{\sqrt{\pi}}{2}.$$

$$235. \int_0^1 \left( \log \frac{1}{u} \right)^n \, du = n!.$$

$$236. \int_0^1 \frac{du}{\left( \log \frac{1}{u} \right)^{\frac{1}{2}}} = \sqrt{\pi}.$$

$$237. \int_0^1 u^m \log \left( \frac{1}{u} \right)^n \, du = \frac{\Gamma(n+1)}{(m+1)^{n+1}}, \text{ if } m+1 > 0, n+1 > 0.$$

$$238. \int_0^\infty \log \left( \frac{e^u + 1}{e^u - 1} \right) \, du = \frac{\pi^2}{4}.$$